# Antenna Potpourri

**Random Musings on Cats, Physics, Waves, Impedance, Resonance, *K*-Factor, *Q*, Efficiency, and Ground Effects**


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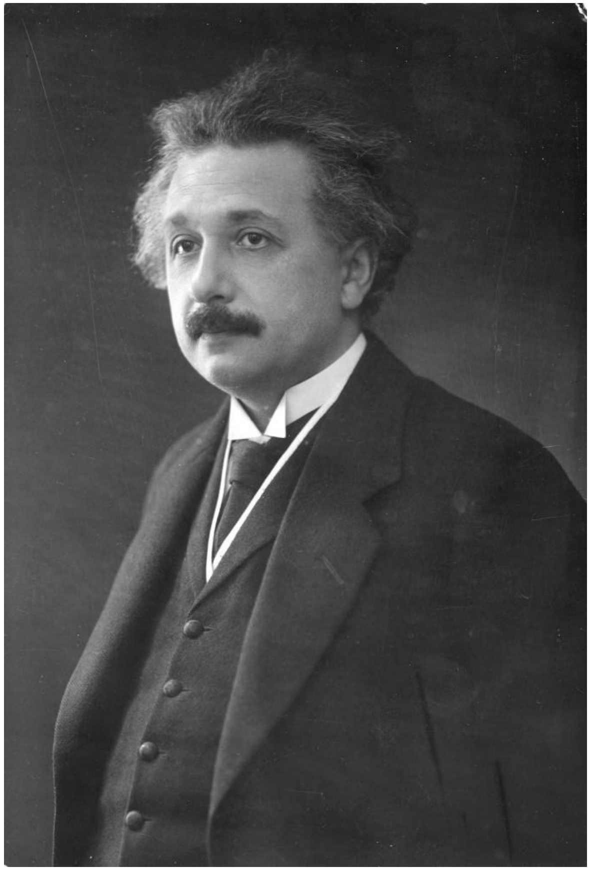
# Abstract

Cats feature in some famous explanations of radio. Examples are given. Facts and fictions about dipole antennas are given. Antenna impedance and equivalent circuits are given. Different definitions of resonance, *K* factor, and *Q* are discussed. A fundamental definition of *Q* is given. Chu's famous 1948 result on antenna *Q* is revisited and shown to be correct but incomplete. A correction is given that generalizes Chu. Apparent "*Q* = 0" antennas show certain formulas for *Q* are wrong. Finally a comparison of HF dipoles versus verticals versus horizontal and vertical loops over ground shows which is better and why. No, it's not just a matter of pattern shape or noise polarization. A list of inexpensive antenna modeling software packages for radio amateurs is updated, and the best current programs are identified.

# Topics

- **Einstein, Eliot, Webber, and Schrödinger on how radio works**
- **The nature of physical laws**
  - Dispelling myths about wave propagation (and Ohm's Law)
- **Antenna impedance and equivalent circuits**
- **Antenna *Q***
  - Apparent "$Q = 0$" antennas
  - Problems with $Q$ formulas
- **Ground loss**
  - Comparison of dipoles vs verticals vs horizontal and vertical loops
- **Recommended antenna modeling software**
- **Good books on antennas**

# Albert Einstein's Explanation of How Radio Works



Albert Einstein, 1879-1955

- **"You see, wire telegraph is a kind of a very, very long cat. You pull his tail in New York and his head is meowing in Los Angeles. Do you understand this? And radio operates exactly the same way: you send signals here, they receive them there. The only difference is that there is no cat." – *Albert Einstein***

# T.S. Eliot and Andrew Lloyd Webber Contrary View of How HF Radio Works

Old Deuteronomy sends Jellicle cats “up, up, up, past the Russell Hotel up, up, up, up to the Heaviside Layer”

Cats handle HF radio communications in the ionosphere

But what explains band openings?

Russell Hotel is a mistake understandable in the context of British topical events. Eliot meant the Hilbert Hotel.

# Schrödinger Cat Hypothesis

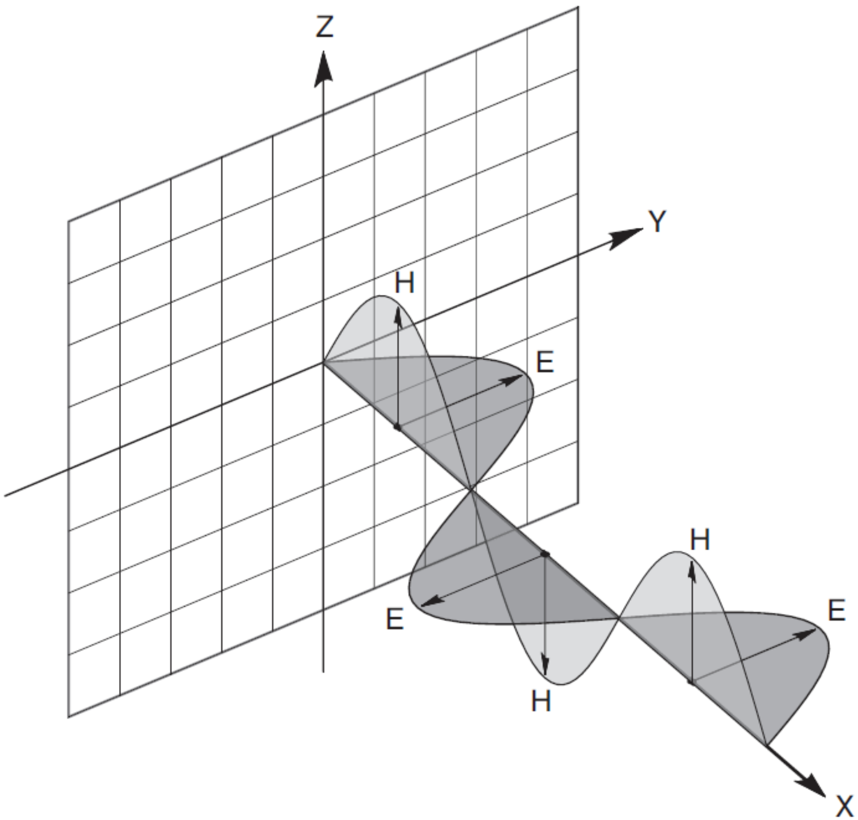
Erwin Schrödinger cat's wave function never collapsed.
The cat is 50% alive.
The cat makes radio work with probability 50%,
which explains band openings.

# The Nature of the Laws of Physics

- **Physical laws state relationships among physical entities**
- **The relationships are "logical," not "causal"**
- **Ohm's Law is descriptive, not causal**
  - Neither voltage nor current is more fundamental
  - Voltage and current co-exist and mutually obey Ohm's Law
- **Maxwell's Equations are descriptive, not causal**
  - Electric and magnetic fields co-exist
  - Waves have electric and magnetic aspects that co-exist
  - Neither aspect "causes" the other

General physical laws state "if…, then…" logical relationships, rather than cause and effect.

# Dispelling a Common Myth About Wave Propagation




- For radiating TEM waves…
- A time-varying electric field does NOT create or produce the magnetic field
- A time-varying magnetic field does NOT create or produce the electric field
- The electric and magnetic waves travel together as a single electromagnetic wave; neither "causes" the other
- Both fields are created by accelerating charge at the source
- Both fields are given by radiation integrals of the source currents

O.D. Jefimenko, *Electricity and Magnetism*, 2e, Electret Scientific, 1966, 1989.
O.D. Jefimenko, *Causality, Electromagnetic Induction, and Gravitation*, 2e, Electret Scientific, 2000.

# Fictions

- A dipole should be one-half wavelength long
- Half-wavelength dipoles are resonant
- Dipoles are 75 ohms
- In free space, a half-wavelength dipole has a real (resistive) feedpoint impedance
- A half-wavelength dipole is 50 ohms
- The feedpoint resistance of a half-wavelength dipole depends on its diameter
- The feedpoint reactance of a half-wavelength dipole depends on its diameter
- Dipoles are anti-resonant at lengths slightly longer than an even number of half-wavelengths

# Truths

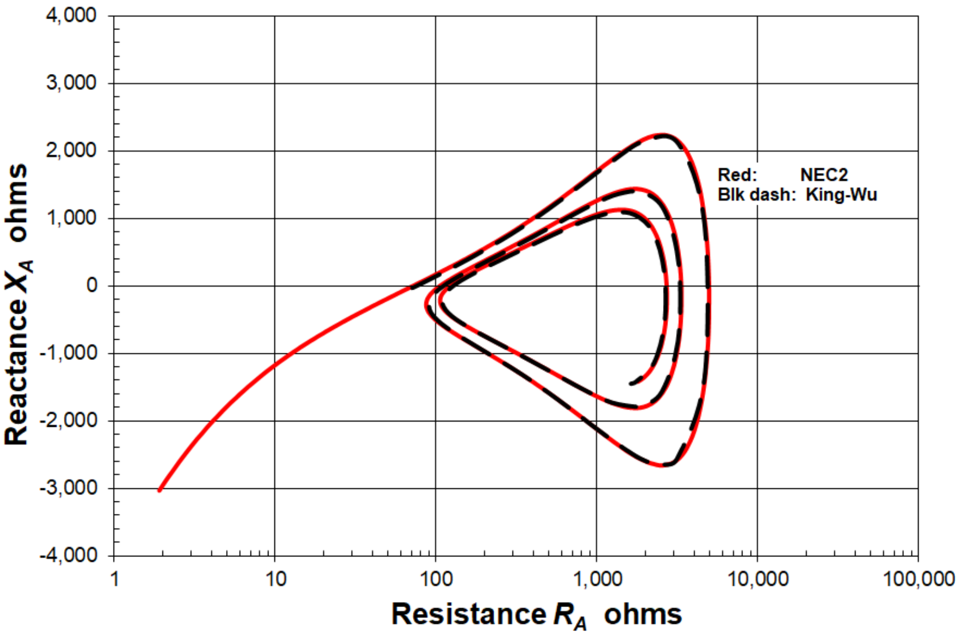
- **A simple dipole is symmetric and center fed**

**A.C. = a cat**

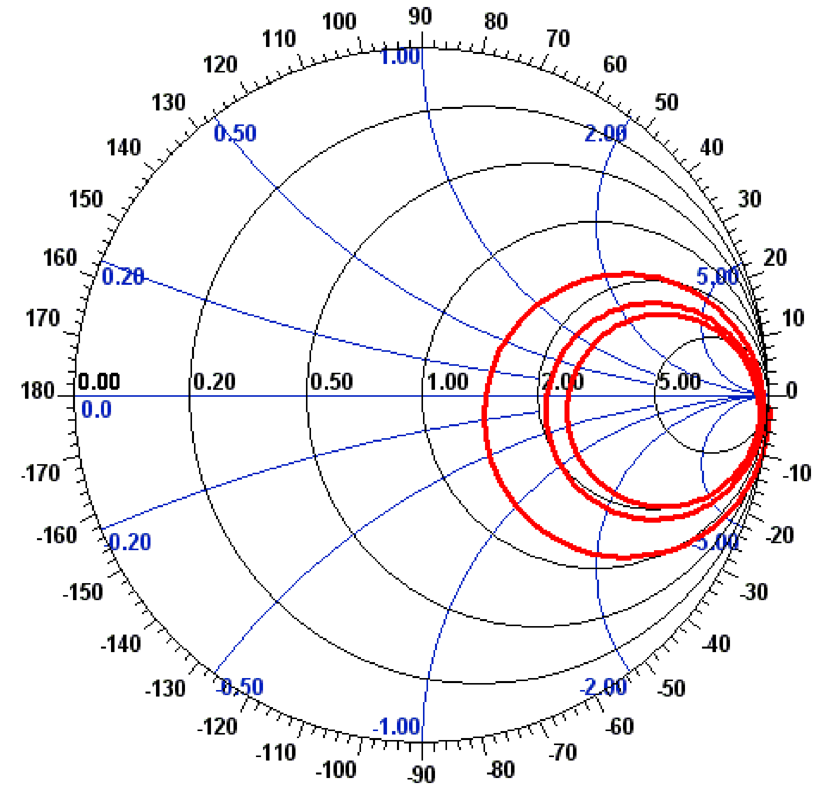
- **For lossless antennas, directivity and gain are the same**
- **An antenna's radiation resistance is not unique.  It depends on a reference current or location**
- **The resonant length of a dipole depends on its diameter**
- **Dipoles are resonant at lengths slightly shorter than an odd number of half-wavelengths**
  - The resonant length of a Hertz dipole or doublet is $L = \frac{K\lambda}{2}$
  - $K$ depends on resonance number and dipole fatness
- **Dipoles are anti-resonant at lengths slightly shorter than an even number of half-wavelengths**
- **If linear antenna is resonant, then its feedpoint impedance is real everywhere along its length**
- **If a dipole is a half-wavelength, then its current phase is −30° (inductive) everywhere along its length (taking feed voltage as reference phase)**

# Complex Impedance as Frequency is Swept

**ARRL Semilog Impedance Plane**

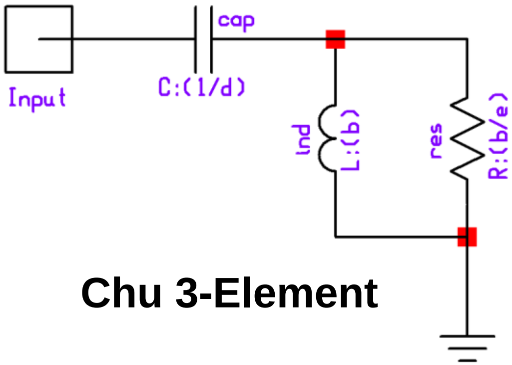



**Smith Chart**

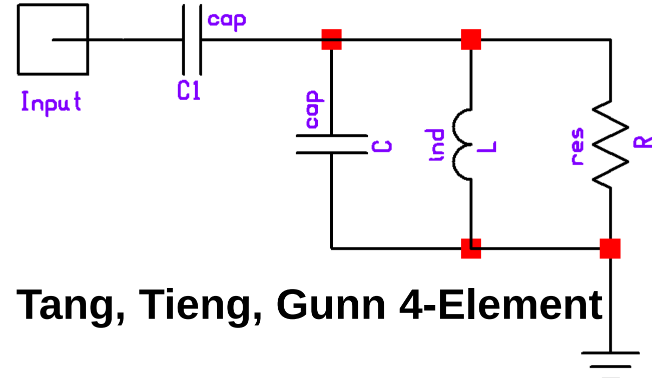



- **Antenna: 98.4-foot dipole in free space**
- **Wire: #10 AWG**
- ***L/d* = 11,000**
- **Frequency: 1 MHz to 30 MHz**

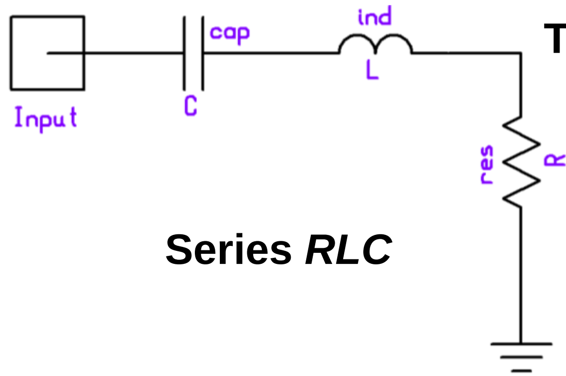
# Equivalent Circuits – Narrowband Models for Electrically-Small Dipoles & Monopoles



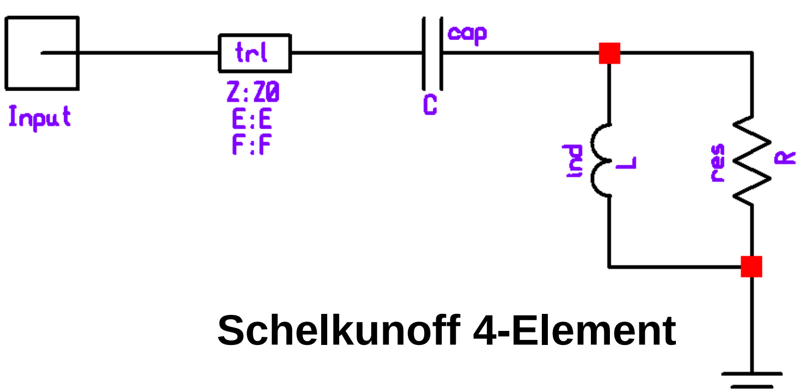

Chu 3-Element



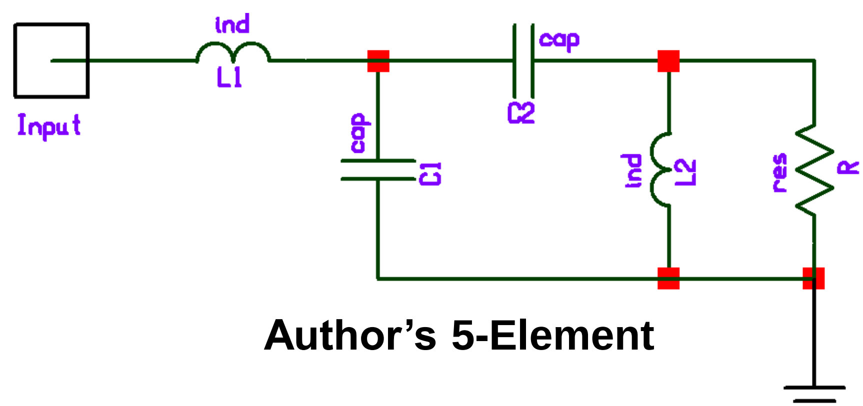

Tang, Tieng, Gunn 4-Element




Series *RLC*

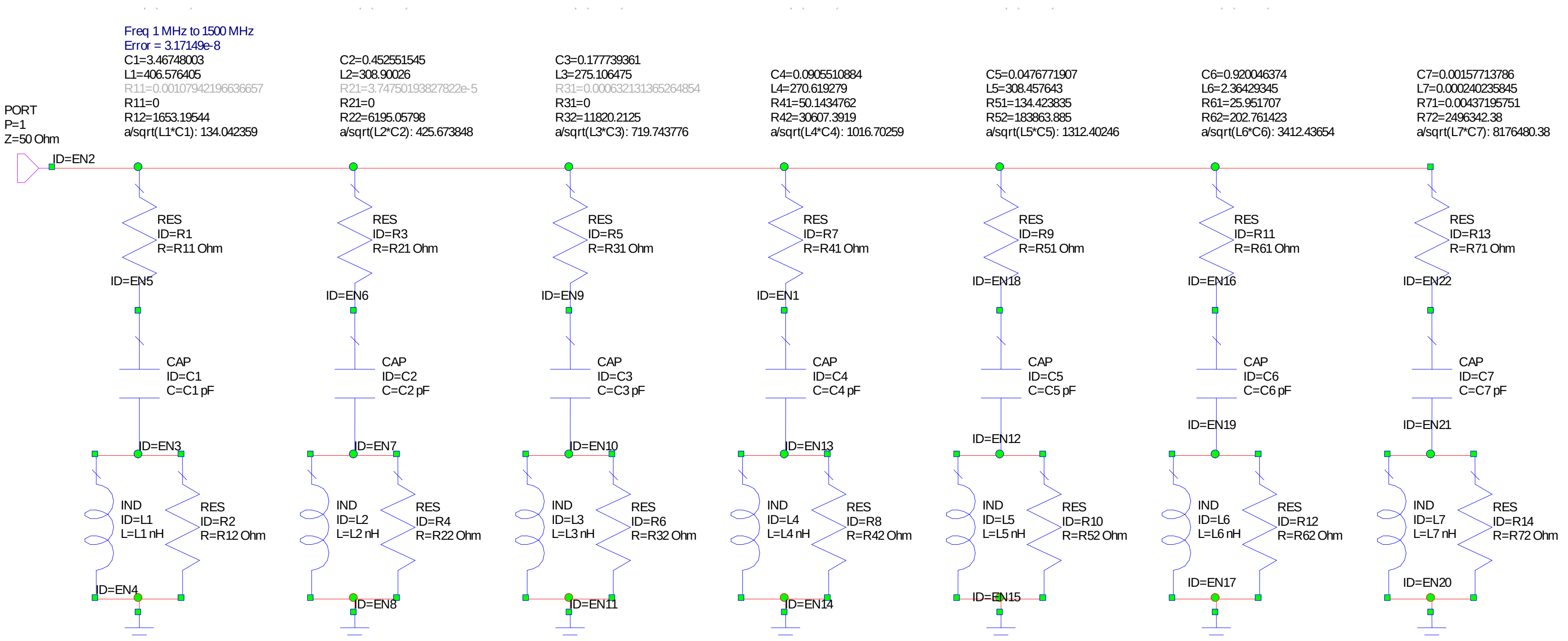



Schelkunoff 4-Element



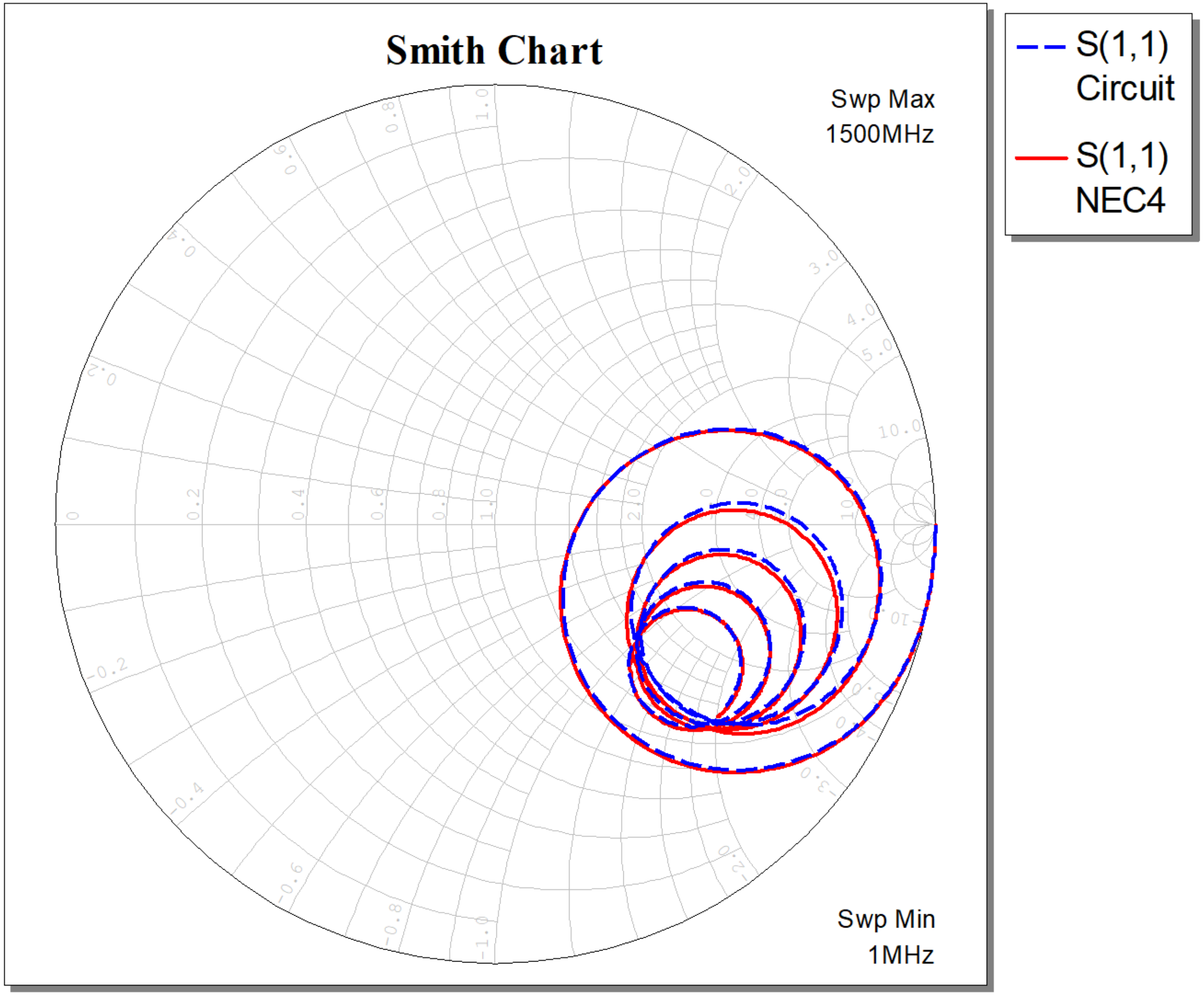

Author’s 5-Element

# Equivalent Circuit – Broadband Model for 1-Meter Long Dipole from DC to 1.5 GHz

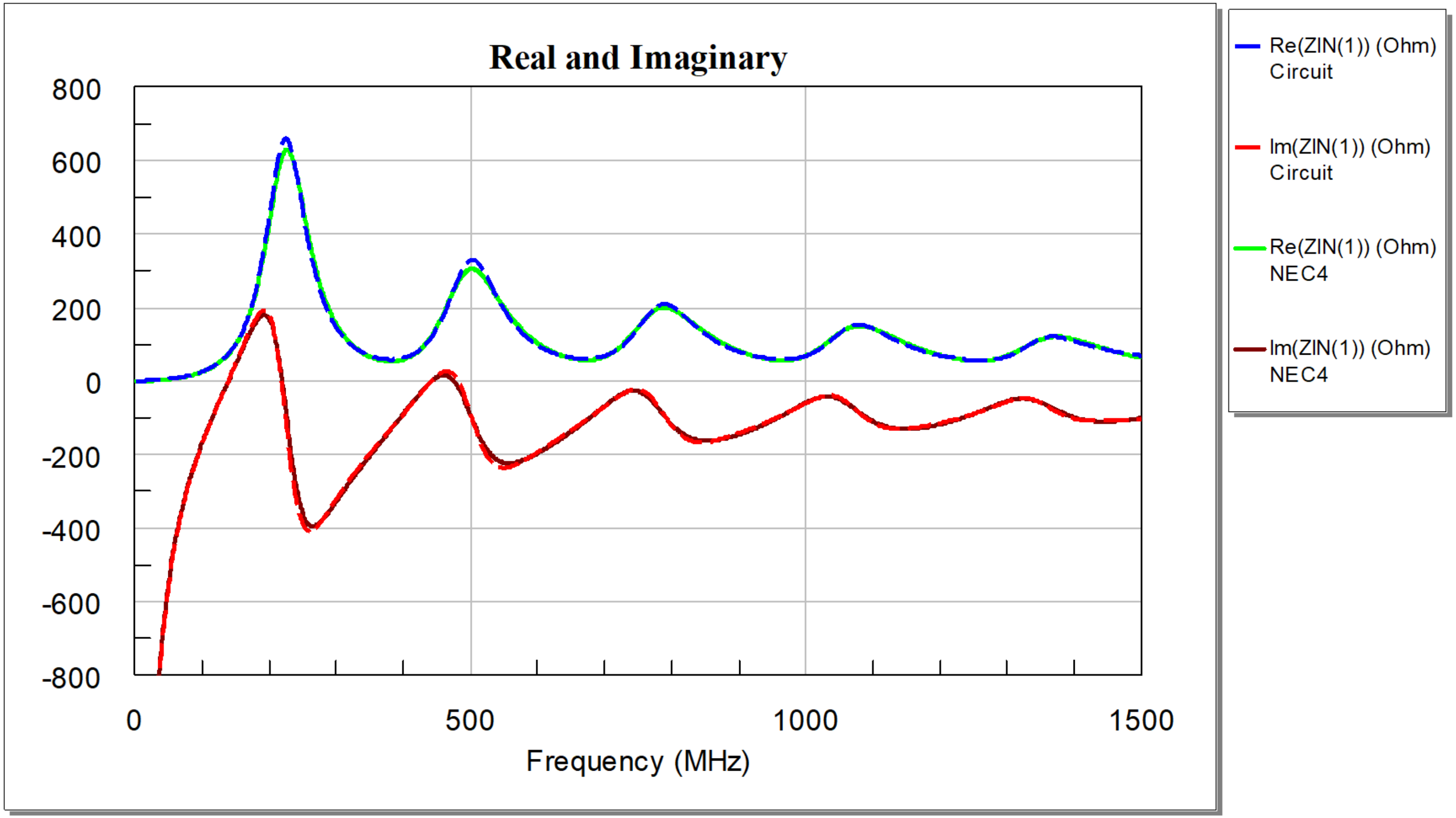



- 1-meter dipole (*L* /*d* = 50)
- Equivalent circuit developed by the author in 2007
- Partial fraction expansion of dipole admittance
- A modification of Foster's 2nd canonical form
- More accurate than other broadband equivalent circuits for dipoles, viz. Hamid-Hamid (1997), Rambabu-Ramesh-Kalghatgi (1999), and Streable-Pearson (1981)
- Six stages sufficient to cover *d-c* to 1.5 GHz

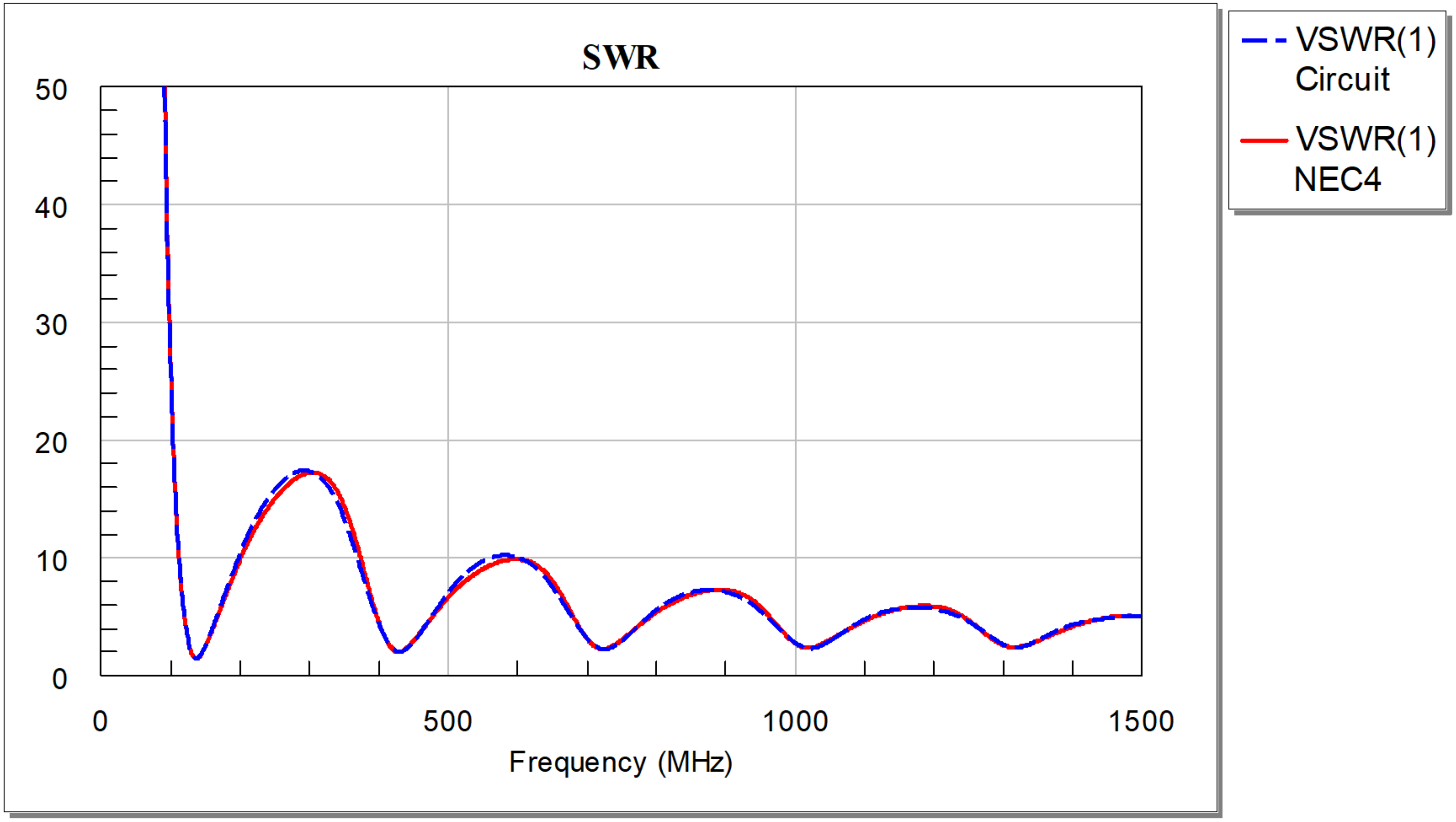
# Impedance Comparison on Smith Chart



# Impedance Comparison:  NEC4 vs Equivalent Circuit



# SWR of Equivalent Circuit



# Dipole Resonance and Resonant Length

## The Mysterious Factor *K*

# Two Definitions of Resonance

- **Electric field energy equals magnetic field energy**

$$\iiint \left( \varepsilon_0 |E|^2 - \mu_0 |H|^2 \right) dV = 0$$

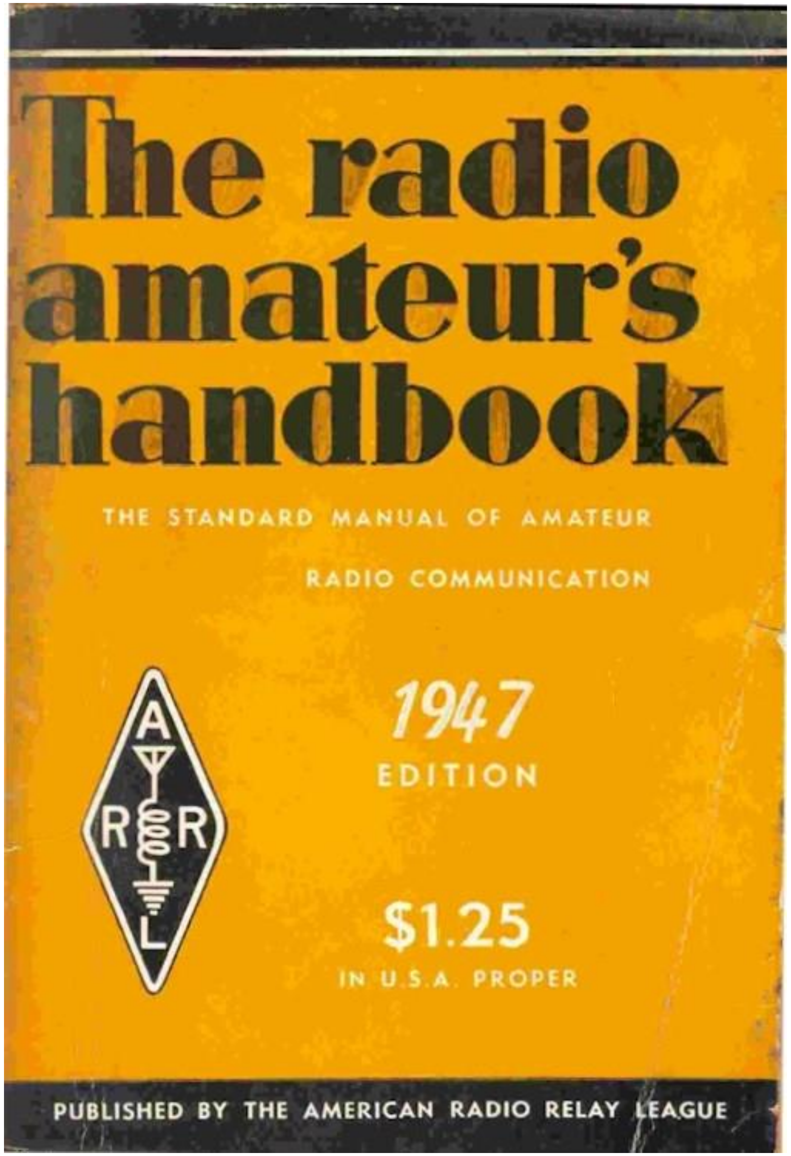
  - Some authors exclude radiation energy and consider only stored energy that is not associated with radiation, i.e. real power delivery to infinity

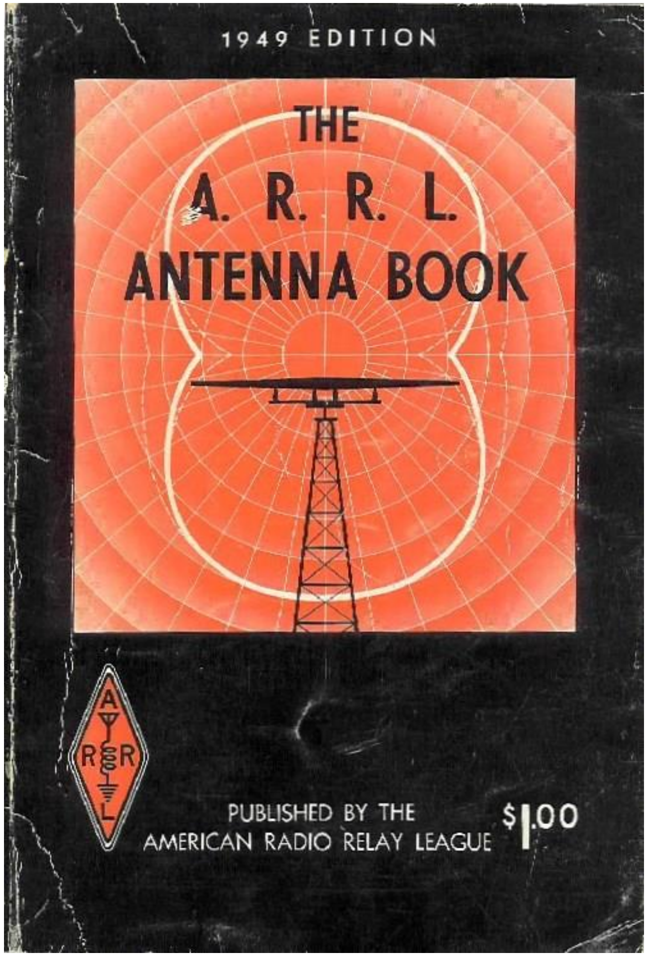
- **Feedpoint reactance is zero**

$$X(f) = 0$$

  - This definition is standard but less fundamental
  - A nonresonant antenna can be made resonant, and vice versa, by incorporating transmission line
  - If an antenna's impedance curve lies entirely in the upper or lower half of the Smith chart and does not cross the horizontal $X$ = 0 midline, then it has no resonances

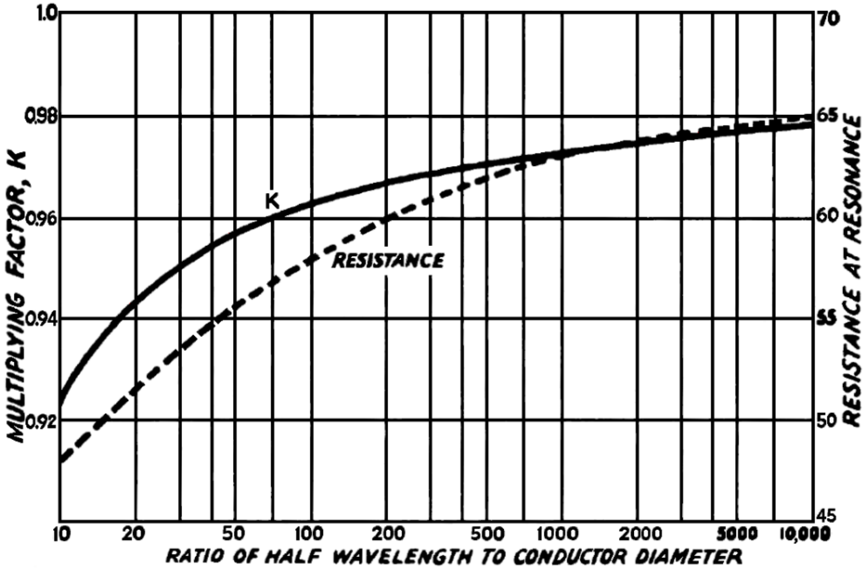
# Dipole Resonant Length



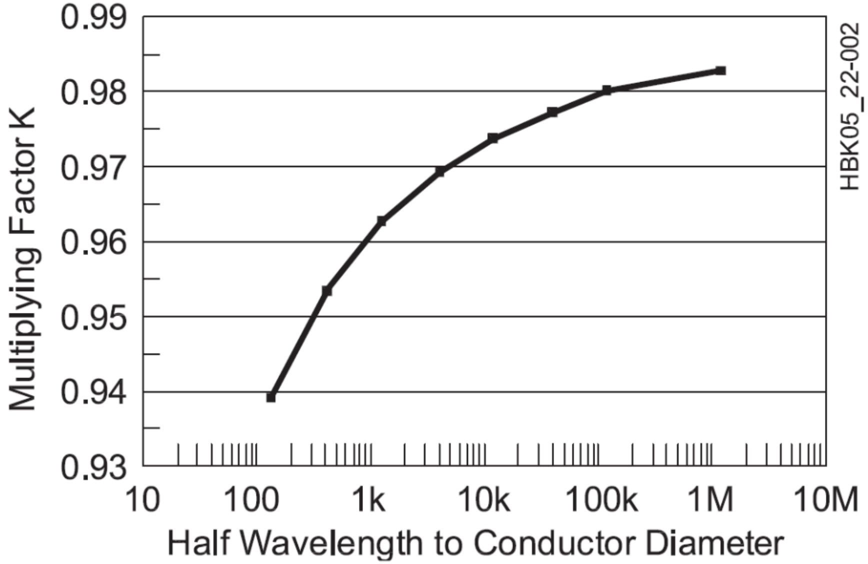



# The Multiplying Factor *K*

ARRL 1947-1997



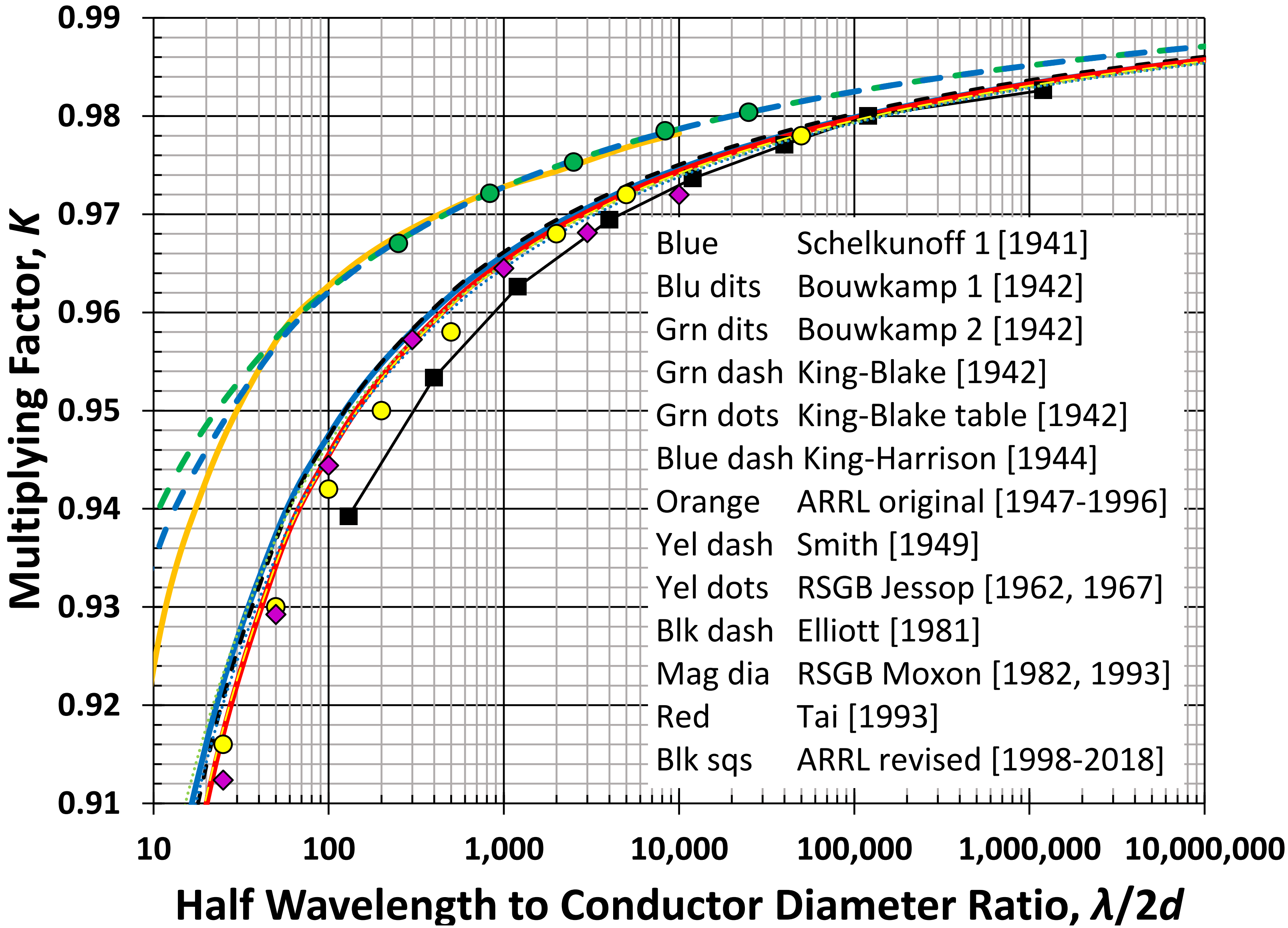

ARRL 1998-2018




$$L_{res} = K \times \frac{\lambda}{2} = K \times \frac{491.786}{f_{MHz}} \quad \text{feet}$$

*K* is not a velocity factor !

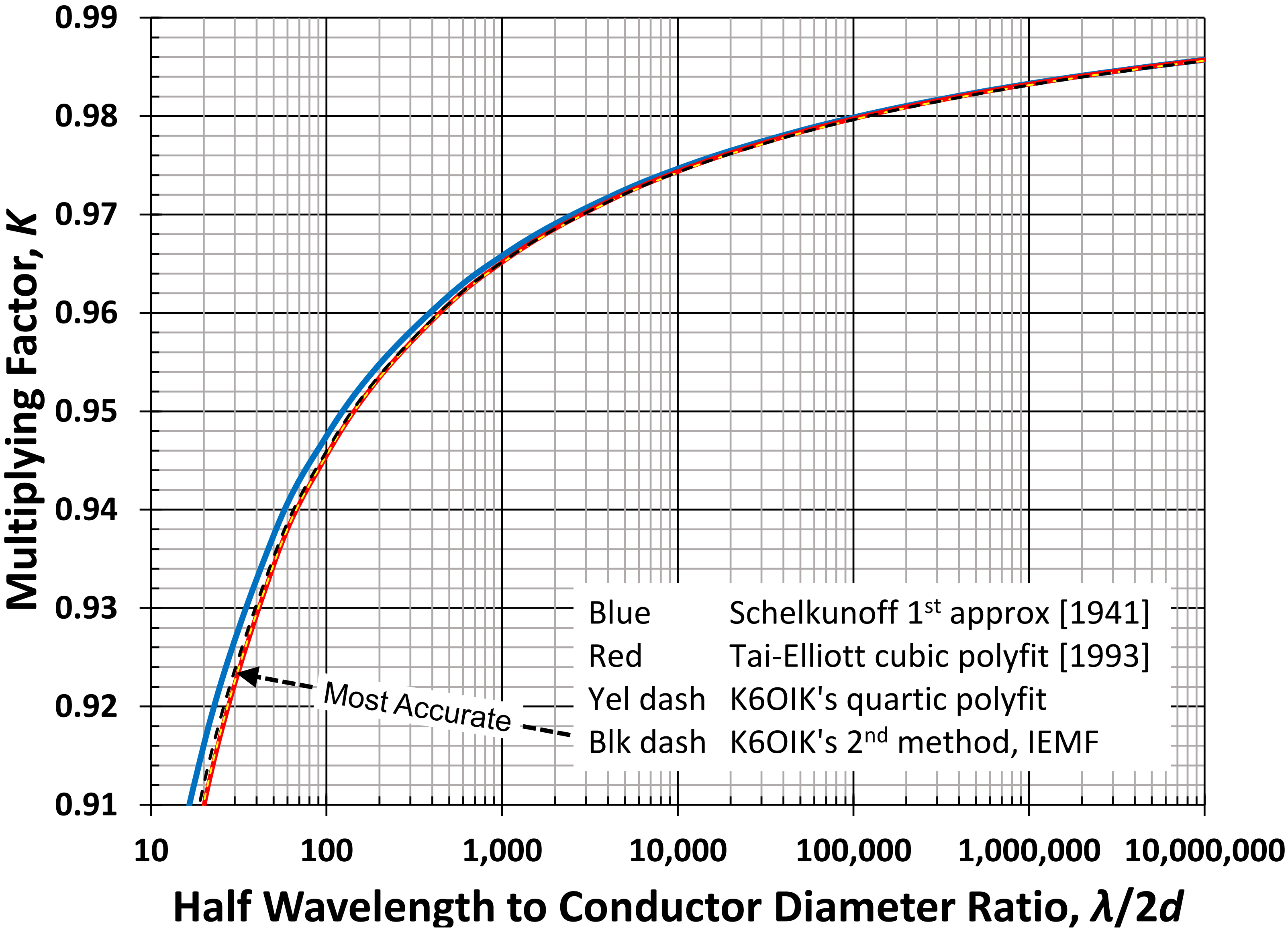
# The "*K*" Universe – Who is Right?



# Three Half-Baked Theories of the Multiplying Factor *K*

- **Theory 1: *K* is a velocity factor**
  - Claim: Dipole is a transmission line and *K* is a velocity factor
  - No physical basis exists for non-unity velocity factor
    - Only materials are PEC metal and vacuum, no dielectric or losses
  - Schelkunoff's wave theory of antennas has velocity factor = unity
- **Theory 2: *K* is an "end" effect**
  - Claim: Dipole is a quarter-wave transmission line, velocity factor = 1, but fringing capacitance at ends transforms to inductance at feedpoint
  - Forward and reverse traveling waves would give a sinusoidal current standing wave, but this is not the case
- **Theory 3: *K* is due to current distribution not being sinusoidal**
  - True, the magnitude of the current distribution deviates from sinusoidal as a dipole gets fatter, but this does not explain *K*
  - *K* is predicted accurately by the induced e.m.f. method which assumes sinusoidal current distribution; so non-sinusoidal current is not the explanation

# The Real *K*



0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
Multiplying Factor, K
10
100
1,000
10,000
100,000
1,000,000
10,000,000
Half Wavelength to Conductor Diameter Ratio, λ/2d
Most Accurate
Blue Schelkunoff 1st approx [1941]
Red Tai-Elliott cubic polyfit [1993]
Yel dash K6OIK's quartic polyfit
Blk dash K6OIK's 2nd method, IEMF

# Comments on the Multiplying Factor *K*

- **Most popular expositions on *K* are partly correct at best**
  - *K* is not a velocity factor
  - *K* is not an "end" effect
  - *K* is not due to departure from sinusoidal current distribution
- ***K* can be determined by rigorous methods**
  - Induced e.m.f. method gives *K* accurately for the 1st resonance
  - Best method: Analyze a dipole as a boundary value problem
  - Solve Pocklington's or Hallén's equations for the current on the antenna
  - *K* is found from the dipole length for which the feedpoint reactance is zero
- **Numerical methods are fine, but MoM has a caveat**
  - Antenna models that use delta-gap sources and MoM do not predict resonance or *K* very accurately

Antenna models that use delta-gap sources and MoM do not predict resonance or *K* very accurately.

# Antenna $Q$

# Heinrich Hertz’s Drawings of Electric Fields of a Dipole circa 1888

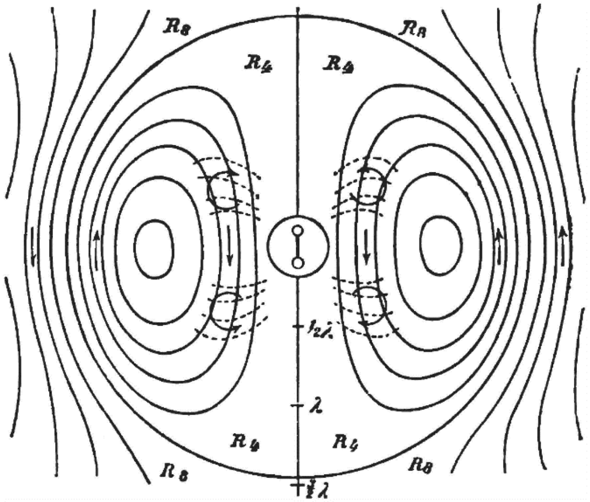



Fig. 27.

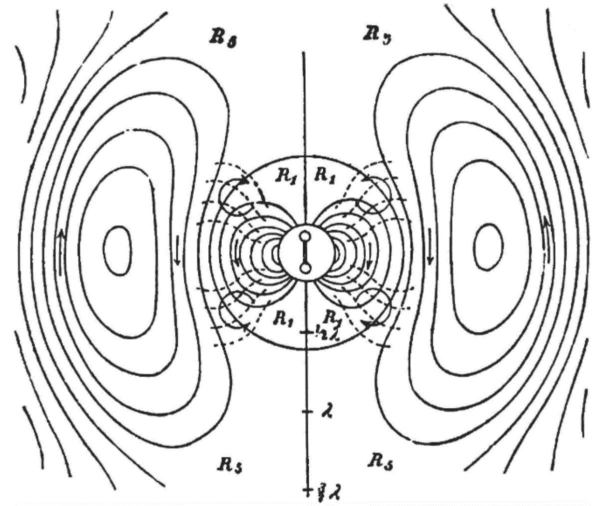



Fig. 28.

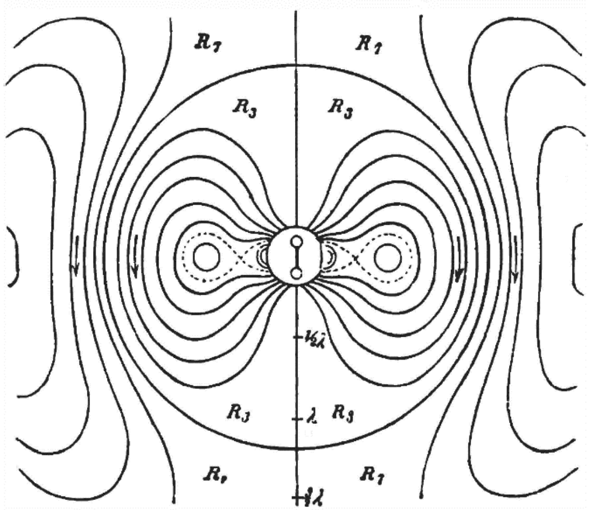



Fig. 30.

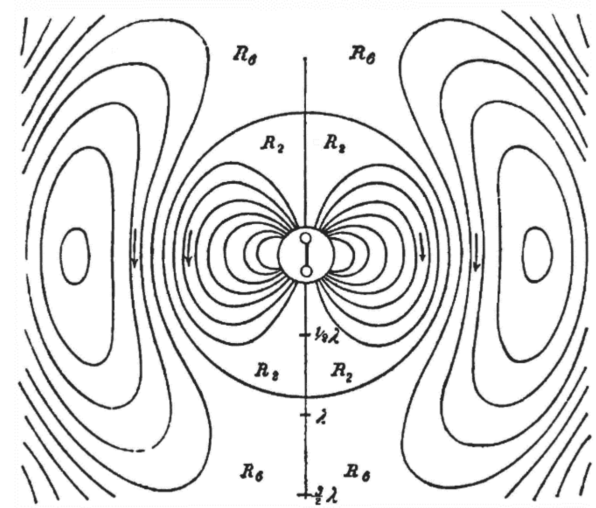
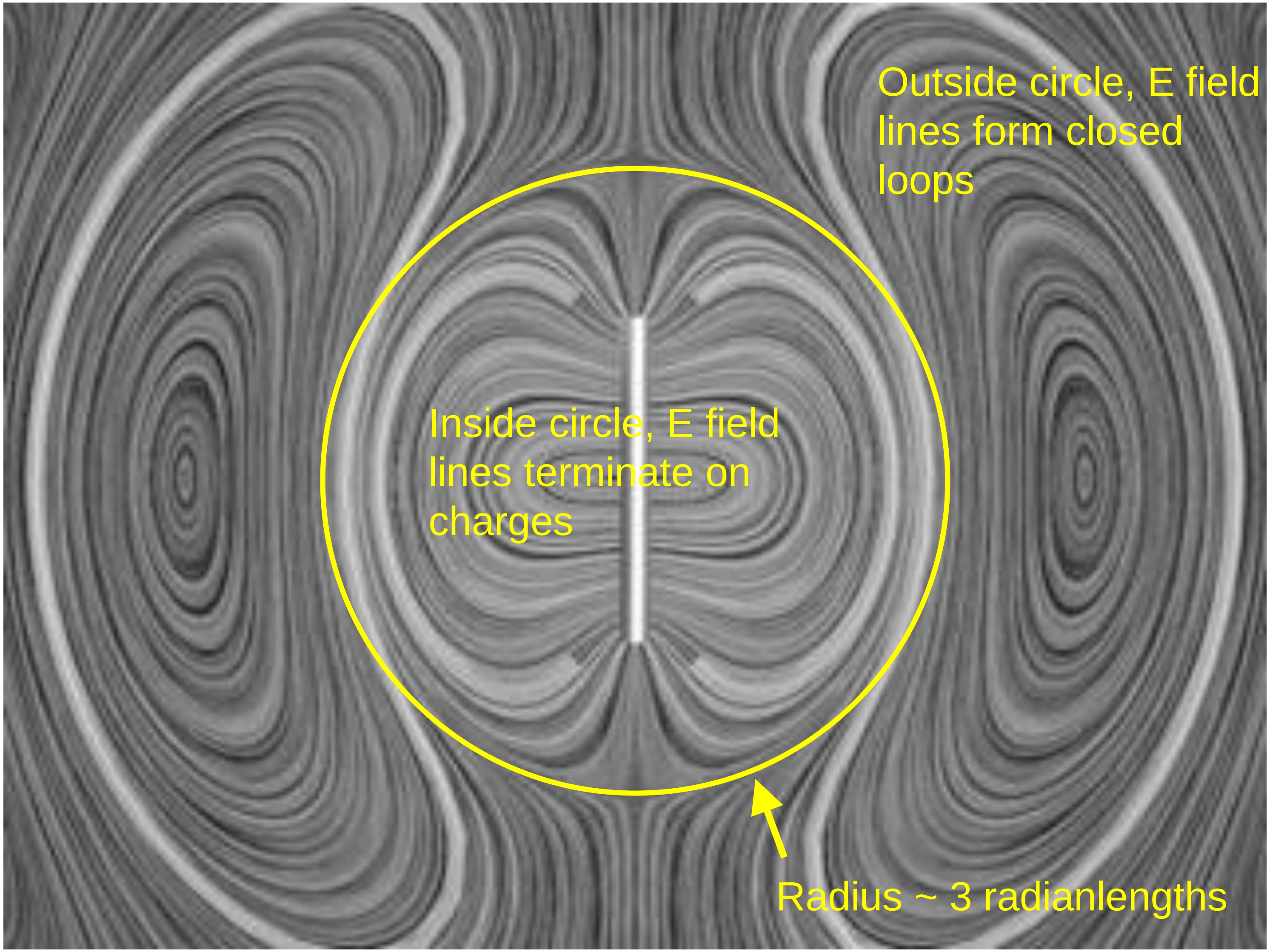



Fig. 29.

# Generation of Dipole Fields



# *Q* – Fundamental Definition

$$Q = 2\pi \left( \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} \right)$$

$$= 2\pi f \left( \frac{\text{Energy stored}}{\text{Power dissipated}} \right)$$

$$= \frac{\omega U_{stored}}{P_{dissipated}}$$

$$\leq \frac{2\omega \max\{U_E, U_H\}}{P_{dissipated}}$$

Electric or magnetic field energy, whichever is greater

# Ambiguity in the Numerator

- **Does stored energy mean total field energy minus energy that is being transported via radiation?**
  - Includes circulating real power, such as knotted waves

$$U_{stored} = U_{total} - U_{radiation}$$

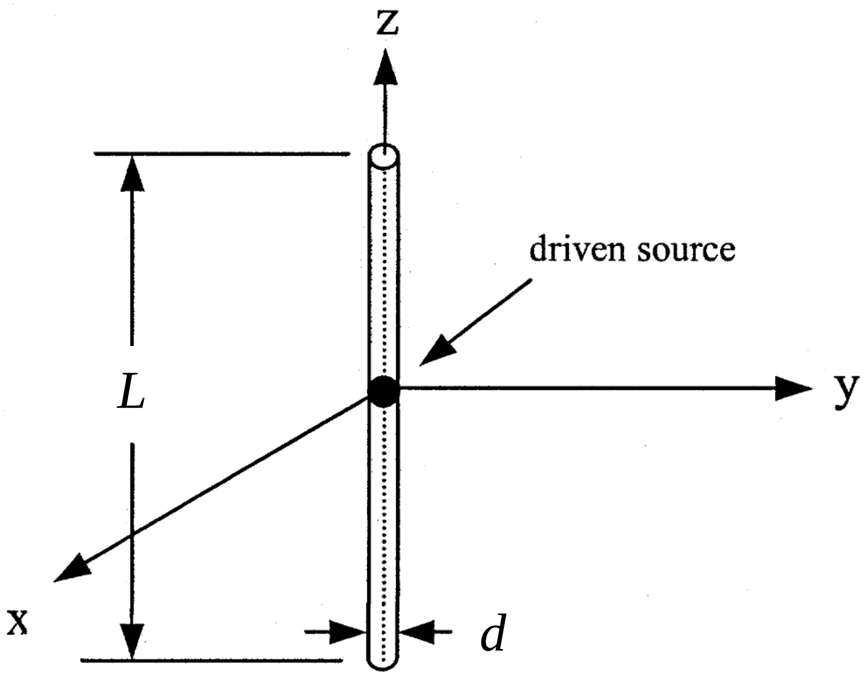
- **Or does stored energy mean reactive energy associated with the imaginary part of the Poynting vector, not total near field energy?**
  - Excludes circulating real power
- **Either way, determining the difference between infinite quantities requires care**

$$U_{stored} = \infty - \infty$$

# Ambiguity in the Denominator

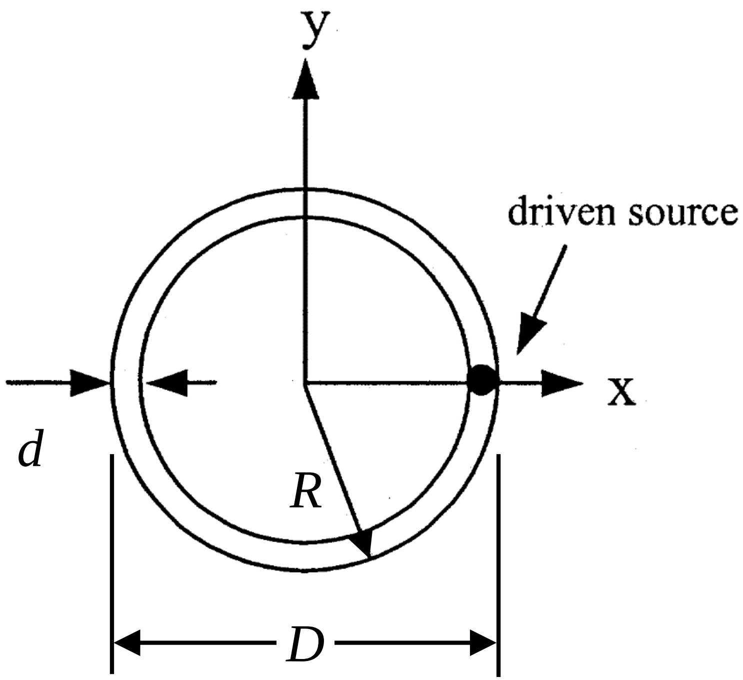
- **Does power lost per cycle mean all dissipated power?**
  - Includes ohmic and ground loss
- **Or does it mean real power that reaches the far field?**
  - Excludes ohmic and ground loss

# $Q$ of Small Dipole from Electromagnetic Field Analysis



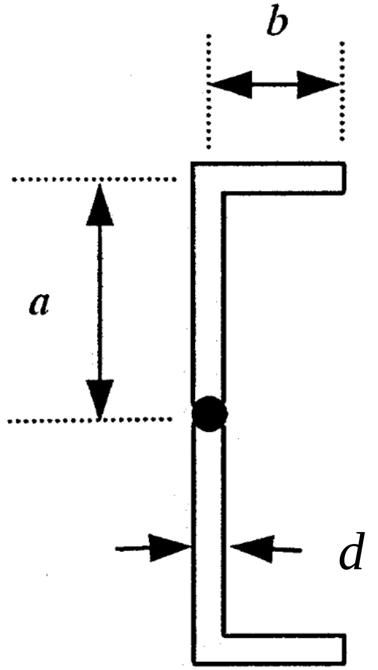

$$Q_{dipole} \approx \frac{6\left[\ln\left(\frac{L}{d}\right)-1\right]}{\pi^{3}\left(\frac{f\,L}{c}\right)^{3}}$$

# $Q$ of Small Loop from Electromagnetic Field Analysis




$$Q_{loop} \approx \frac{6\ln\left(\frac{D}{d}\right)}{\pi^4\left(\frac{fD}{c}\right)^3}$$

# *Q* of Inverted-L from Electromagnetic Field Analysis




$$Q_{inverted-L} \approx \left(\frac{3}{4}\right) \frac{\left[\ln\left(\frac{2a}{d}\right)-1\right] + \frac{b}{a}\left[\ln\left(\frac{4b}{d}\right)-1\right]}{\pi^3\left(\frac{f}{c}\right)^3 a(a+b)^2}$$

# Proposed Formulas for *Q* in Terms of Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

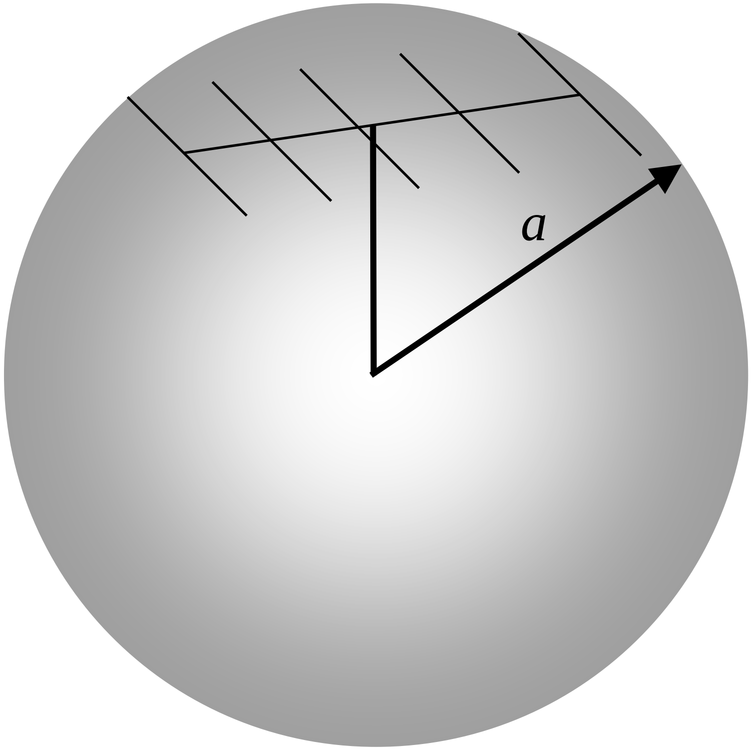
$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

# Fundamental Bounds on Antenna *Q*

L.J. Chu, *Physical Limitations of Omni-Directional Antennas*, tech rept. 64, MIT, May 1948. Also in MIT J. Appl. Phys., Dec 1948.




Smallest sphere that circumscribes antenna

- **Chu (1948)**

$$Q_{Chu} \geq \frac{1}{ka} + \frac{1}{(ka)^3}$$

- **Hansen and Collin (2007)**

$$Q_{New} \geq \frac{0.71327}{ka} + \frac{1.49589}{(ka)^3}$$

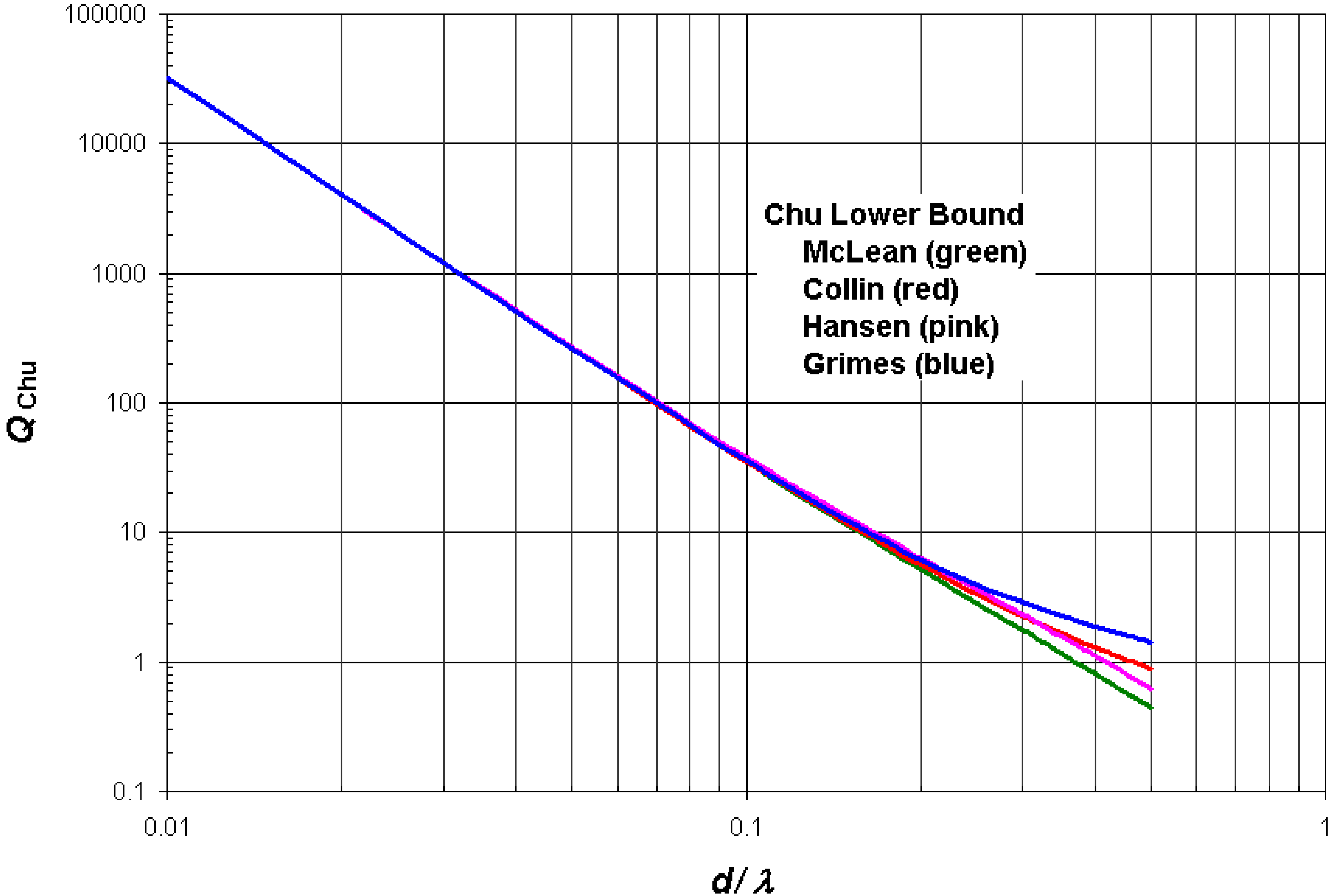
**where**

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$a = \text{radius of sphere}$$

Reducing an antenna's size in half increases its *Q* by 8 and reduces its bandwidth by 8.

# Chu *Q*



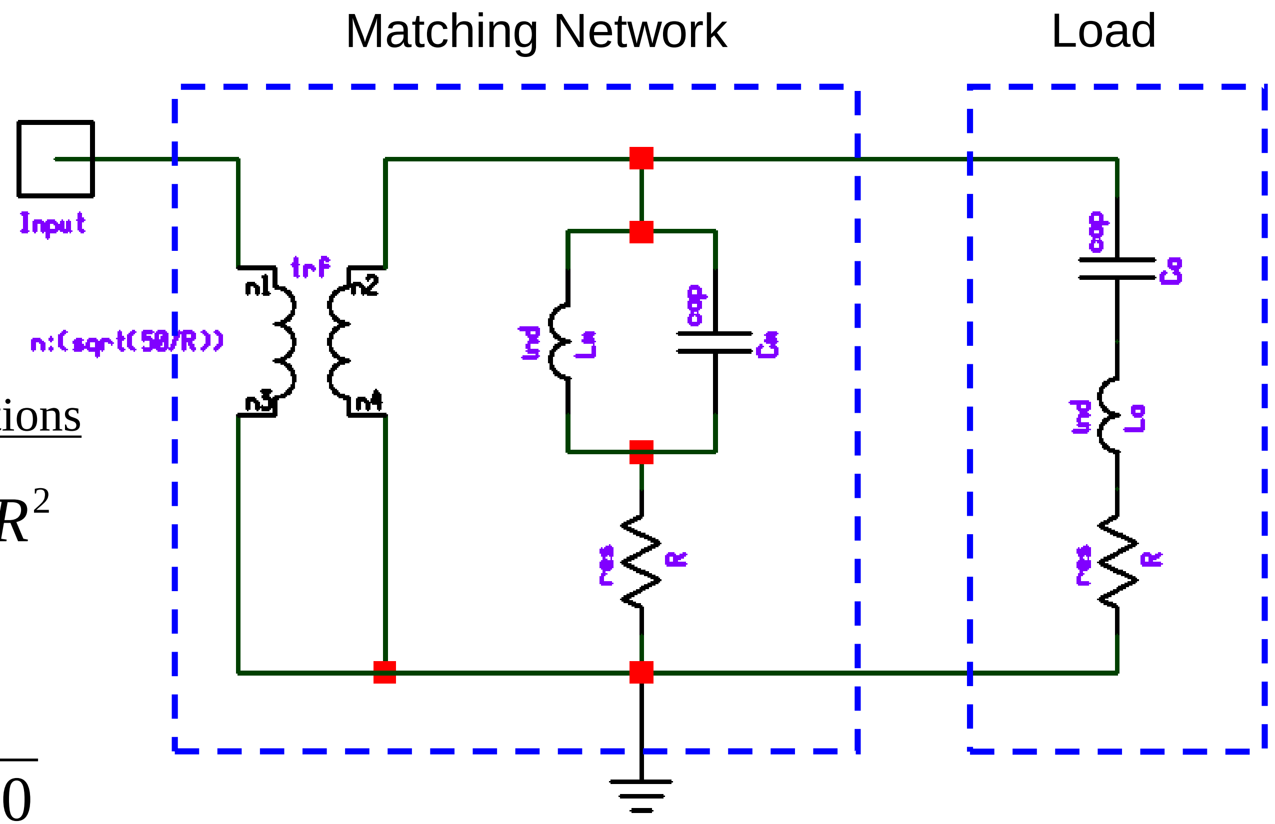
100000
10000
1000
100
10
1
0.1
Q Chu
0.01
0.1
1
d/ λ
Chu Lower Bound
McLean (green)
Collin (red)
Hansen (pink)
Grimes (blue)

# Apparent $Q = 0$ Antennas

# Four Ways to Make Apparent $Q = 0$ Antennas

- **Combine two complementary antennas that are impedance duals**
  - A small dipole and a small loop
  - Cross-field antennas
- **Use an integrated reflectionless match network**
  - Restricted to antennas whose Darlington reactance 2-port is a simple series or shunt reactance connected to load resistor
- **Use an integrated broadband non-Foster match network**
  - Requires clone of Darlington reactance 2-port and two negative impedance converters or inverters
- **Use a metamaterial radome shell that does external non-Foster matching**
  - Shell must enclose the reactive near field region

# Constant Resistance Network for a Series RLC Load




<u>Design Equations</u>

$$L_M = C_A R^2$$

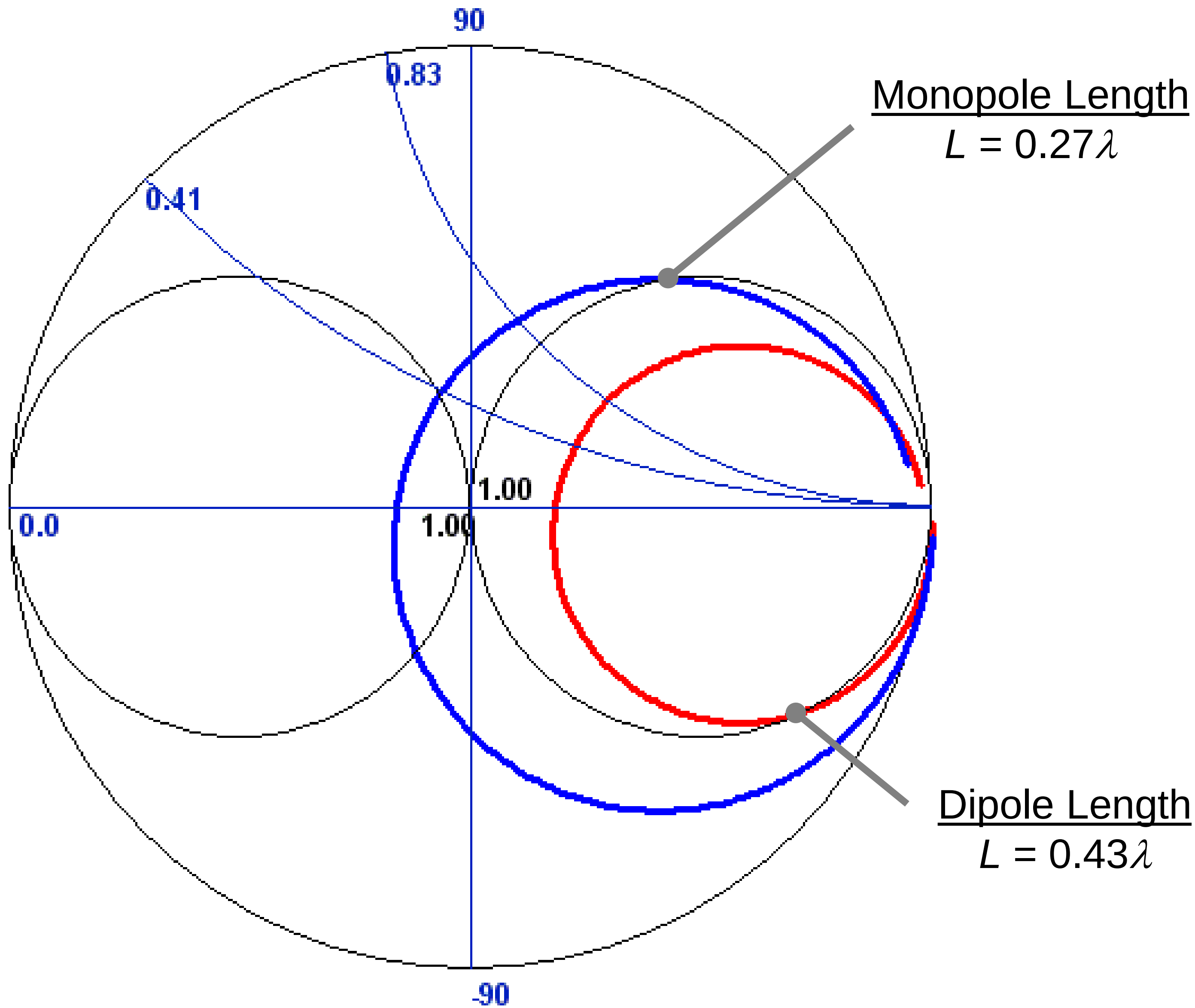
$$C_M = \frac{L_A}{R^2}$$

$$N = \sqrt{\frac{50}{R}}$$

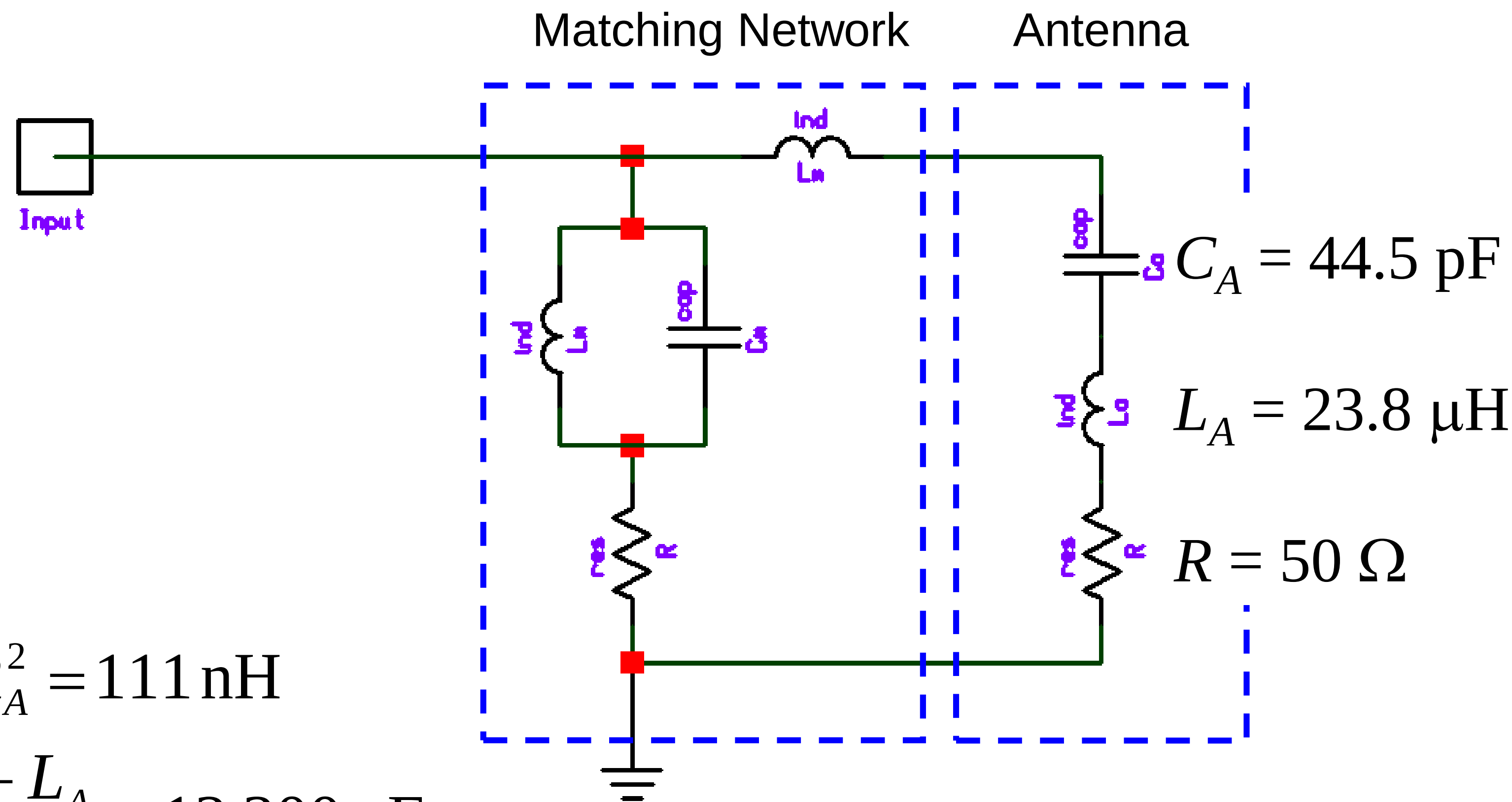
# Reflectionless Matching for Dipoles and Monopoles

- **Set length so that $R_A$ = 50 ohms**
  - Dipoles: Use $K \approx 0.86$, or $L \approx 0.43\lambda$
  - Monopoles: Use $K \approx 1.07$, or $L \approx 0.27\lambda$
- **Add a series reactance to cancel feedpoint reactance**
  - Dipoles: Add a series inductor
  - Monopoles: Add a series capacitor
- **Add a shunt network to yield a 50-ohm constant-resistance (CR) network**

# Eliminating the Transformer



# Example: Reflectionless Match to 0.43λ Dipole



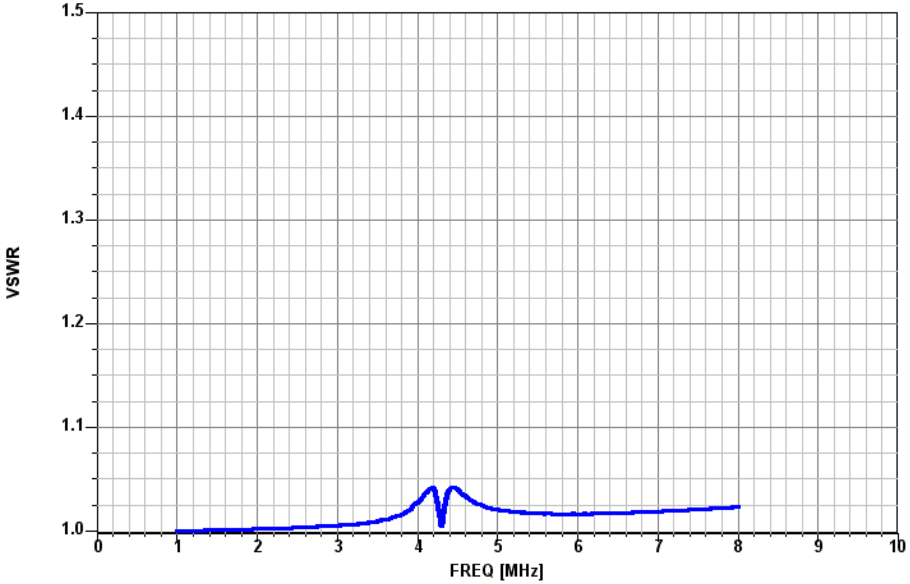

$$L_S = 7\ \mu\text{H}$$

$$L_M = C_A R_A^2 = 111\ \text{nH}$$

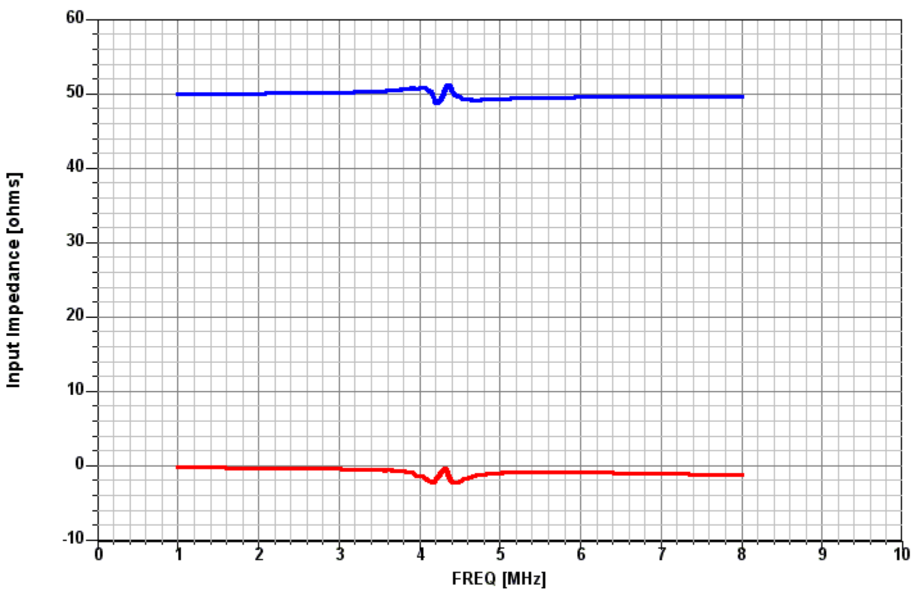
$$C_M = \frac{L_S + L_A}{R_A^2} = 12{,}300\ \text{pF}$$

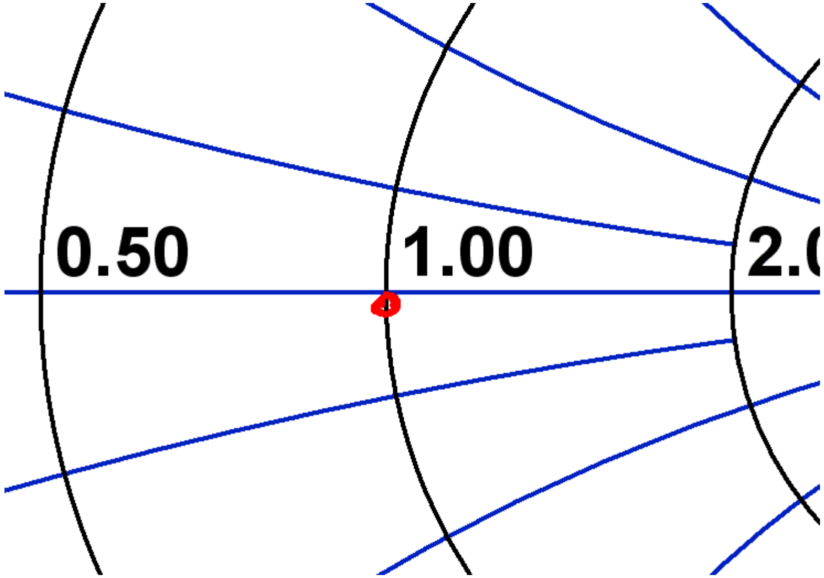
$$R = R_A = 50\ \Omega$$

# Network Performance on Dipole Impedance Data

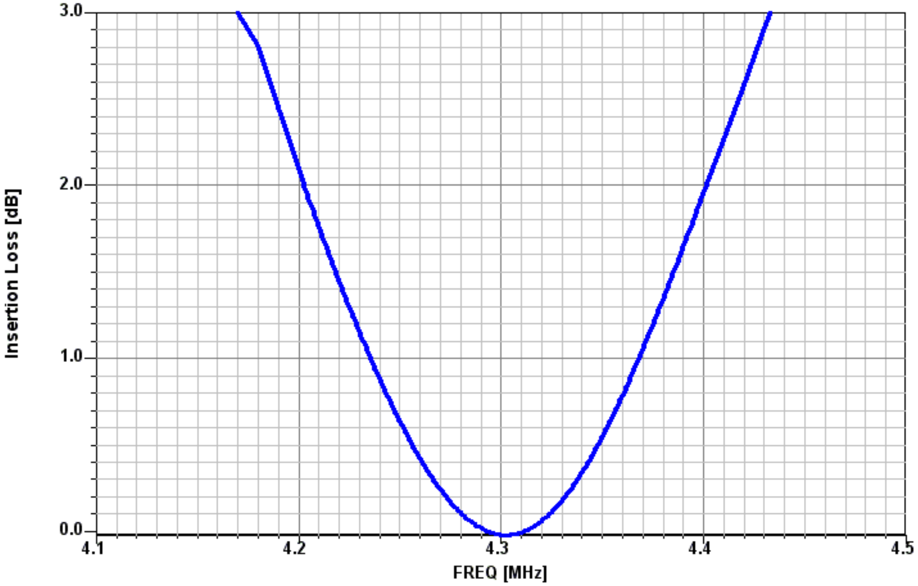



- Frequency sweep 1 to 8 MHz
- Maximum SWR = 1.04
- Input resistance:  48.8 to 51.2 ohms
- Input reactance:  -2.1 to 0 ohms



# Power Delivered to the 0.43λ Dipole



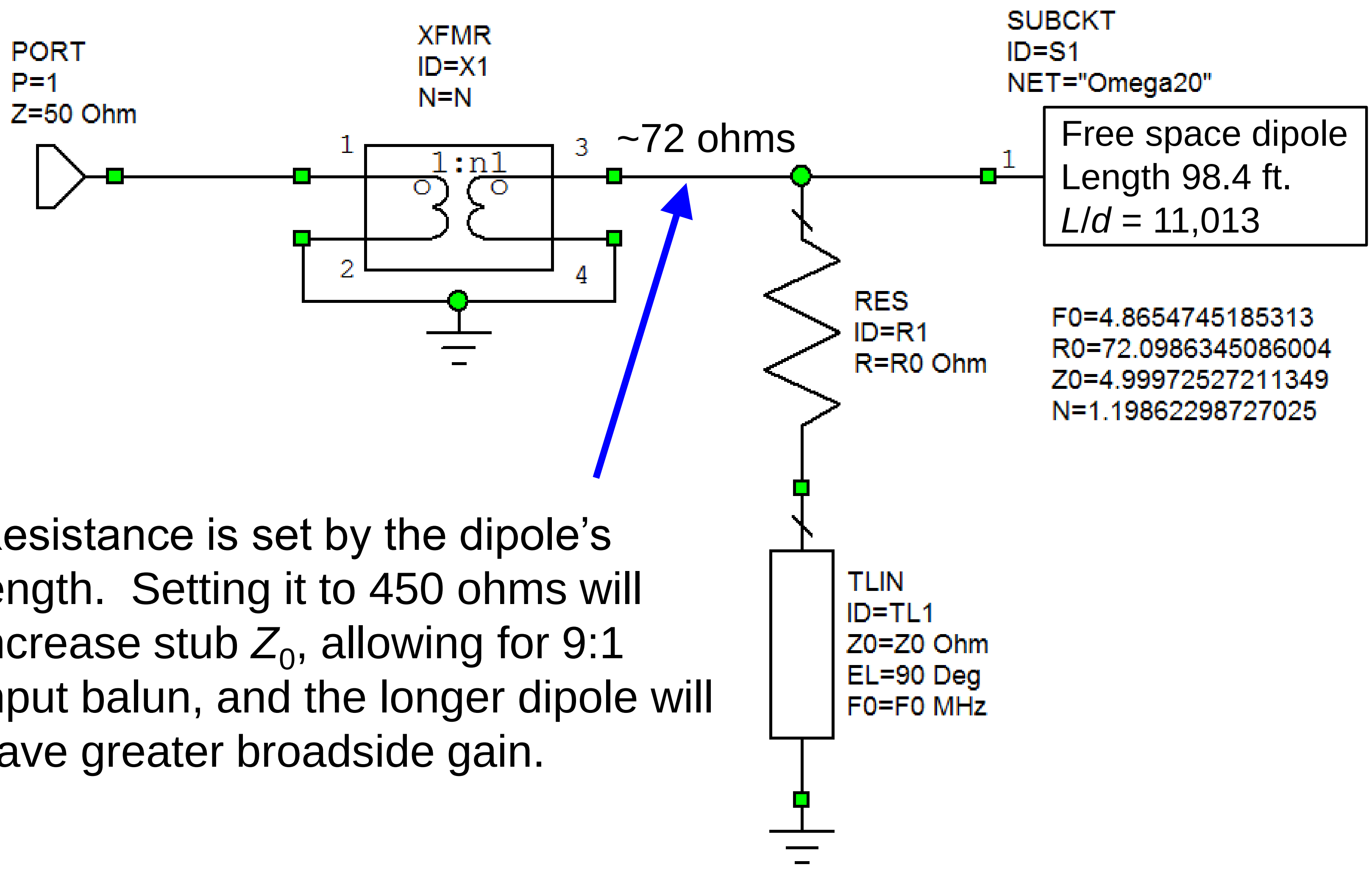

- Pattern gain = - 0.11 dBd
- Minimum insertion loss = 0 dB
- 100% power delivery at 4.3 MHz
- 3-dB Bandwidth = 259 kHz (6.0%)
- 0.51-dB Bandwidth = 91 kHz (2.1%)

| Bandwidths to Compare | | |
|---|---|---|
| Lossless Networks | | Reflectionless Networks |
| $BW_{SWR\ 5.83:1}$ | ⟺ | $BW_{IL\ 3\text{-}dB}$ |
| $BW_{SWR\ 2:1}$ | ⟺ | $BW_{IL\ 0.51\text{-}dB}$ |

# Reflectionless Matching Using a Complementary Stub

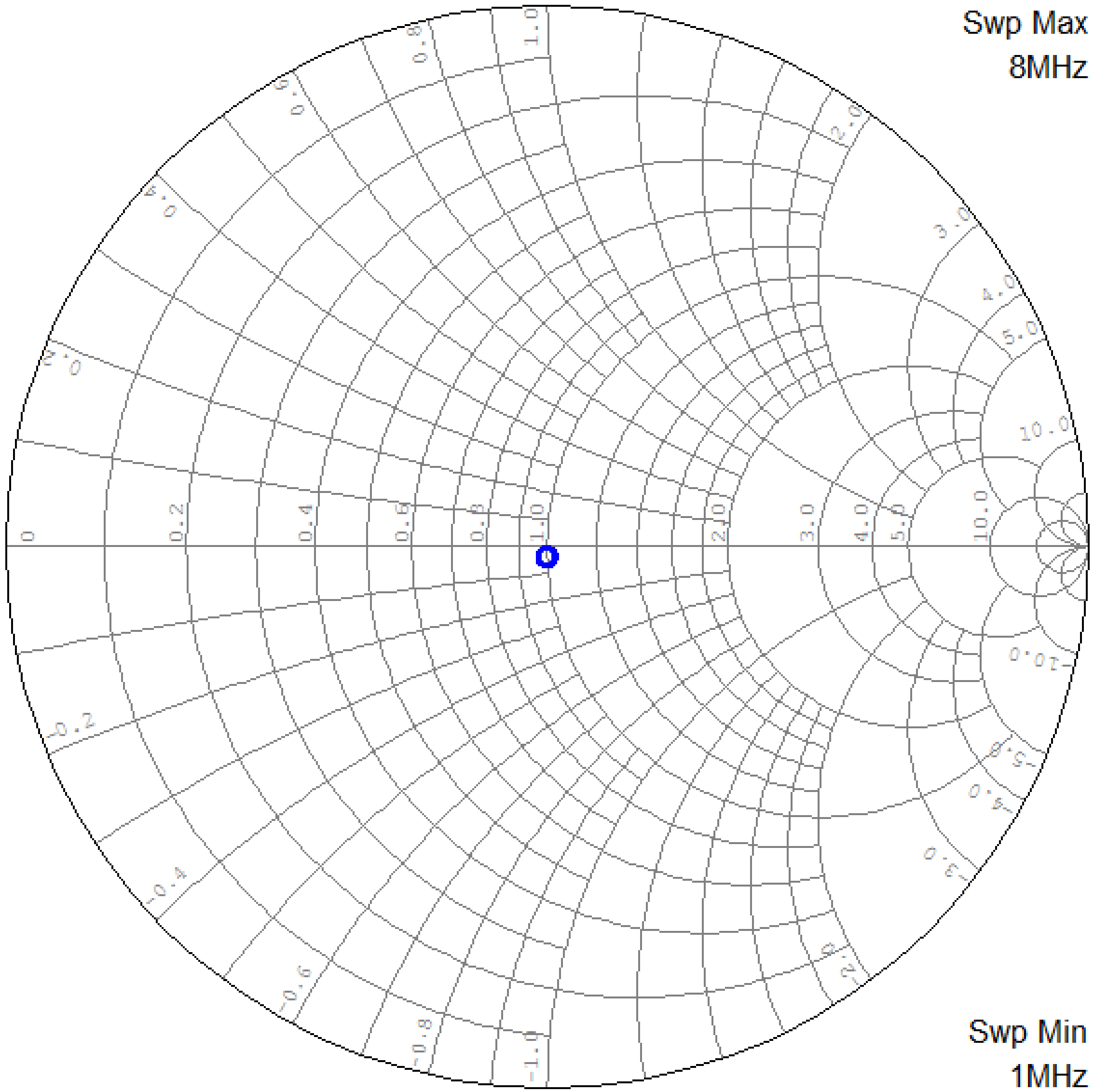
- Assume the resistance varies slowly across a band
- A reflectionless match network is obtained by putting a complementary admittance in parallel with the dipole or monopole
- We obtain a constant-resistance (CR) match network made from a stub and a resistor

# Parallel Connection of Resistor-Stub and Dipole

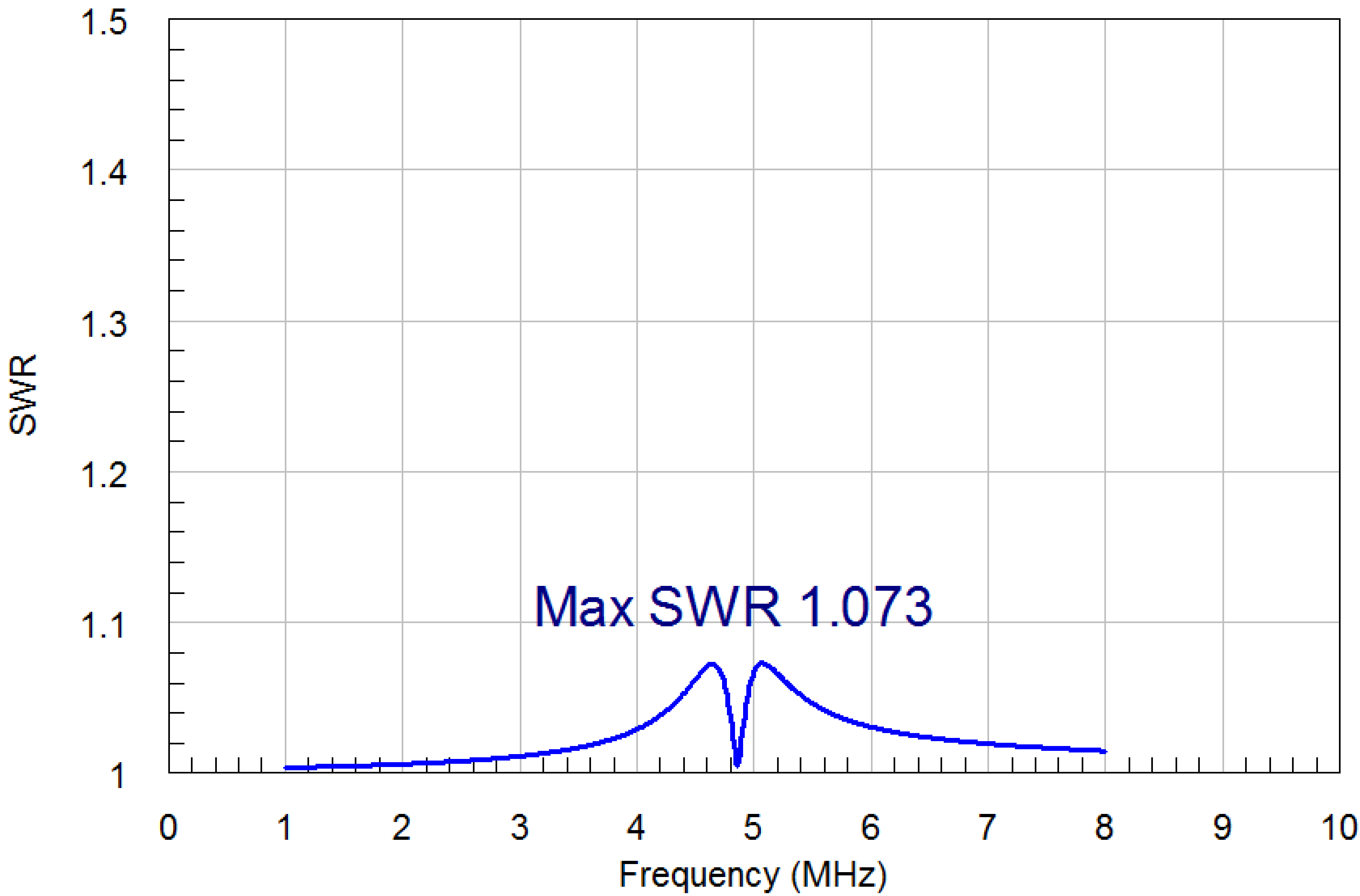



Resistance is set by the dipole's length. Setting it to 450 ohms will increase stub $Z_0$, allowing for 9:1 input balun, and the longer dipole will have greater broadside gain.

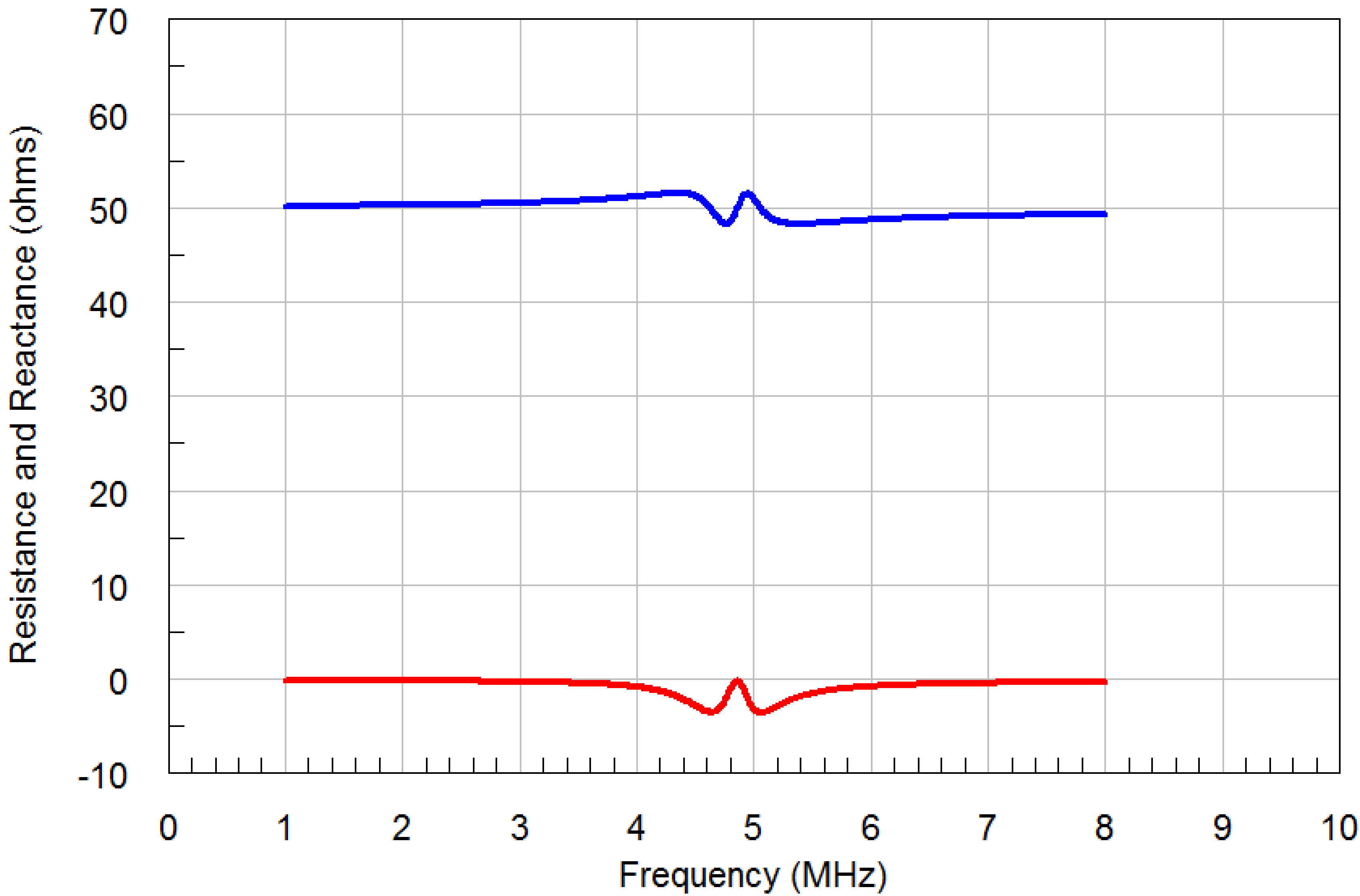
# Multi-Octave Match from 1 MHz to 8 MHz



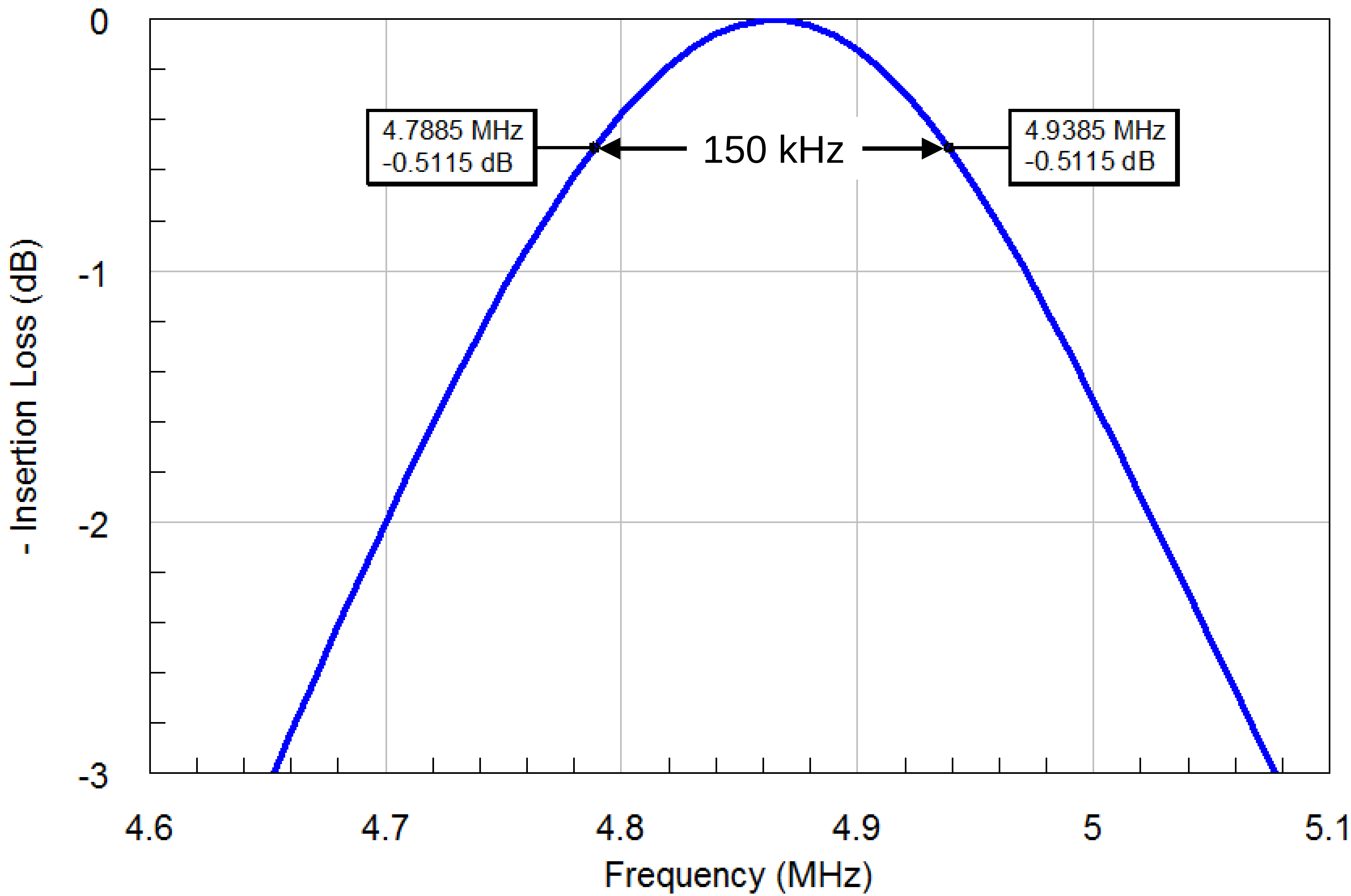
# SWR



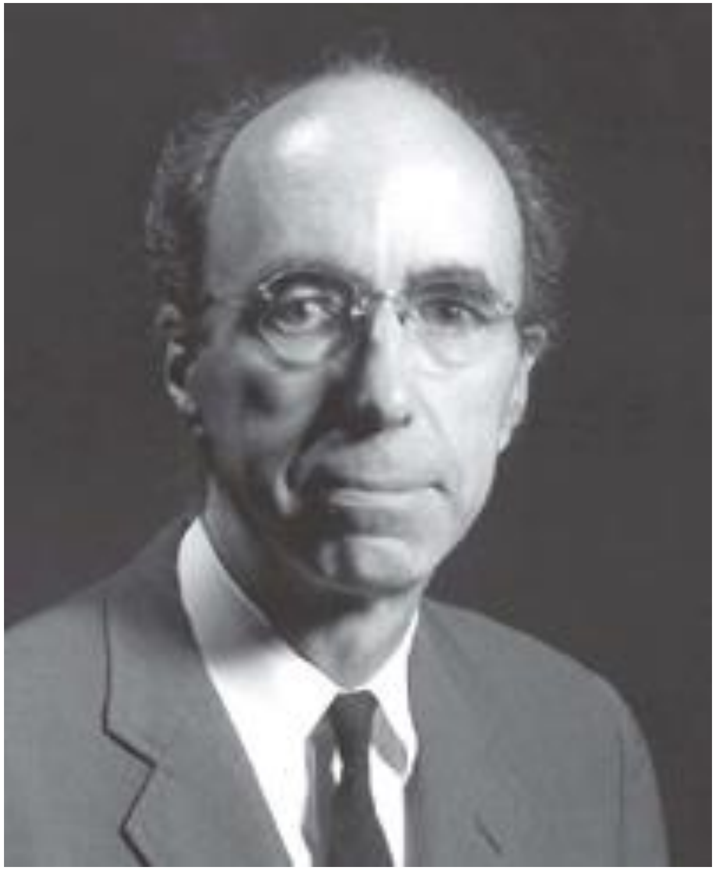
# Resistance and Reactance



# – Insertion Loss



# Sidney Darlington, 1906-1997



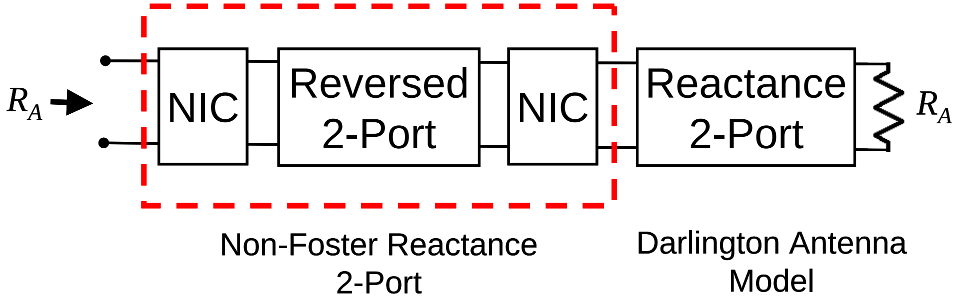
# Darlington Forms (1939)

- **Every positive-real, rational immittance function can be realized as a lossless lumped 2-port terminated by a resistor**

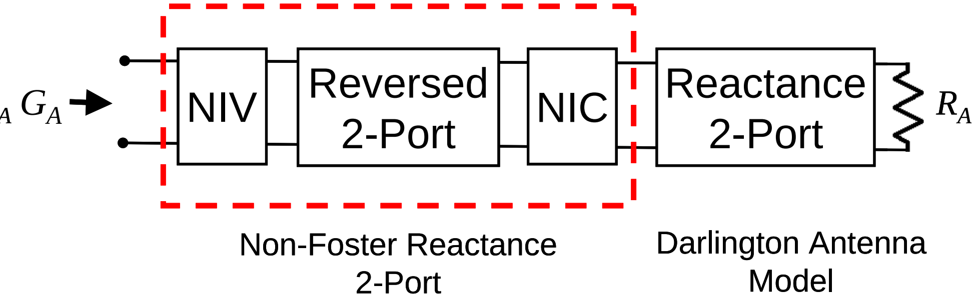

- **For antennas, a meromorphic antenna impedance function can be represented by a convergent sequence of equivalent circuits in Darlington form**
- **The Darlington form is a starting point for a theory of non-Foster matching of antennas**

# Cascade Inversions Using NICs and NIVs

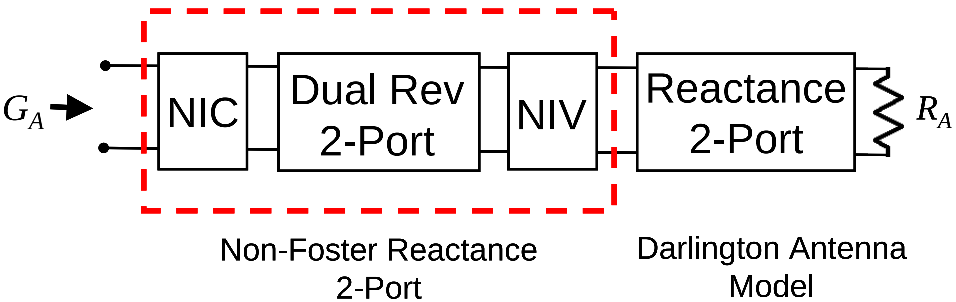
## 1st Canonical Form



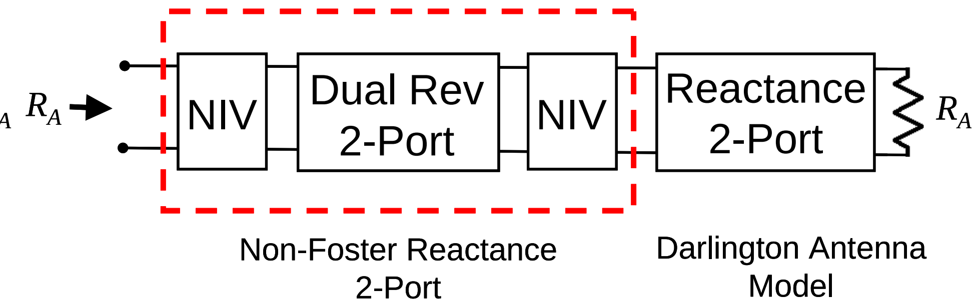

## 2nd Canonical Form




## 3rd Canonical Form




## 4th Canonical Form




Formal inverse match networks achieve perfect impedance matching and infinite match bandwidth.

# Proposed Formulas for *Q* are Incorrect

- **Series *RLC* equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

*Q* cannot be determined reliably from *Z*.
*Q* must be computed from field formulas!

# Antenna *Q* Formulas Are Wrong !

- **Apparent *Q* = 0 antennas are counterexamples which show proposed *Q* formulas can be very wrong**
- **The feedpoint impedance can be a constant real number independent of frequency**
  - Reactance and its derivatives can be made zero

$$Z_A = R_A + j0 \qquad \frac{\partial R_A}{\partial \omega} = 0 \qquad X_A = 0 \qquad \frac{\partial X_A}{\partial \omega} = 0$$

- ***Q* formulas based on feedpoint *Z* yield zero**
  - *Q* by formula is zero
  - But antenna *Q* is not zero

No general formula for computing *Q* from *Z* exists.
Antenna *Q* must be computed from field formulas.

# More on Antennas and Radiation

# Every Ham Wants a Big Antenna!

**WLW**
**Mason, OH**
**831 feet**

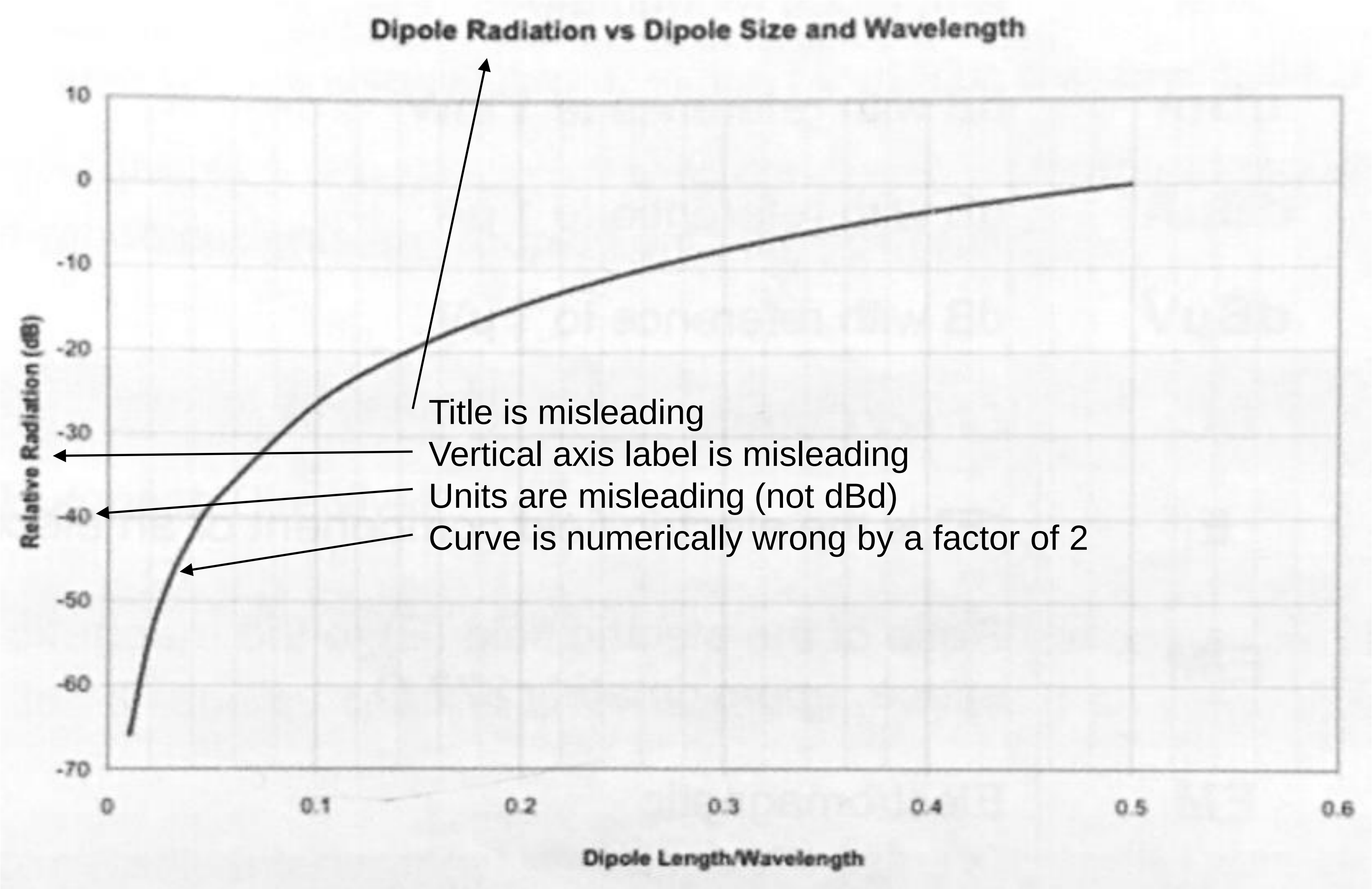
**GVARC Full Band HF Discone**
**Green Valley, AZ**
**80 feet**

**Sutro Tower**
**San Francisco, CA**
**977 feet**

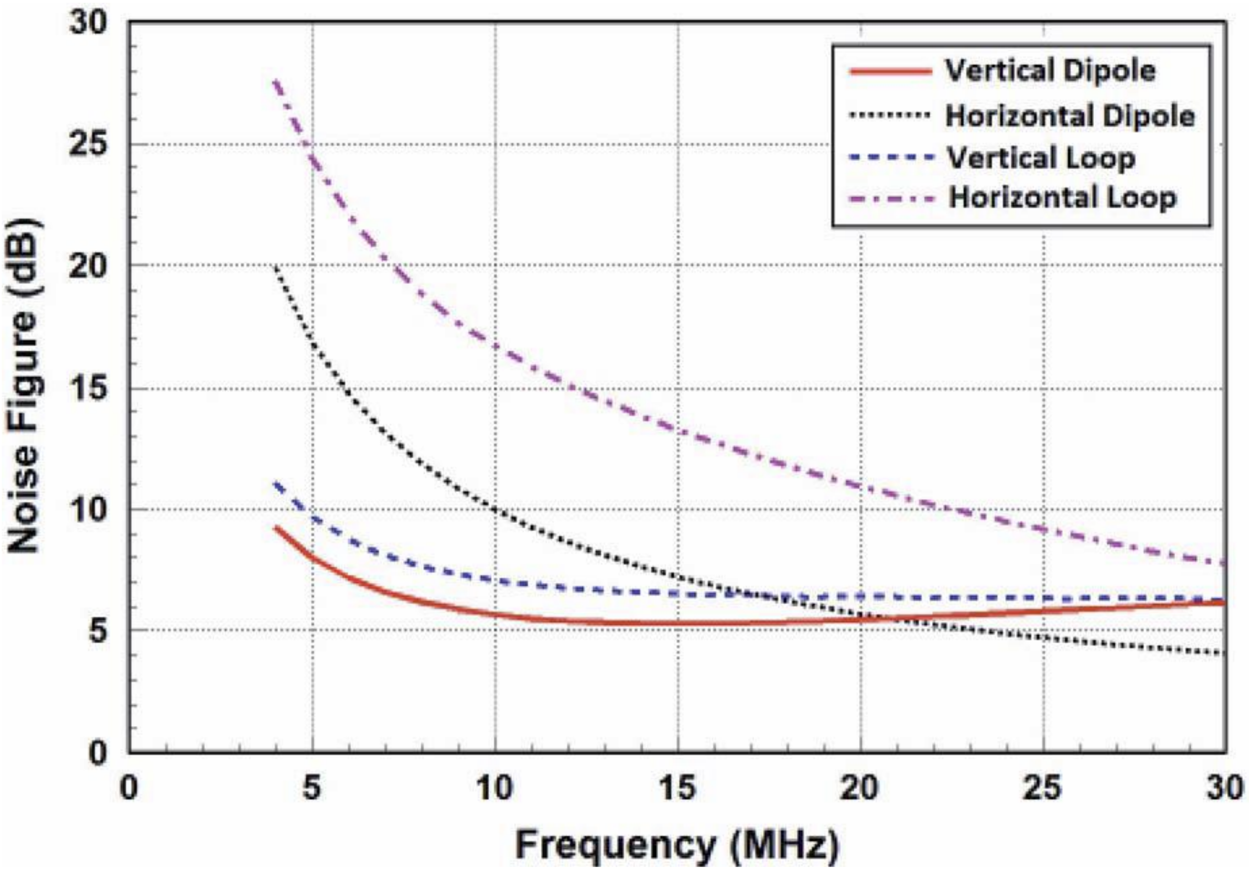
# Or Even Bigger!

**Duga, in Ukraine (former Soviet Union), 2,300 feet x 490 feet**

# Information that Misleads



# Noise Figures of Four Antennas with Ground Loss



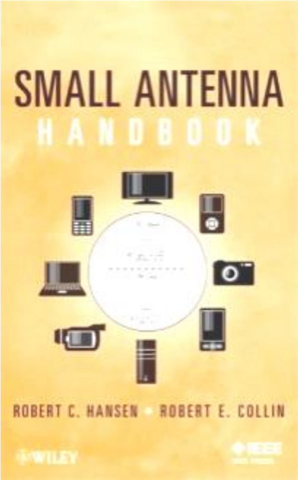

- Noise figure comparison of a 2-meter long dipoles and 1-meter diameter loops located 2 meters above average soil. The external noise is assumed to be galactic.
- Low frequency ranked order: V dipole, V loop, H dipole, H loop
- High frequency ranked order: H dipole, V dipole, V loop, H loop

S.R. Best, "Optimizing the Receiving Properties of Electrically Small HF Antennas," *Radio Science Bulletin*, Dec. 2016.

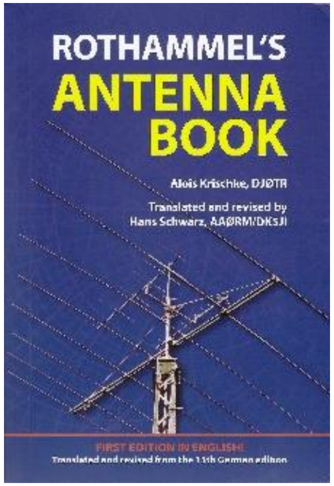
# Inexpensive Antenna Modeling Software Packages

- **Best method-of-moments thin-wire programs**
  - AN-SOF ($350 to $450 depending on grade)
    - from Tony Golden, Montevideo, Uruguay
  - NEC-5 package
    - NEC-5 from Lawrence Livermore $110
    - EZNEC GUI program, free
    - AutoEZ from Dan Maguire, AC6LA, $80
- **Best method-of-moments surface program**
  - HOBBIES (no longer available)
- **Older software (more restrictions and limitations)**
  - NEC-2 and NEC-4
  - Old versions of EZNEC
  - MiniNEC and MMANA-GAL
  - AWAS 2.0
  - WIPL-D Lite

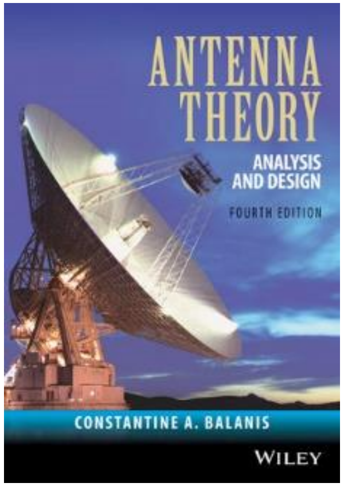
# Recommended Antenna Books



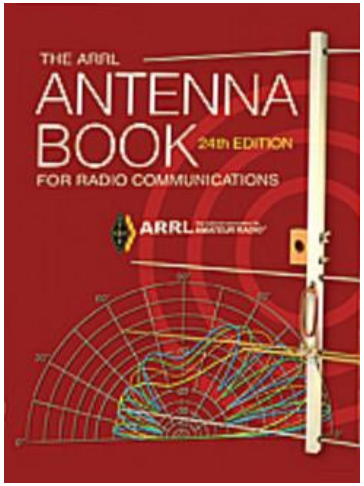

R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011




A. Krischke, DJ0TR, ed., *Rothammel's Antenna Book*, 13e, English transl., DARC, 2019




C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016




H.W. Silver, N0AX, ed., *ARRL Antenna Book*, 24e, ARRL, 2019

New 25th edition coming in 2023

## The End