Bode, Chu, Fano, Wheeler: Antenna Q and Match Bandwidth

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ARRL Pacificon Presentations by K6OIK

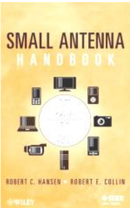
Archived at
http://www.fars.k6ya.org

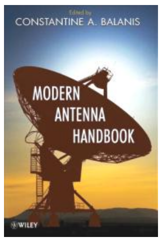
1999	Mysteries of the Smith Chart	
2000	Jam-Resistant Repeater Technology: Signal Separation, Identification, Routing, Control, and Automatic Logging	
2001	Mysteries of the Smith Chart	✓
2002	How-to-Make Better RFI Filters Using Stubs	
2003	Twin-Lead J-Pole Design	
2004	Antenna Impedance Models – Old and New	✓
2005	Novel and Strange Ideas in Antennas and Impedance Matching	
2006	Novel and Strange Ideas in Antennas and Impedance Matching 2	✓
2007	New Results on Antenna Impedance Models and Matching	✓
2008	Antenna Modeling for Radio Amateurs	✓
2009	(convention held in Reno)	
2010	Facts About SWR, Reflected Power, and Power Transfer on Real Transmission Lines with Loss	✓
2011	Conjugate Match Myths	✓
2012	Transmission Line Filters Beyond Stubs and Traps	✓
2013	Bode, Chu, Fano, Wheeler: Antenna Q and Match Bandwidth	✓

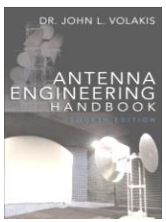
Outline

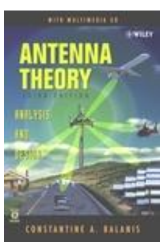
- **Antenna books**
- **Software and instruments**
  - Antenna modeling
  - Circuit design
  - Smith chart
  - Antenna analyzers
- **Antenna impedance models (equivalent circuits)**
- **Bode, Chu, Fano, Wheeler**
- **Antenna Q**
- **Chu bound**
- **Fano bound**
- **References**

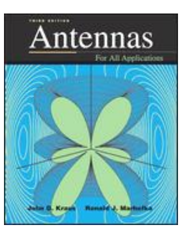
Favorite Antenna Books





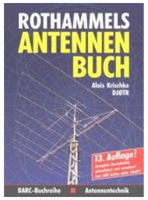


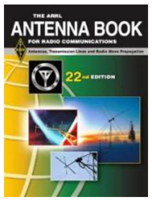


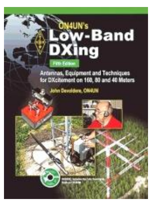


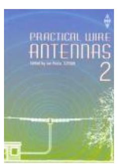
- **Books for antenna engineers and students**
  - R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011, ISBN 0470890835
  - *Modern Antenna Handbook*, C.A. Balanis editor, Wiley, 2008, ISBN 0470036346
  - *Antenna Engineering Handbook*, 4th ed., J.L. Volakis editor, McGraw-Hill, 2007, ISBN 0071475745. First published in 1961, Henry Jasik editor
  - C.A. Balanis, *Antenna Theory*, 3rd ed., Wiley, 2005, ISBN 047166782X. First published in 1982 by Harper & Row
  - J.D. Kraus and R.J. Marhefka, *Antennas*, 3rd ed., McGraw-Hill, 2001, ISBN 0072321032. First published in 1950
  - S.J. Orfanidis, *Electromagnetic Waves and Antennas*, draft textbook online at http://www.ece.rutgers.edu/~orfanidi/ewa/
  - E.A. Laport, *Radio Antenna Engineering*, McGraw-Hill, 1952 http://snulbug.mtview.ca.us/books/RadioAntennaEngineering
  - G.V. Ayzenberg, *Shortwave Antennas*, 1962, transl. from Russian, DTIC AD0706545. http://www.dtic.mil/dtic/tr/fulltext/u2/706545.pdf
- **Antenna research papers**
  - IEEE AP-S Digital Archive, 2001-2009 (1 DVD), JD0307
  - IEEE AP-S Digital Archive, 2001-2006 (1 DVD), JD0304
  - IEEE AP-S Digital Archive, 2001-2003 (1 DVD), JD0301
  - IEEE AP-S Digital Archive, 1952-2000 (2 DVDs), JD0351

Favorite Antenna Books continued

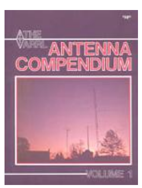


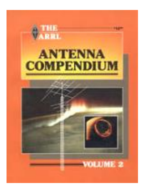


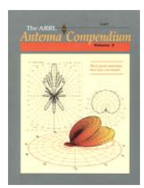


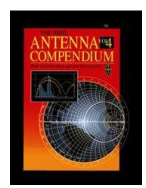


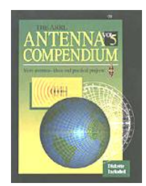

- **Books for Radio Amateurs**
  - *Rothammel's Antennenbuch*, 13th ed., A. Krischke (DJ0TR) editor, DARC Verlag, 2013
  - *ARRL Antenna Book*, 22nd ed., H.W. Silver (N0AX) editor, American Radio Relay League, 2011, ISBN 087259694X
  - J. Devoldere (ON4UN), *ON4UN's Low-Band Dxing*, 5th ed., American Radio Relay League, 2011, ISBN 087259856X
  - *Practical Wire Antennas 2*, I. Poole (G3YWX) editor, Radio Society of Great Britain, 2005, ISBN 1905086040
  - J. Sevick (W2FMI), *The Short Vertical Antenna and Ground Radial*, CQ Communications, 2003, ISBN 0943016223
  - L. Moxon (G6XN), *HF Antennas for All Locations*, 2nd ed., Radio Society of Great Britain, 1983, ISBN 1872309151
- **ARRL Antenna Compendium series – Volumes 1 through 8**

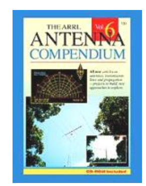


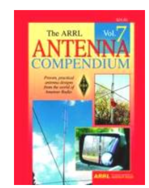


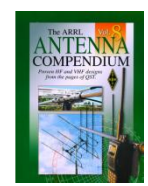




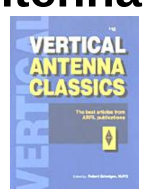


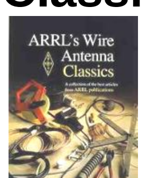


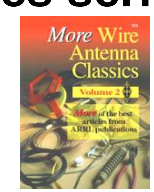


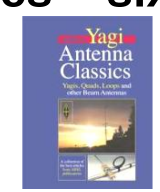


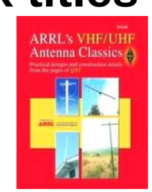
- **ARRL Antenna Classics series – six titles**

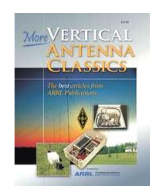




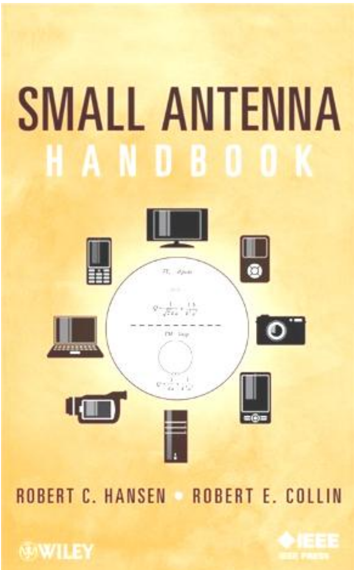




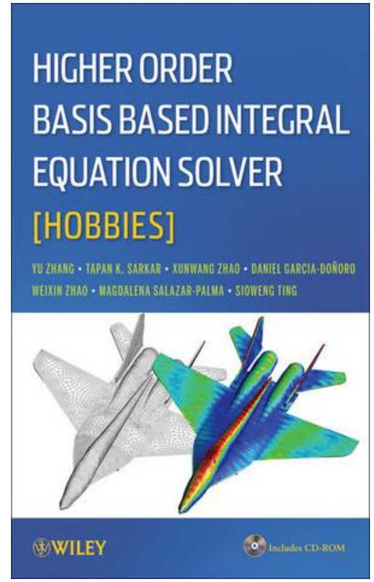




New Antenna Books of Interest

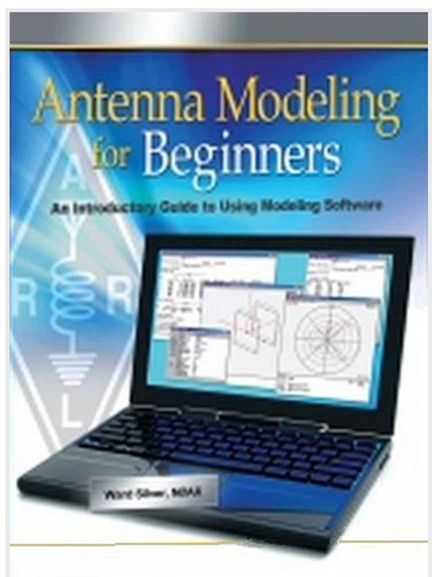


R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011, ISBN 0470890835

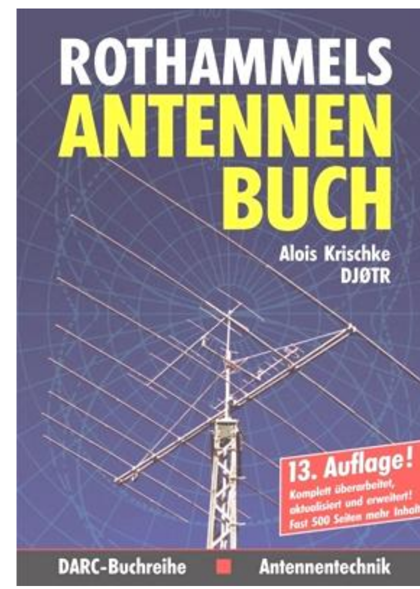


Y. Zhang et al., *Higher Order Basis Based Integral Equation Solver* [*HOBBIES*], Wiley, 2012, ISBN 1118140656

H.W. Silver, N0AX, *Antenna Modeling for Beginners*, ARRL, 2012, ISBN 9780872593961



A. Krischke, DJ0TR, ed., *Rothammels Antennen Buch*, 13th ed, DARC, 2013

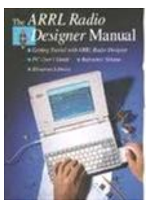


Software and Tools

Antenna Modeling Programs for Radio Amateurs

- ***EZNEC*** **http://www.eznec.com**
  - EZNEC v.5 demo program Free
  - EZNEC-ARRL v.3 & v.4 $50 (on ARRL Antenna Book CD-ROM)
  - EZNEC v.5 $90
  - EZNEC+ v.5 $140
  - EZNEC Pro/2 v.5 $500
  - EZNEC Pro/4 v.5 $650 (sold only to NEC-4 licensees)
- ***4nec2*** **http://home.ict.nl/~arivoors/**
  - Free, 11,000 segments, two optimizers
- ***NEC-4*** **https://ipo.llnl.gov/data/assets/docs/nec.pdf**
  - Noncommercial user license $300
- ***FEKO Lite*** **http://www.feko.info**
  - Free LITE version
- ***WIPL-D Lite*** **http://www.wipl-d.com**
  - Free Demo version, more capable Lite version from Artech House $450
- ***HOBBIES*** **http://em-hobbies.com**
  - Similar to WIPL-D Lite but more capability, $162 to $210

General RF Circuit Design, Analysis, and Optimization



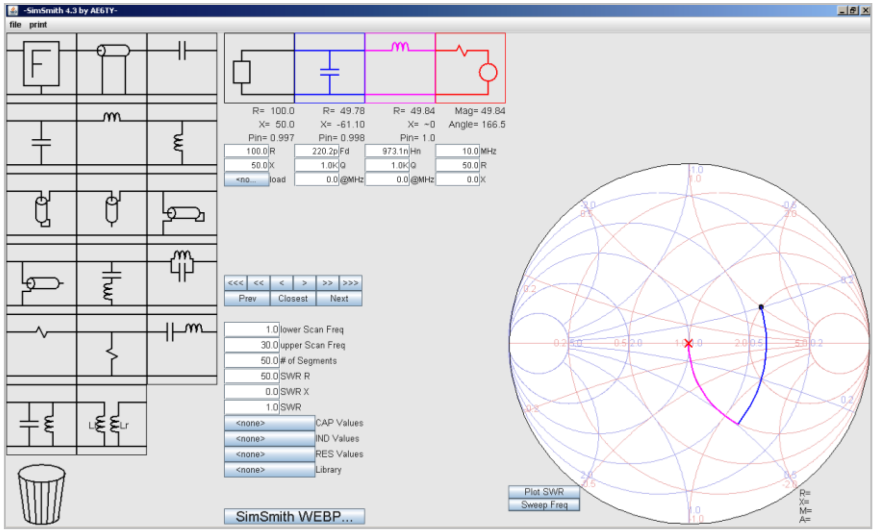
- **Software for Radio Amateurs**
  - *Quite Universal Circuit Simulator* (QUCS) 0.0.16, 2011, $0 (free)
    - Download from http://qucs.sourceforge.net
  - *Ansoft Serenade SV* 8.5 (student version), Ansoft, 2000, $0 (free). No longer available.
  - *ARRL Radio Designer* 1.5, ARRL, 1995. No longer available.
- **Professional electronic design automation (EDA) software**
  - *Advanced Design System* (ADS), Agilent
  - *Microwave Office* (MWO), Applied Wave Research
  - *Designer*, ANSYS

Software for Smith Charting and Network Design

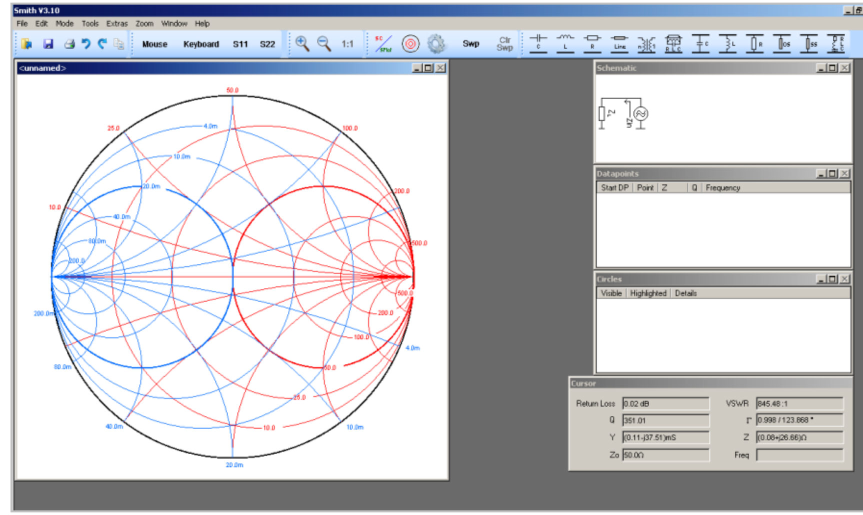
- **Match network analysis with Smith Chart display**
  - *SimSmith* by Ward Harriman AE6TY, 2011, $0 (free)
    - Download from http://ae6ty.com/Smith_Charts.html
  - *Smith* by Fritz Dellsperger HB9AJY, 2010, $0 (free)
    - Download from http://fritz.dellsperger.net
  - *QuickSmith* by Nathan Iyer KJ6FOJ, 2009, $0 (free)
    - Download from http://www.nathaniyer.com/qsdw.htm
  - *linSmith* by James Coppens ON6JC/LW3HAZ, $0 (free)
    - Download from http://jcoppens.com/soft/linsmith/index.en.php
  - *SuperSmith* by J. Bromley K7JEB and J. Tonne W4ENE, $0 (free)
    - Download from http://www.tonnesoftware.com/supersmith.html
  - *XLZIZL* by Dan Maguire AC6LA, 2005. No longer available.
  - *WinSMITH* 2.0, Noble Publishing, 1995. No longer available.
  - *MicroSmith* 2.3, ARRL, 1992. No longer available.

Smith Chart Programs for Ladder Network Design

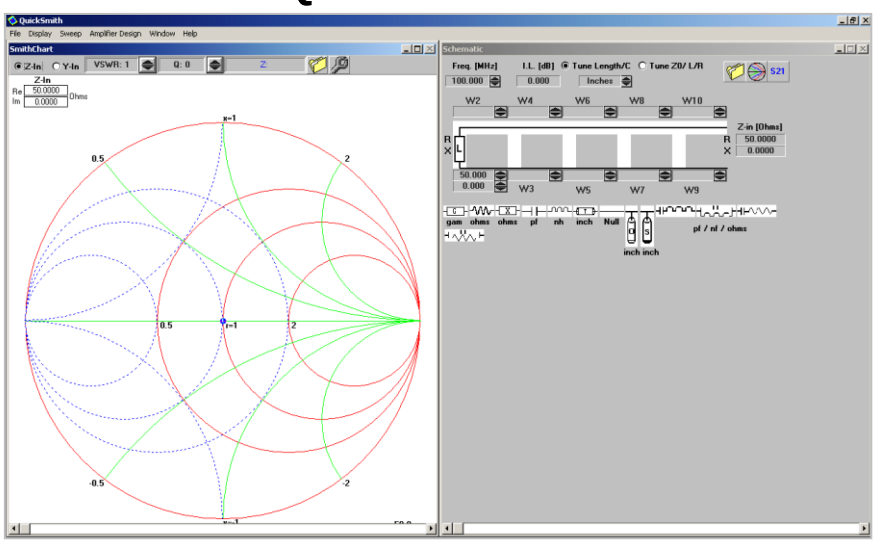
SimSmith 7.8



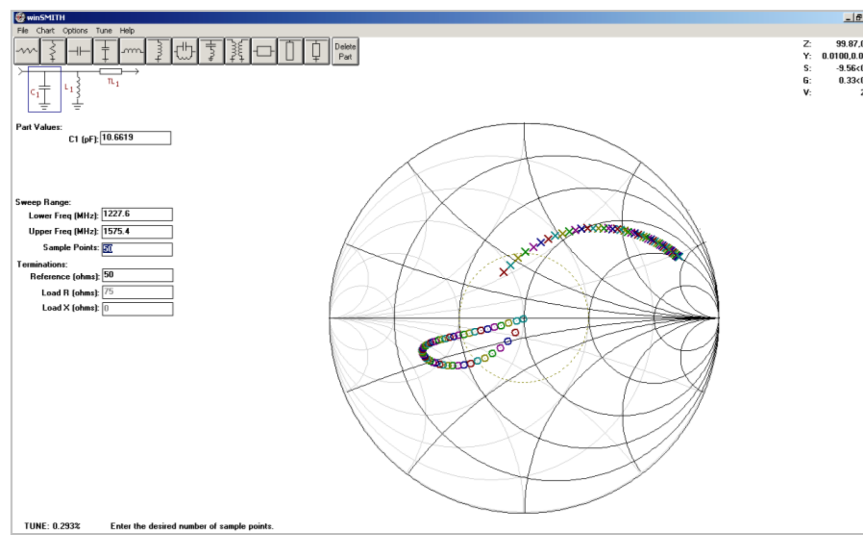
Smith 3.10



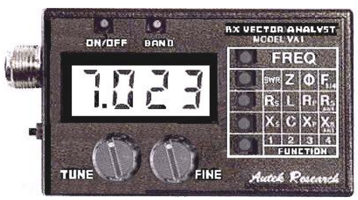
QuickSmith 4.5



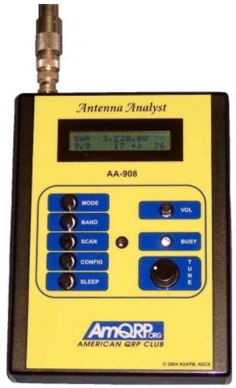
winSMITH 2.0



Instruments for Measuring Impedance



Autek VA1 Vector R-X Analyst



Micro908

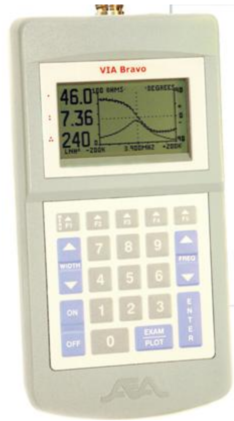
MFJ 269

Palstar ZM30

Kuranishi BR-210

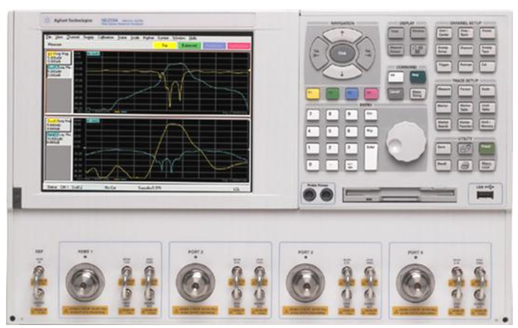
Comet
CAA-500

Timewave TZ-900



AEA VIA Bravo

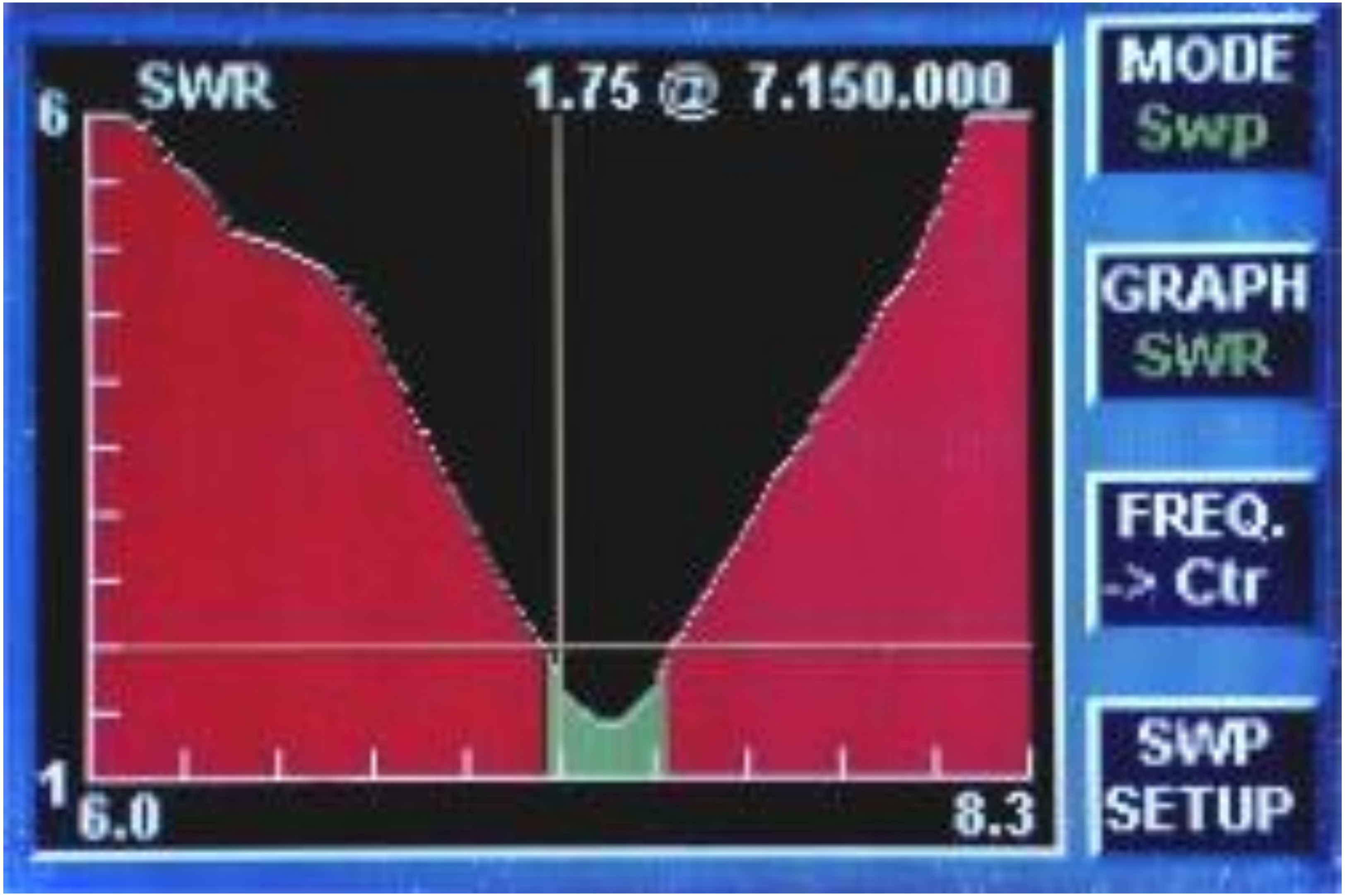
Anritsu Site Master S361E



Agilent N5230A

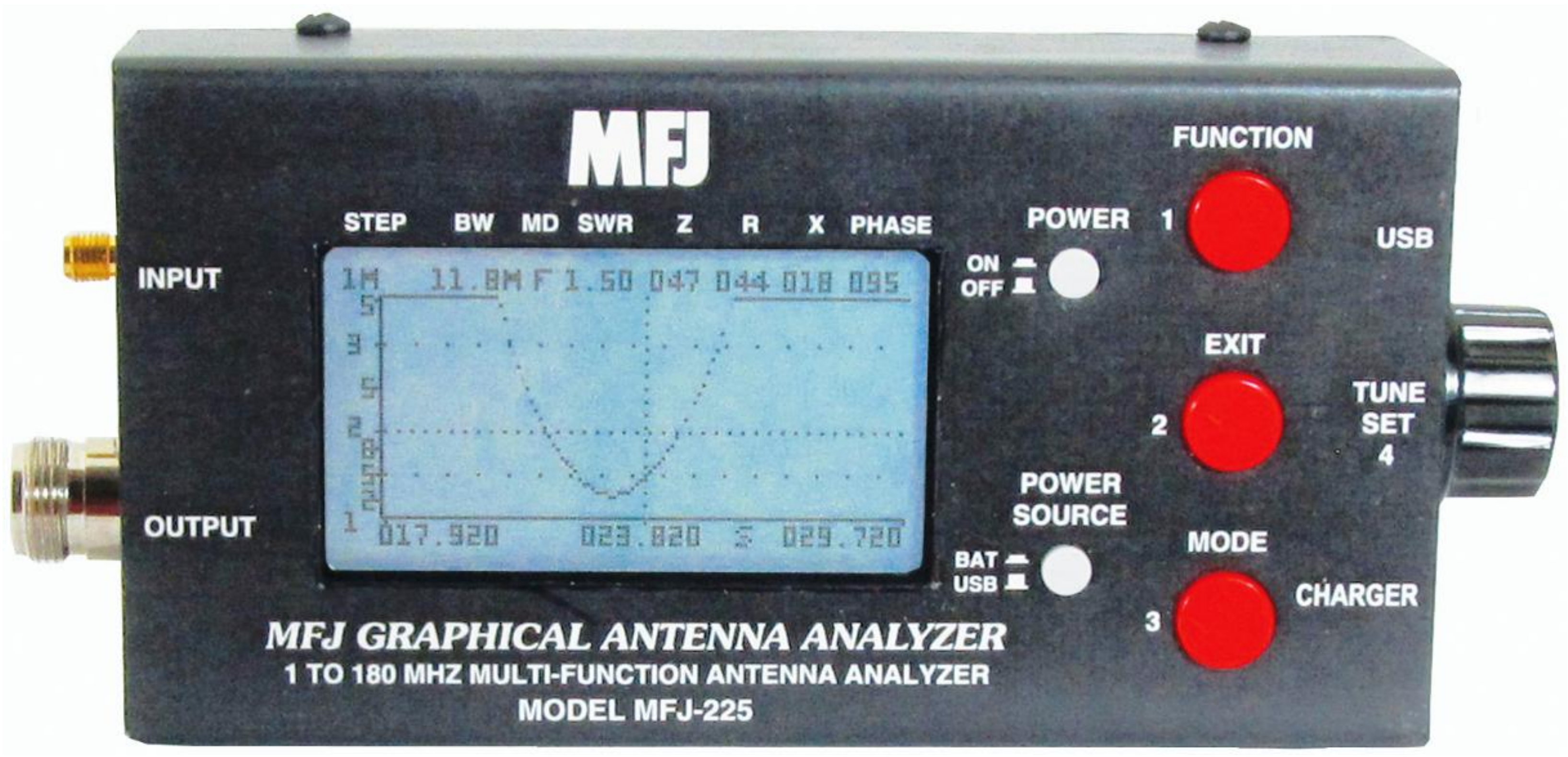
Timewave TZ-900 AntennaSmith

SWR Curve of 40 Meter Loop Antenna
Measured by Timewave TZ-900 AntennaSmith



Impedance Curve of 40 Meter Loop Antenna
Measured by Timewave TZ-900 AntennaSmith

MFJ-225 Graphical Antenna Analyzer



mini Radio Solutions miniVNA PRO and Extender

miniVNA PRO at Antenna Feedpoint

SDR-Kits DG8SAQ Vector Network Analyzer

SDR-Kits Vector Network Analyzer and Accessories

Bridge Instruments for Measuring Impedance

- **Antenna analyzers**

American QRP Micro908	$250
Anritsu Site Master S361E	$8,500
Autek Research VA1	$200
AEA Technology VIA Bravo	$1,300
Comet CAA-500	$420
Kuranishi Instruments BR-210	$500
MFJ-259B to MFJ-269 Pro	$240 to $400
MFJ-225 Graphical Analyzer	$370
Palstar Inc. ZM30	$400
Rig Expert, 7 models	$270 to $1190
Timewave AntennaSmith TZ-900	$1,000

- **Noise bridge (less accurate)**

MFJ-202B	$80
Make your own. See *QST*, Aug. 1989	

- **Impedance analyzer (1 port)**

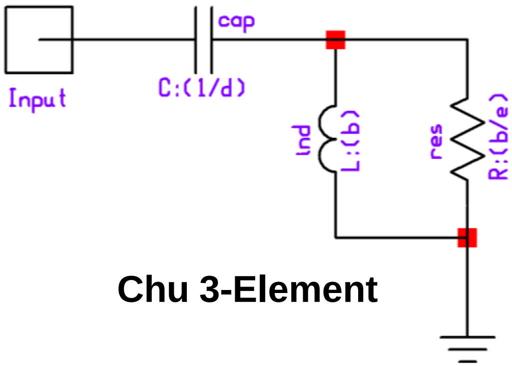
Agilent, 4 models	$28k to $48k

- **Network analyzer (2+ ports)**

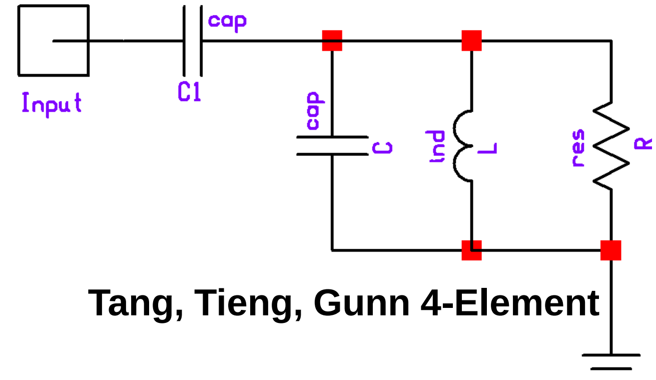
mini Radio Solutions miniVNA	$400 to $550
SDR-Kits VNWA3, various accessories	$560 to $740
Agilent, 4 models	$29k to $100k

Antenna Impedance Models

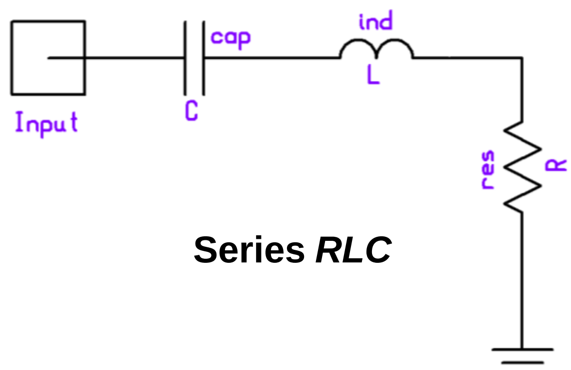
Impedance Models for Electrically-Small Dipoles & Monopoles



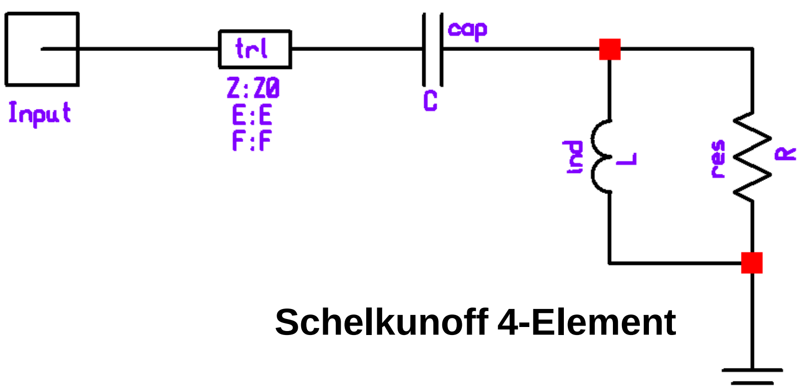
Chu 3-Element



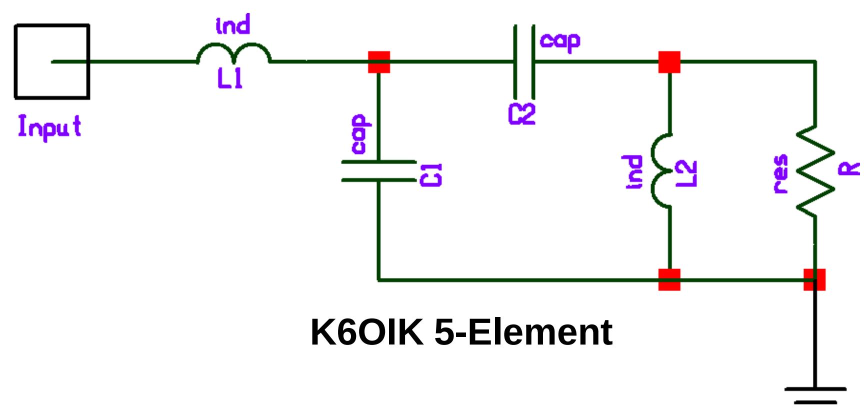
Tang, Tieng, Gunn 4-Element



Series *RLC*

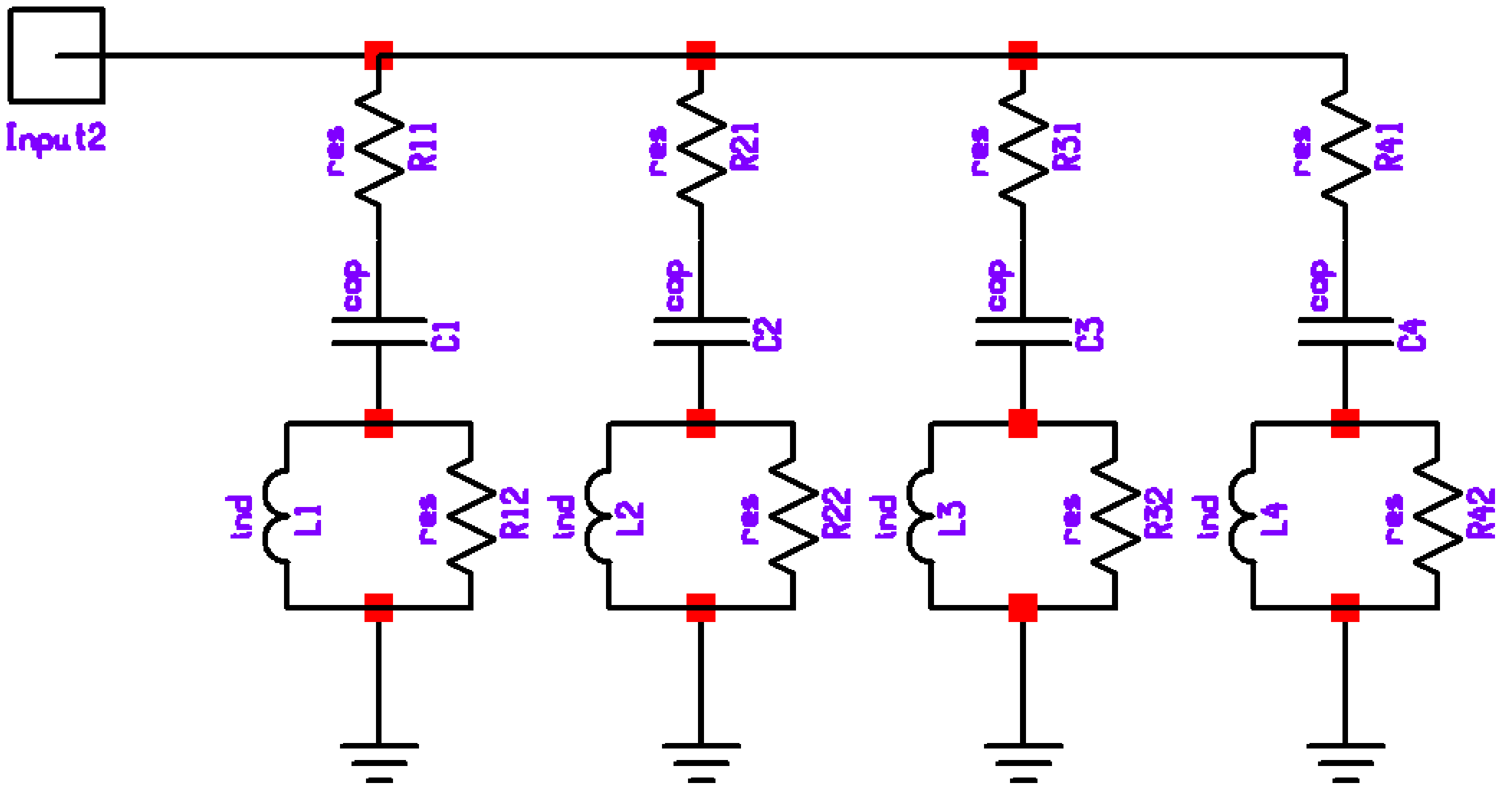


Schelkunoff 4-Element



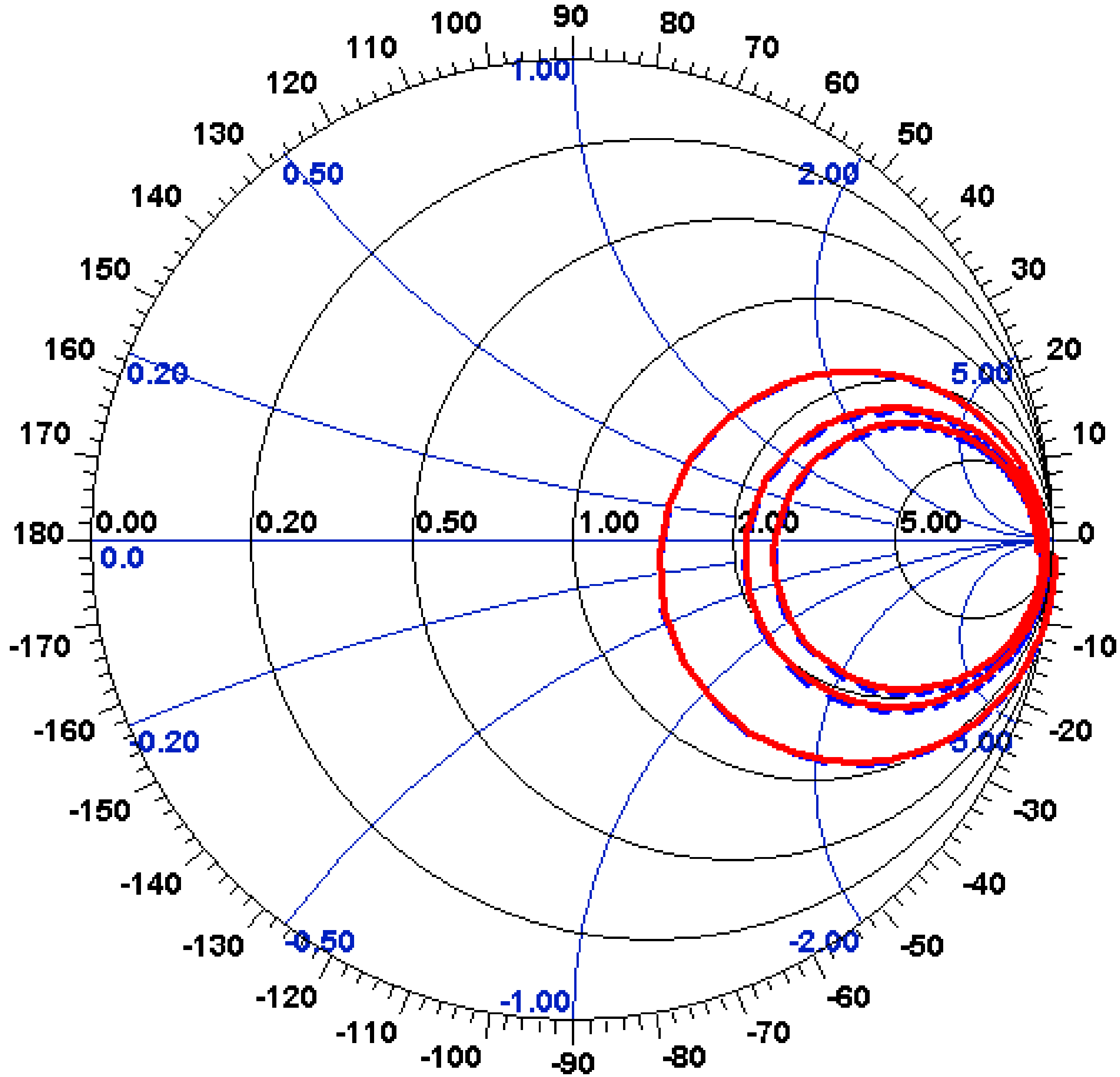
K6OIK 5-Element

Example 1: K6OIK's Broadband Equivalent Circuit for 98-ft Dipole from d-c to 30 MHz

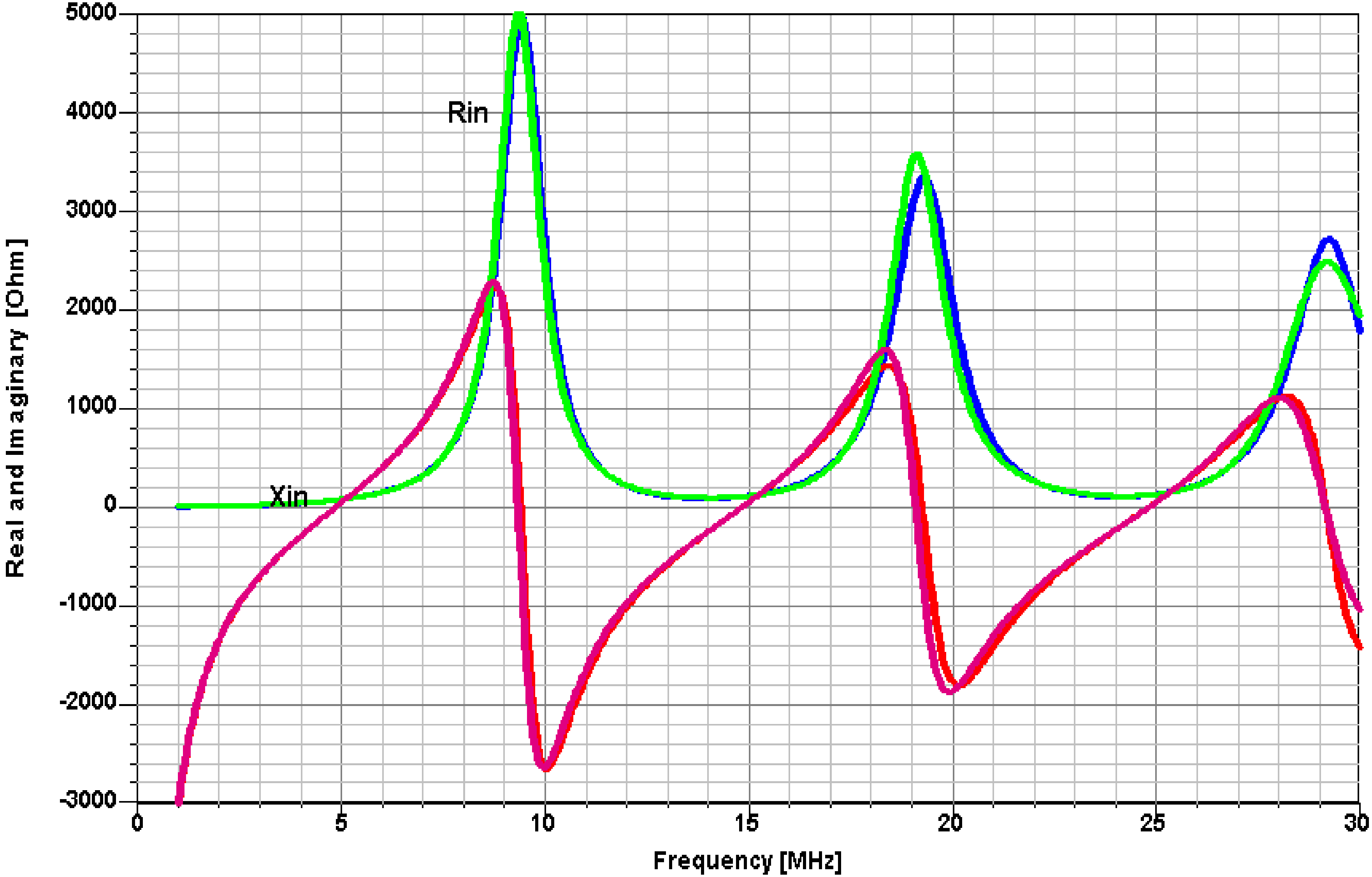


$R11 = 5.06\ \Omega$	$R21 = 0\ \Omega$	$R31 = 25.5\ \Omega$	$R41 = 0\ \Omega$
$C1 = 39.9$ pF	$C2 = 4.64$ pF	$C3 = 4.69$ pF	$C4 = 1.68$ pF
$L1 = 27.1\ \mu$H	$L2 = 24.9\ \mu$H	$L3 = 2.26\ \mu$H	$L4 = 24.5\ \mu$H
$R12 = 10.1$ kΩ	$R22 = 50.1$ kΩ	$R32 = 2.68$ kΩ	$R42 = 116$ kΩ

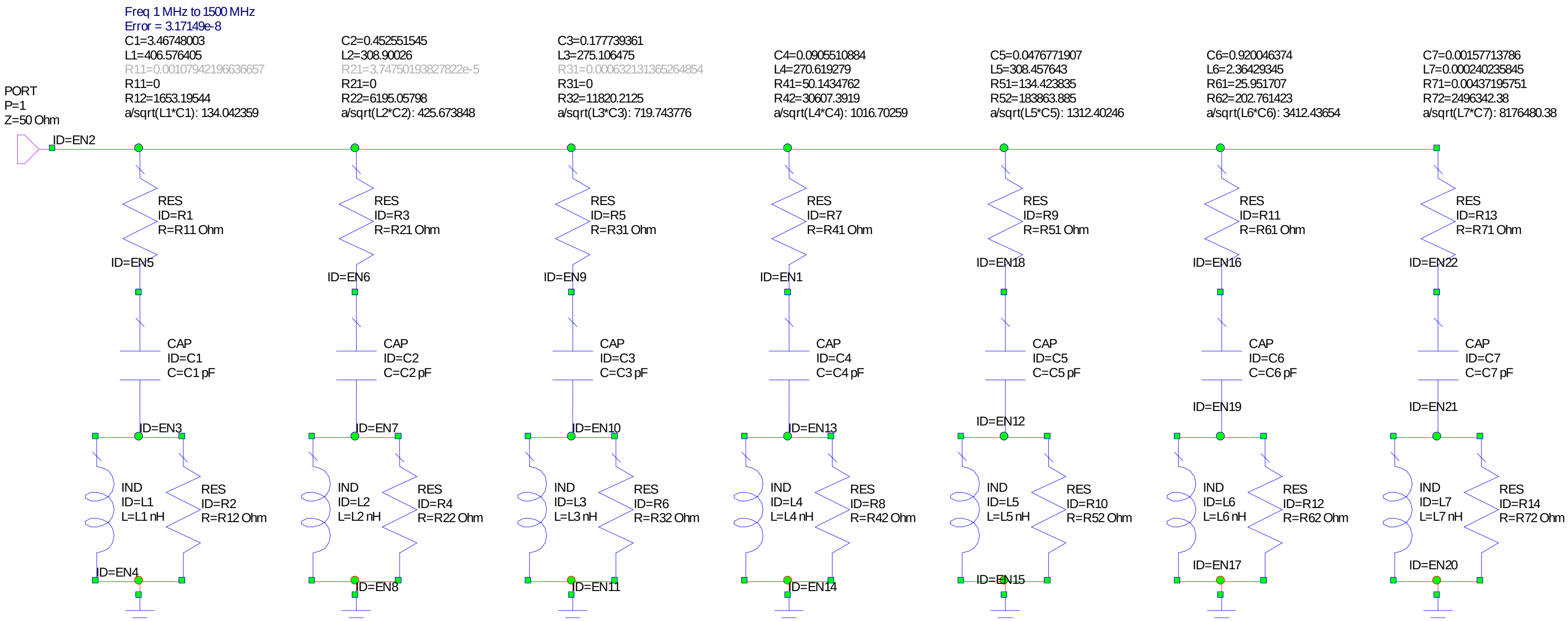
Performance of Equivalent Circuit



Impedance of Equivalent Circuit

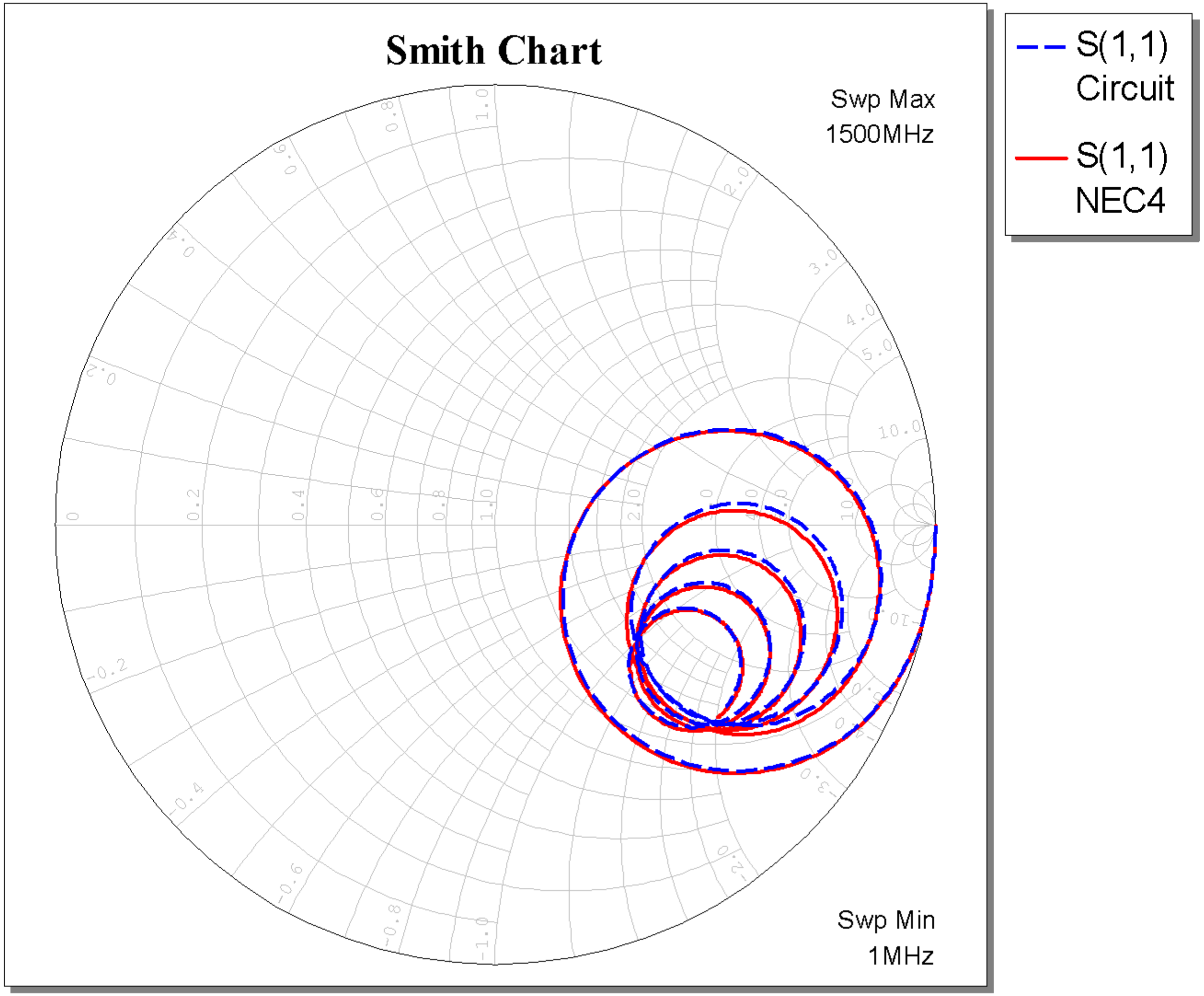


Example 2: K6OIK Broadband Equivalent Circuit for 2-meter Dipole (*L/d* = 50) from *d-c* to 1.5 GHz

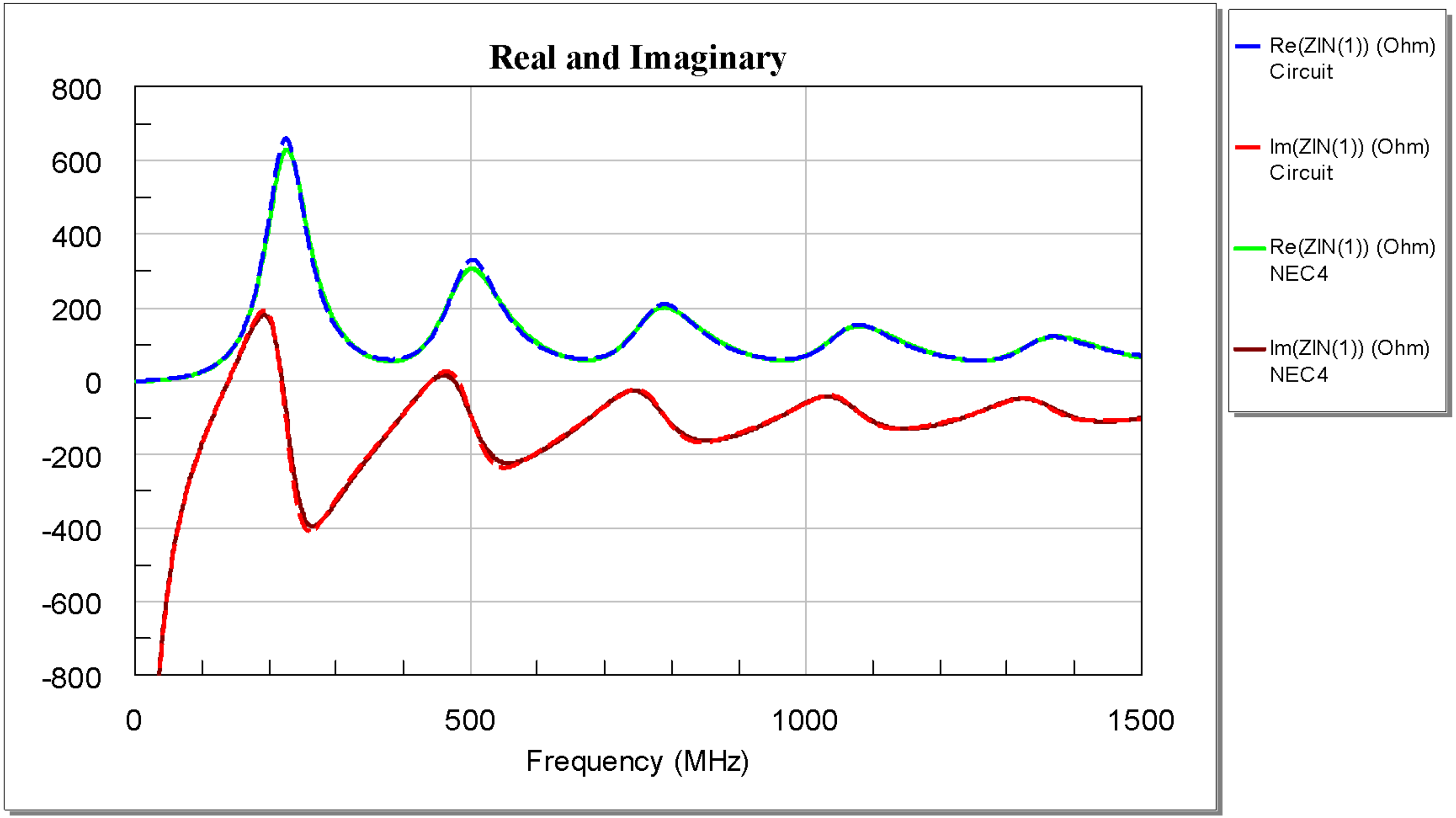


- **Introduced by the author (2007)**
- **Partial fraction expansion of dipole admittance**
- **A modification of Foster's 2nd canonical form**
- **More accurate than other broadband equivalent circuits for dipoles, viz. Hamid-Hamid (1997), Rambabu-Ramesh-Kalghatgi (1999), and Streable-Pearson (1981)**
- **Six stages sufficient to cover *d-c* to 1.5 GHz**

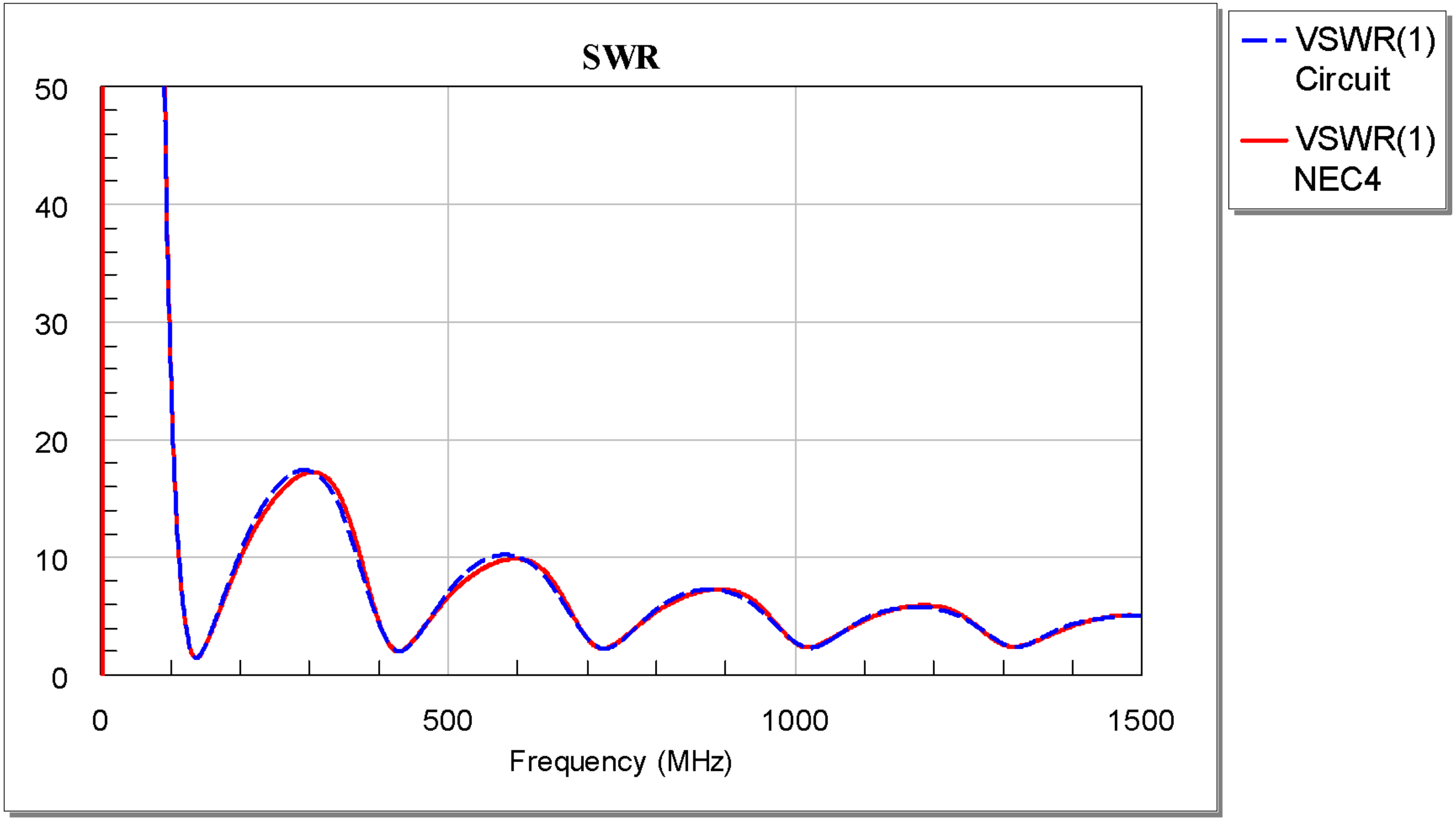
Performance of Equivalent Circuit



Impedance of Equivalent Circuit



SWR of Equivalent Circuit



Pioneers of Antenna Q and Impedance Matching

Harold Alden Wheeler
1903-1996

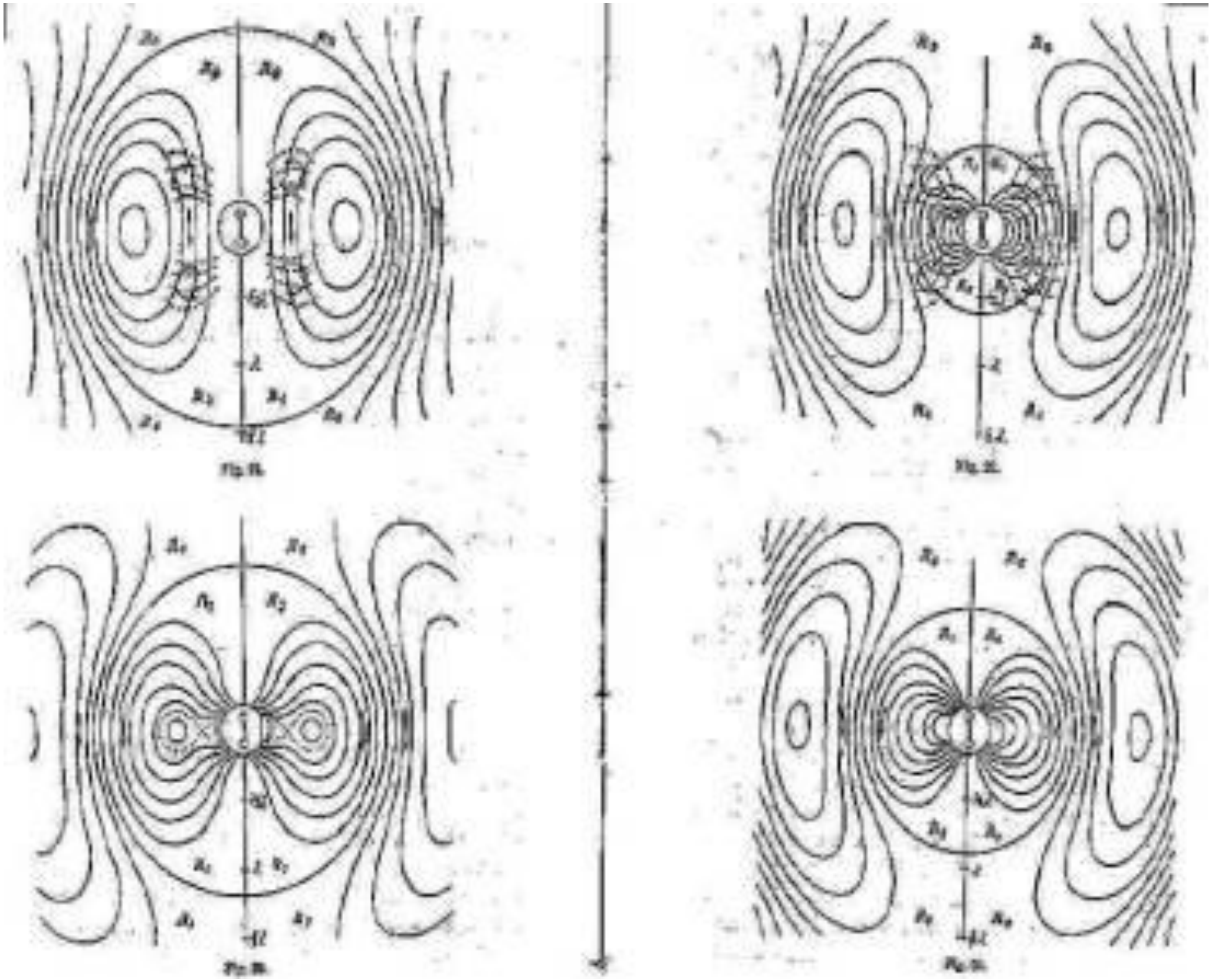
Lan Jen Chu
1913-1973

Hendrick Wade Bode
1905-1982

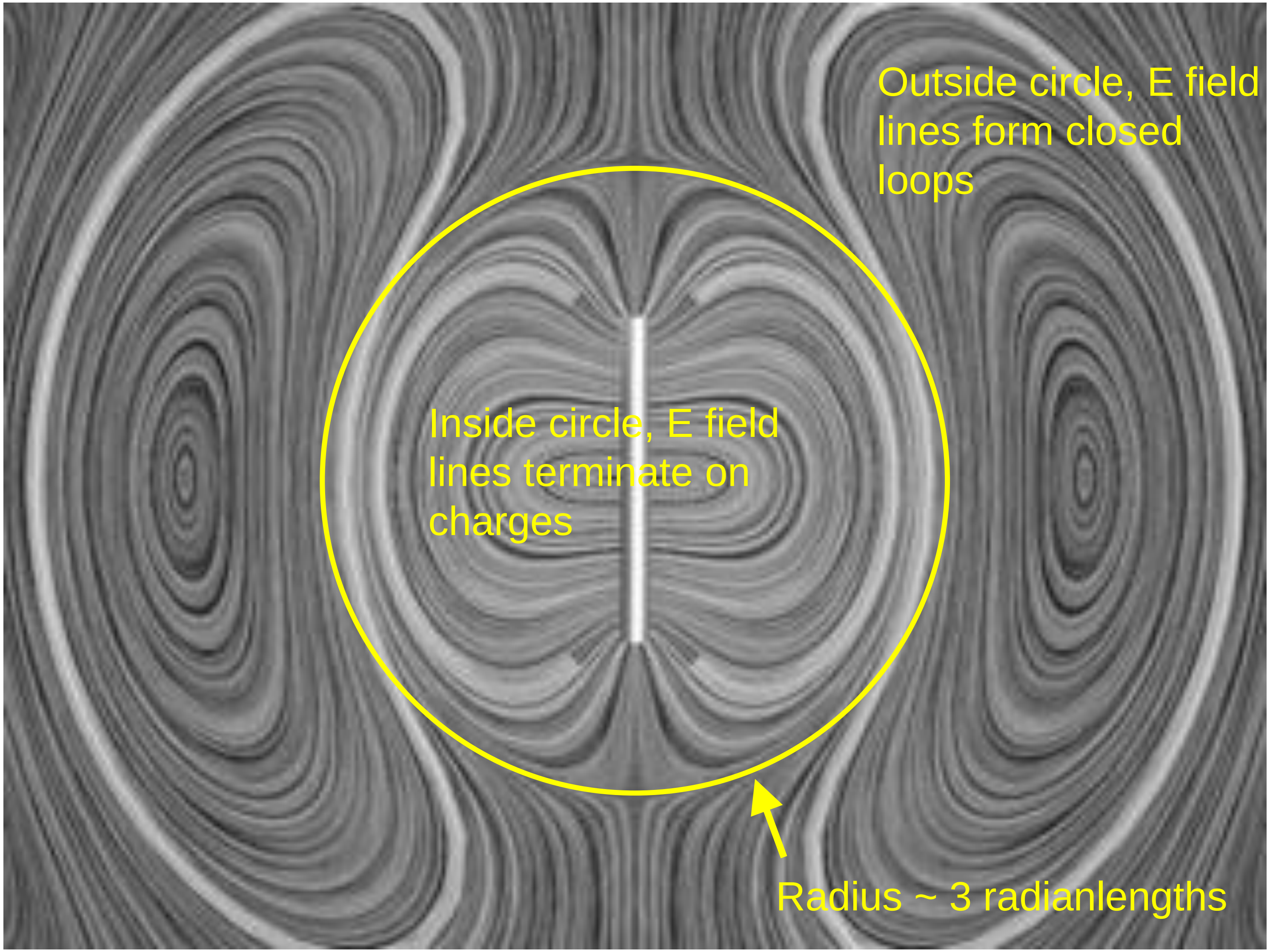
Robert Mario Fano
1917-

Antenna Q

Electric Fields of a Dipole
Heinrich Hertz (1889)



Generation of Dipole Fields



Approximations to Q from Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

If feedpoint loading or matching is allowed, Q cannot be computed from Z. Q must be computed directly from field formulas!

Q – Fundamental Definition

$$Q = 2\pi \left(\frac{\text{Energy stored}}{\text{Energy radiated per cycle}} \right)$$

$$= 2\pi f \left(\frac{\text{Energy stored}}{\text{Power radiated}} \right)$$

$$= \frac{\omega U_{stored}}{P_{radiated}}$$

$$\leq \frac{2\omega \max\{U_E, U_H\}}{P_{radiated}}$$

Electric or magnetic field energy, whichever is greater

Electric and Magnetic Fields of a Dipole

- **Fields of dipole source in free space, or monopole over perfect electrical conductor (PEC) plane**

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{kI_0 l\cos\theta}{4\pi r}\left(\frac{2}{kr}\right)\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Near field terms

One *radianlength* defined as $r = 1/k = \lambda/2\pi$ is the distance at which far field and near field terms are equal.

How Not to Compute the Q of a Dipole
Method "A" – Using Total Field Energy

$$Q = 2\pi \left(\frac{\text{Energy stored}}{\text{Energy radiated per cycle}} \right)$$

$$= 2\pi f \frac{\frac{1}{4} \iiint\limits_{\text{All space}} (\varepsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu_0 \mathbf{H} \cdot \mathbf{H}^*) dV}{\frac{1}{2} I_0^2 R_{rad}}$$

$$= \frac{\pi f}{I_0^2 R_{rad}} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} (\varepsilon_0 \mathbf{E} \cdot \mathbf{E}^* + \mu_0 \mathbf{H} \cdot \mathbf{H}^*) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \infty$$

How Not to Compute Dipole Q
Method "B" – Using Near Field Terms Only

$$U_H = \frac{\mu_0}{4} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \mathbf{H}\cdot\mathbf{H}^* r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\mu_0}{4} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \left| j \frac{k I_0 l \sin\theta}{4\pi r} \left[\frac{1}{jkr} \right] e^{-jkr} \right|^2 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\mu_0 I_0^2 l^2}{64\pi^2} \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{1}{r^2} \sin^3\theta \, dr \, d\theta \, d\phi$$

$$= \frac{\mu_0 k^2 I_0^2 l^2}{32\pi} \int_0^{\infty} \frac{1}{r^2} dr \int_0^{\pi} \sin^3\theta \, d\theta$$

$$= \frac{3\mu_0 k^2 I_0^2 l^2}{8\pi} \int_0^{\infty} \frac{1}{r^2} dr$$

$$= \infty$$

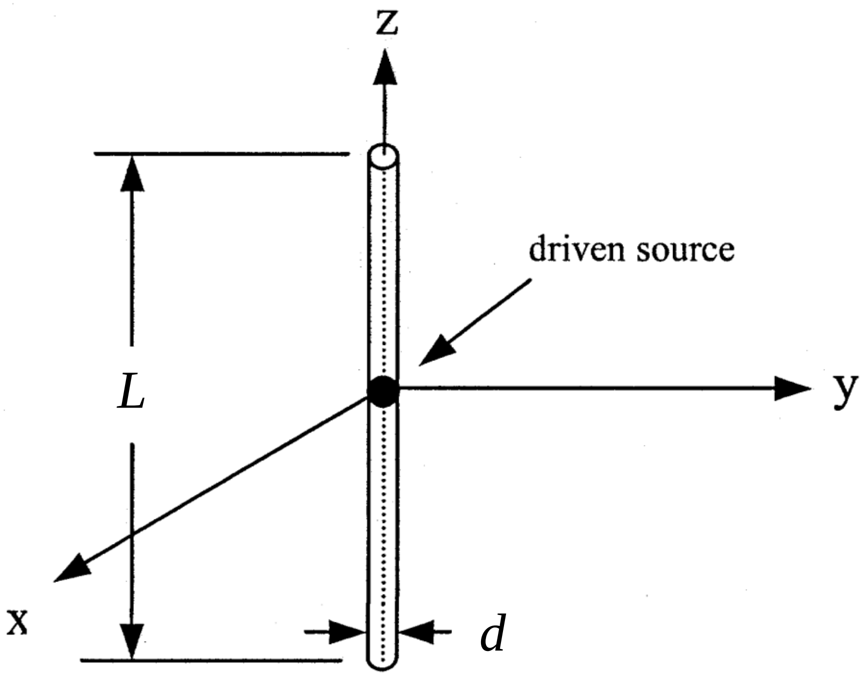
How to Calculate the Q of a Dipole Correctly

- **Method “A” fails because it incorrectly includes radiated field energy as stored energy**
- **Method “B” fails because it does not include interactions between radiated and reactive field components**
- **“Stored” energy means reactive energy associated with the imaginary part of the Poynting vector, not total field energy**
- **Stored energy is total field energy minus energy that is being transported via radiation**

$$U_{stored} = U_{total} - U_{radiation}$$
$$= \infty - \infty$$

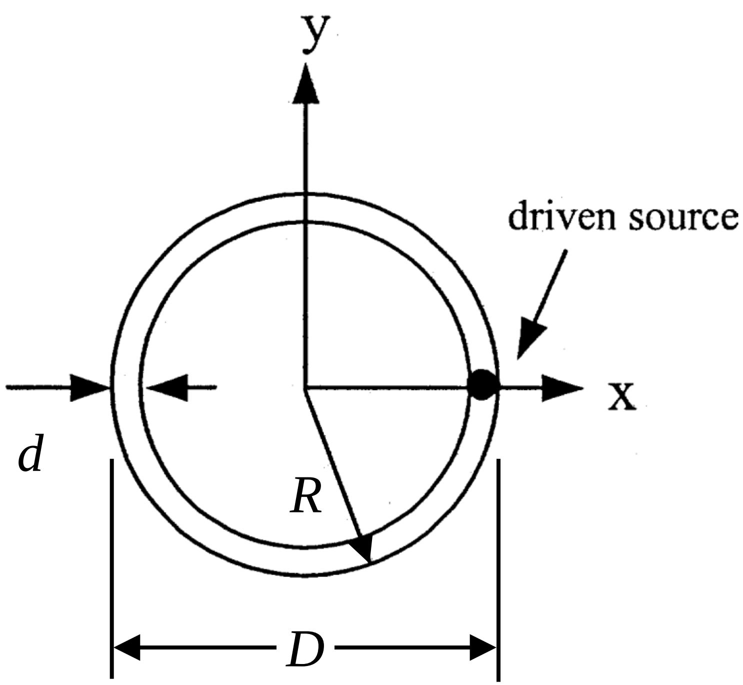
- **Determining the difference between two infinite quantities requires care**

Q of Small Dipole from Electromagnetic Field Analysis



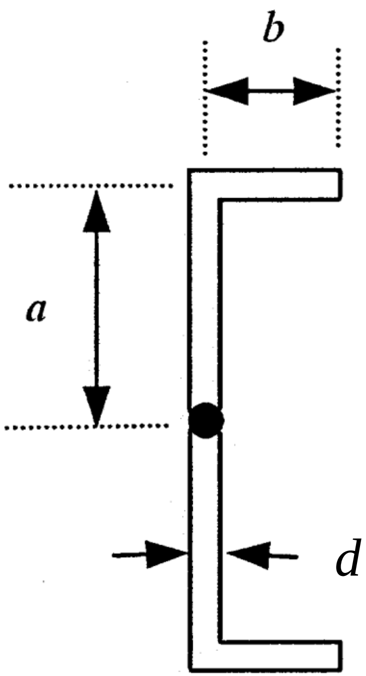
$$Q_{dipole} \approx \frac{6\left[\ln\left(\frac{L}{d}\right)-1\right]}{\pi^3\left(\frac{f\,L}{c}\right)^3}$$

Q of Small Loop from Electromagnetic Field Analysis



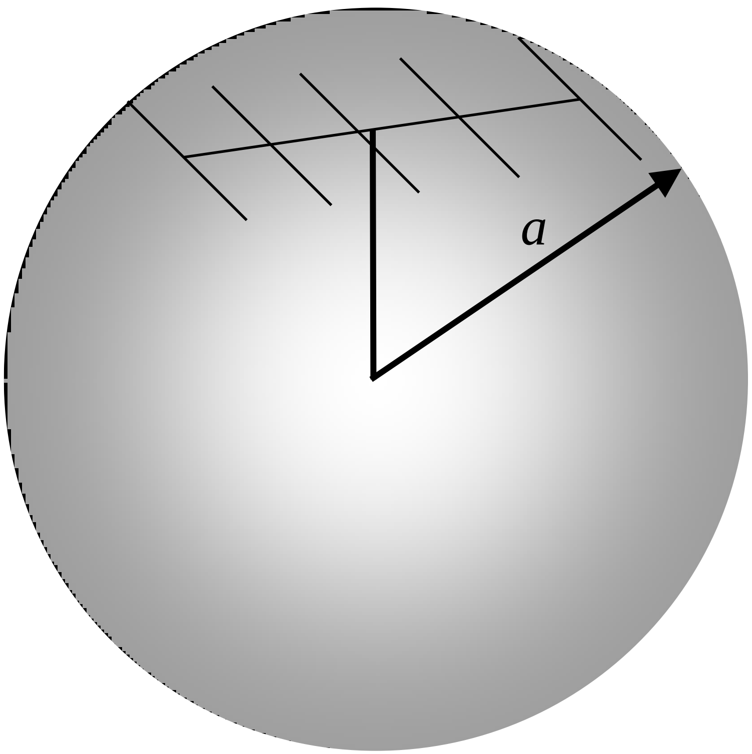
$$Q_{loop} \approx \frac{6 \ln\left(\frac{D}{d}\right)}{\pi^4 \left(\frac{f\,D}{c}\right)^3}$$

Q of Inverted-L from Electromagnetic Field Analysis



$$Q_{inverted-L} \approx \left(\frac{3}{4}\right) \frac{\left[\ln\left(\frac{2a}{d}\right)-1\right] + \frac{b}{a}\left[\ln\left(\frac{4b}{d}\right)-1\right]}{\pi^3\left(\frac{f}{c}\right)^3 a(a+b)^2}$$

Fundamental Bounds on Antenna Q



Smallest sphere that circumscribes antenna

- **Chu (1948)**

$$Q_{Chu} \geq \frac{1}{ka} + \frac{1}{(ka)^3}$$

- **Hansen and Collin (2007)**

$$Q_{New} \geq \frac{0.71327}{ka} + \frac{1.49589}{(ka)^3}$$

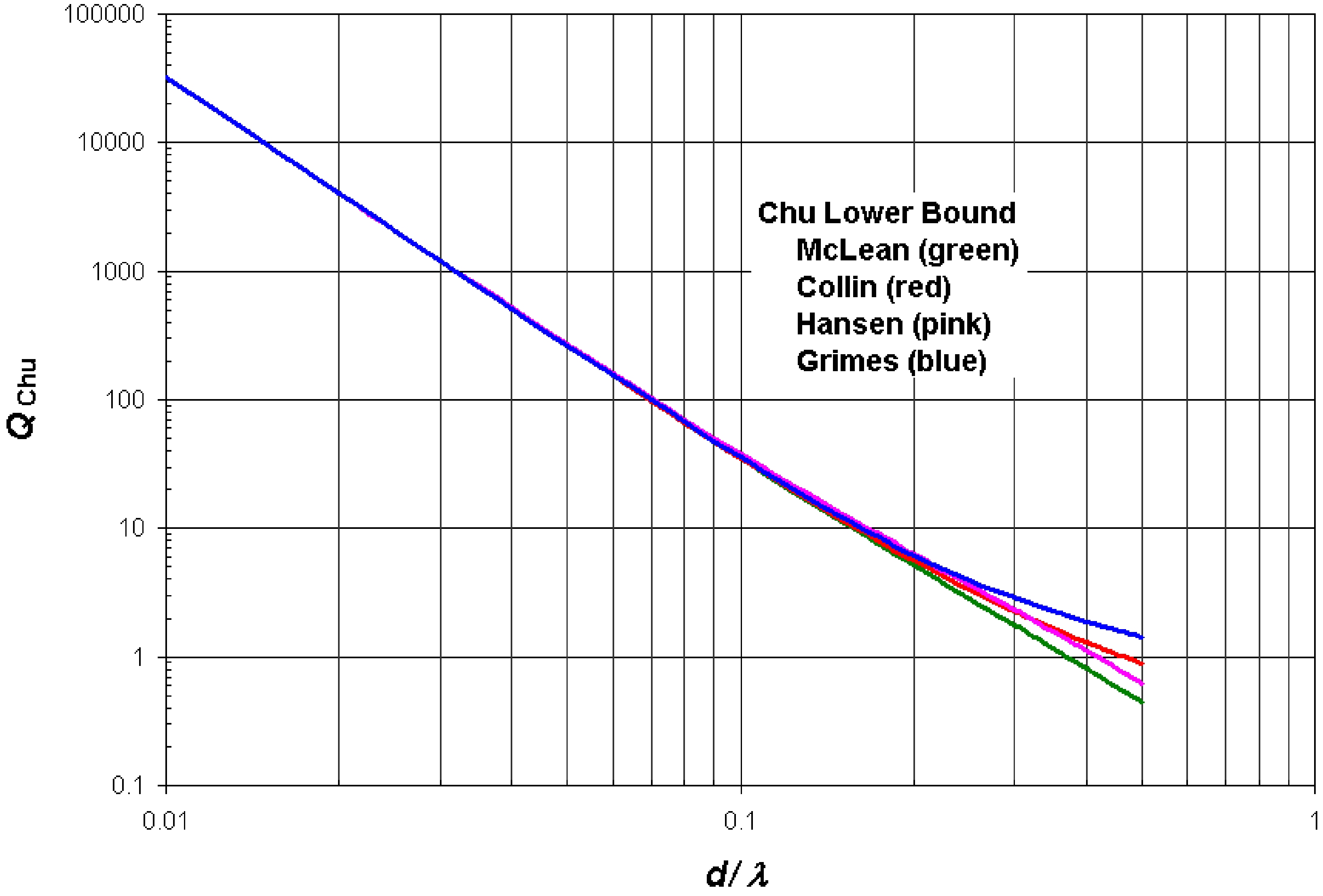
where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$a = \text{radius of sphere}$$

Reducing an antenna's size in half increases its Q by 8 and reduces its bandwidth by 8!

Chu Q


100000
10000
1000
100
10
1
0.1
0.01
0.1
1
Q_{Chu}
d/λ
Chu Lower Bound
McLean (green)
Collin (red)
Hansen (pink)
Grimes (blue)

Fano Bound on Match Bandwidth

R.M. Fano's Bound (1947)

- **Limits how well an arbitrary impedance can be matched by a passive lossless network of any complexity – even infinite**
- **Bounds the return loss bandwidth product**
  - Limits the SWR for a given bandwidth
  - Limits the match bandwidth for a given SWR
- **The bound is fundamental; it cannot be overcome**
- **But it can be bypassed by using match networks outside the class that the bound governs**
  - Not passive
  - Not lossless

Non-Foster match networks and reflectionless match networks are not subject to the Fano bound!

Fano's Bound (1947)

- **Bounds the area under the return loss curve of all passive lossless impedance-matching networks**

$$\int_0^\infty \log\left(\frac{1}{\rho(\omega)}\right) d\omega \le \min\{A_1, A_2, \cdots, A_n\}$$

where

$$\rho(\omega) = \left|\Gamma(\omega)\right| = \left|s_{11}(\omega)\right|$$

and A_1, ... , A_n are constants that depend on the load impedance function $Z_L(f)$

- **Proved in Fano's Ph.D. dissertation at MIT in 1947**
- **Republished in the *Journal of the Franklin Institute*, 1950**

Fano Bound for Matching Series RLC Loads

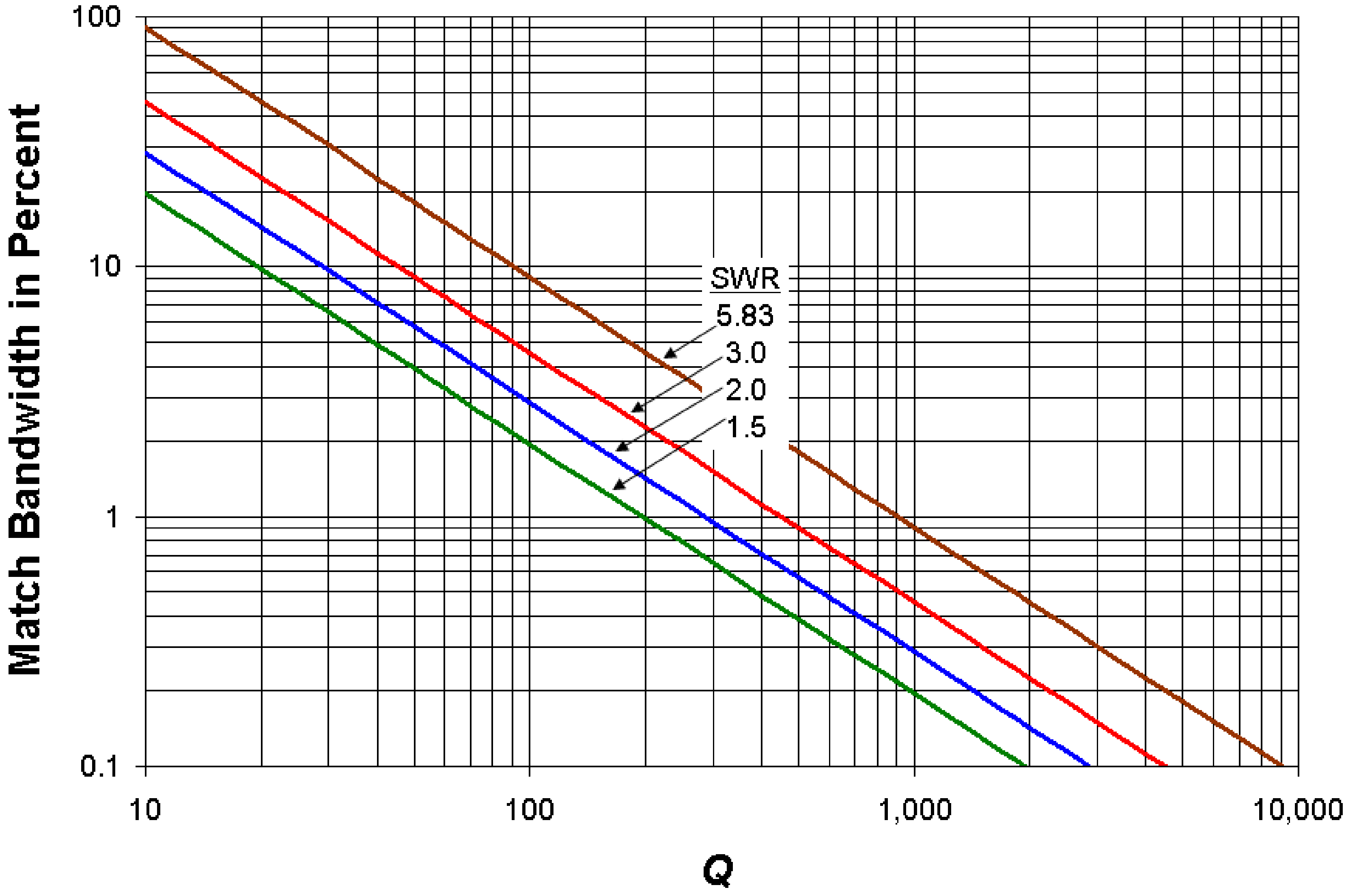
- **Worst-case SWR for a given match bandwidth assuming lossless matching of a series RLC load**

$$SWR \geq \frac{1}{\tanh\left(\dfrac{\pi f}{2QB_\infty}\right)}$$

- **Match bandwidth limit in terms of maximum SWR**

$$B_\infty \leq \frac{\pi f}{2Q \tanh^{-1}\left(\dfrac{1}{SWR}\right)}$$

Maximum Possible Match Bandwidth vs *Q*



Combined Chu-Fano Bound for Electrically Small Antennas

- **Maximum match bandwidth of small antennas assuming infinite match complexity**

$$B_\infty \leq \frac{4\pi^4 f \left(\frac{h}{\lambda}\right)^3}{\tanh^{-1}\left(\frac{1}{SWR}\right)}$$

where *h* is the monopole height or dipole half-length

- **Bandwidth limit for λ/20 monopoles and λ/10 dipoles**

$$B_\infty \leq \frac{f}{20.53\tanh^{-1}\left(\frac{1}{SWR}\right)}$$

Match Bandwidth of Finite-Order Match Networks
Fano's Exact Solution (1947)

- **Q-bandwidth product**

$$\frac{QB_n}{f} = \frac{2\sin\left[\frac{\pi}{2n}\right]}{\sinh(a) - \sinh(b)}$$

where a and b are found from two equations

$$\frac{\tanh(na)}{\cosh(a)} = \frac{\tanh(nb)}{\cosh(b)}$$

$$|\Gamma| = \frac{SWR - 1}{SWR + 1} = \frac{\cosh(nb)}{\cosh(na)}$$

- **The second equation is equivalent to**

$$SWR = \frac{\cosh(na) + \cosh(nb)}{\cosh(na) - \cosh(nb)}$$

Fano's exact solution requires numerical solution.

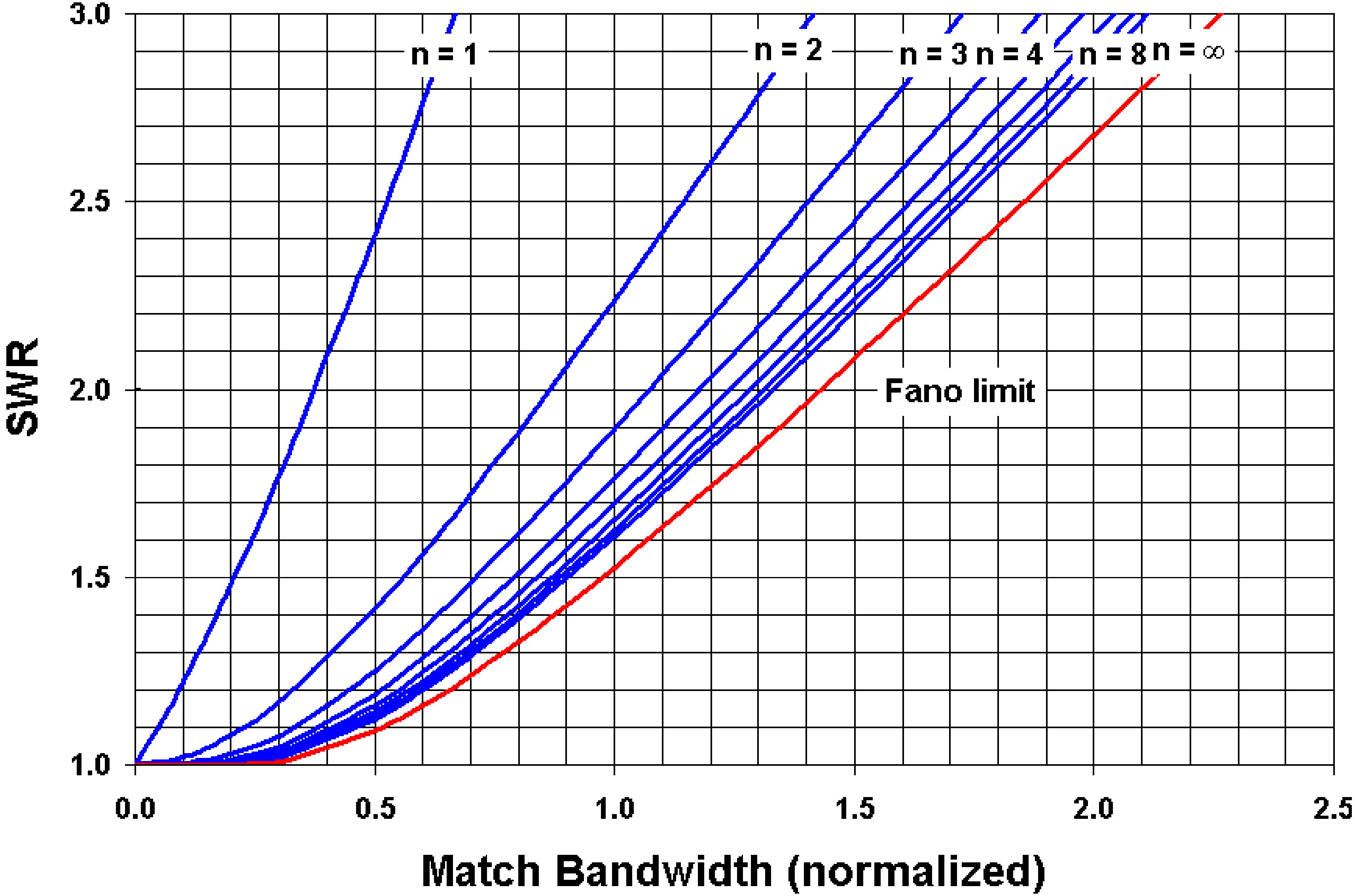
Match Bandwidth of Finite-Order Match Networks Lopez Approximate Solution (2004)

$$\frac{QB_n}{f} = \frac{1}{b_n \sinh\left[\frac{1}{a_n}\ln\frac{1}{|\Gamma|}\right] + (1-b_n)\frac{1}{a_n}\ln\frac{1}{|\Gamma|}}$$

a_n and b_n are tabulated coefficients (different from Fano's a and b)

n	a_n	b_n
1	1	1
2	2	1
3	2.413	0.678
4	2.628	0.474
5	2.755	0.347
6	2.838	0.264
7	2.896	0.209
8	2.937	0.160
∞	π	0

Matching with Chebyschev Networks of Finite Order



References (A Small List)

- H.A. Wheeler, "Fundamental Limitations of Small Antennas," *Proc. IRE*, Dec 1947
- L.J. Chu, "Physical Limitations of Omni-Directional Antennas," Tech Rept. 64, MIT, May 1948; also *J. Applied Physics*, Dec 1948
- R.E. Collin and S. Rothschild, "Evaluation of Antenna *Q*," *IEEE Trans. Antennas and Propagation*, Jan 1964
- R.L. Fante, "Quality Factor of General Ideal Antennas," *IEEE Trans. Antennas and Propagation*, Mar 1969
- R.C. Hansen, "Fundamental Limitations in Antennas," *Proc. IEEE*, Feb 1981
- R.E. Collin, "Minimum Q of Small Antennas," *J. Electromagnetic Waves and Applications*, 1998
- W. Geyi, P. Jarmuszewski, and Y. Qi, "The Foster Reactance Theorem for Antennas and Radiation *Q*," *IEEE Trans. Antennas and Propagation*, Mar 2000
- W. Geyi, "A Method for the Evaluation of Small Antenna Q," *IEEE Trans. Antennas and Propagation*, Aug 2003
- S.R. Best, "A Discussion on the Properties of Electrically Small Self-Resonant Wire Antennas," *IEEE Antennas and Propagation Mag*., Dec 2004
- A.D. Yaghjian and S.R. Best, "Impedance, Bandwidth, and *Q* of Antennas," *IEEE Trans. Antennas and Propagation*, Apr 2005
- R.C. Hansen and R.E. Collin, "A New Chu Formula for *Q*," *IEEE Antennas and Propagation Mag*., Oct 2009
- H.W. Bode, "A Method of Impedance Correction," *Bell Syst. Tech. J*., Oct. 1930
- R.M. Fano, *Theoretical Limitations on the Broadband Matching of Arbitrary Impedances*, Ph.D. dissertation, MIT, 1947; also *J. Franklin Inst*., Jan and Feb 1950
- A.R. Lopez, "Review of Narrowband Impedance-Matching Limitations," *IEEE Antennas and Propagation Mag*., Aug 2004

The End

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