Universal Equivalent Circuits for All Antennas

Stephen D. Stearns

Consulting Engineer
stearns@ieee.org

Abstract

Series and parallel RLC resonant circuits have long been the staple equivalent circuits for dipole and loop antennas despite being narrowband approximations. This paper shows how to make universal equivalent circuits for any antenna over any bandwidth. Part 1 introduces the history of classical electric network synthesis and Smith charts and reviews antenna impedance and admittance properties.

Part 2 explains the modes of vibration of continuous structures, natural frequencies, feedpoint current, and impedance resonances. The impedance function of any antenna can be accurately modeled by two of four universal equivalent circuits given computed or measured impedance data and a circuit optimizer. Examples of broadband universal equivalent circuits are shown for dipole, circular loop, and discone antennas over multi-octave and decade bandwidths.

Broadband equivalent circuits are useful for interpolating between data points and performing lab tests without radiating. 1-port equivalent circuits are useful for making dummy loads for reflection experiments or match network testing. 2-port equivalent circuits are useful for making emulators for transmission tests.

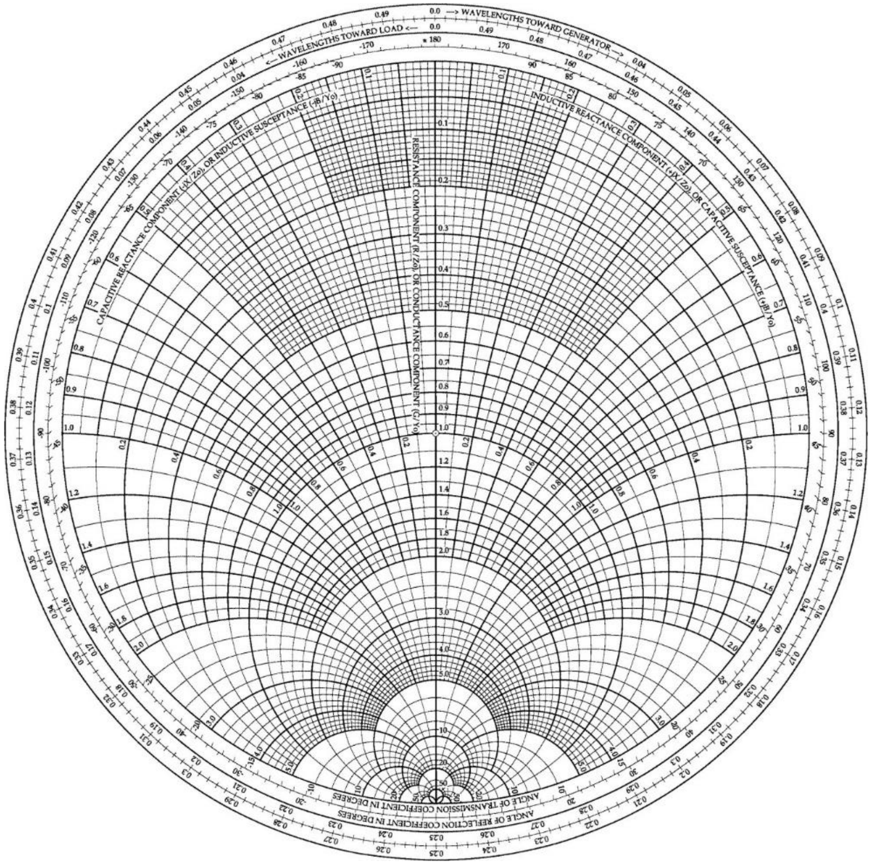
Topics

- The Smith chart – little known facts
- Results from classical network theory
- Antenna impedance functions
- Low-order narrowband equivalent circuits
- Defective equivalent circuits
- Universal equivalent circuits
- Examples demonstrating UEC theory

The Smith Chart

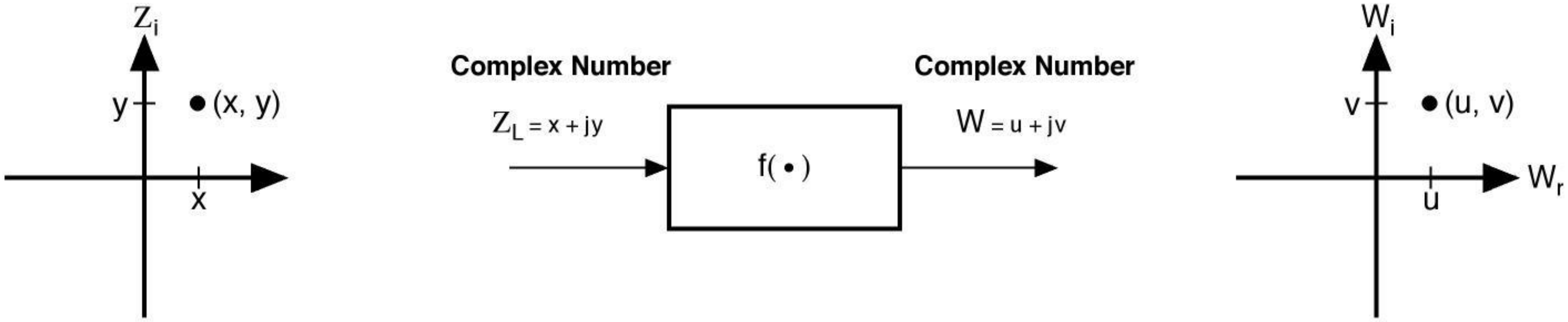
The Smith Chart

Phillip Hagar Smith, 1905-1987

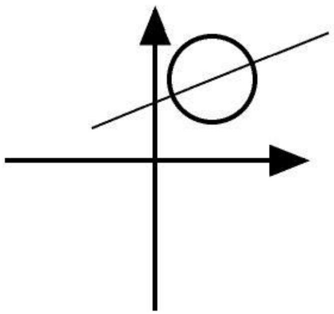


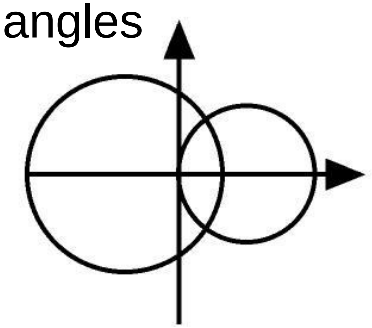
Developed by Phillip H. Smith at Bell Labs 1936
Published in *Electronics*, Jan. 1939 and Jan. 1944
Mrs. Smith sold copyright to IEEE MTT-S in 2015

Complex Functions

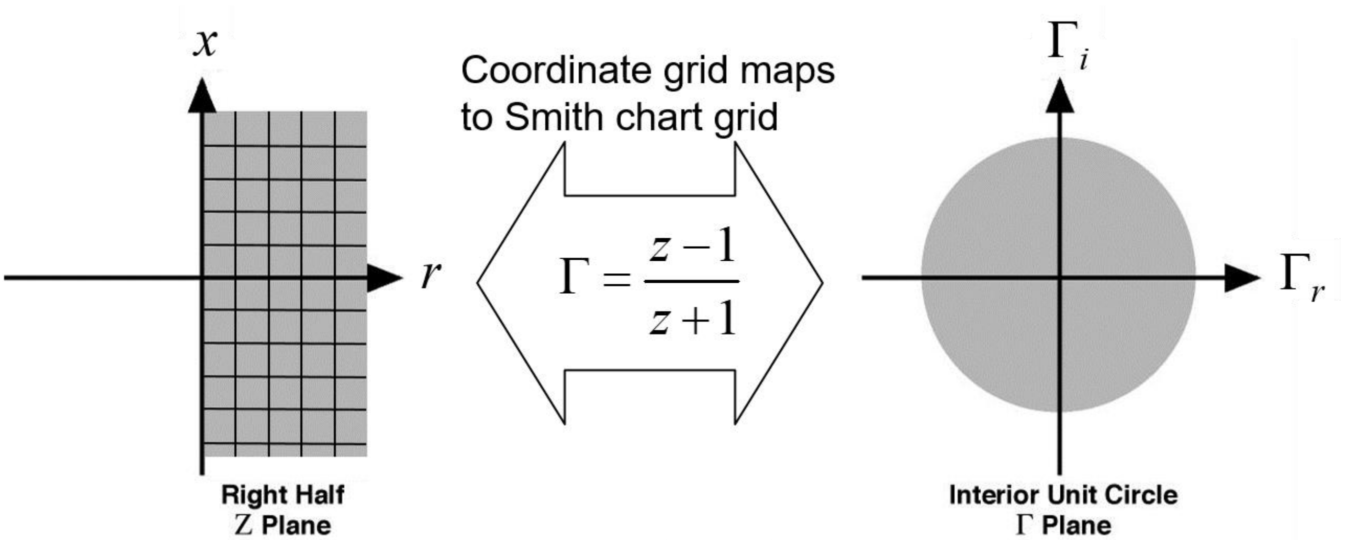


- **Basic types of complex functions**
  - Global Properties
    - Linear – lines map to lines
    - Bilinear – circles map to circles
  - Local Properties
    - Conformal – right angles map to right angles





Smith Chart Coordinate Grid

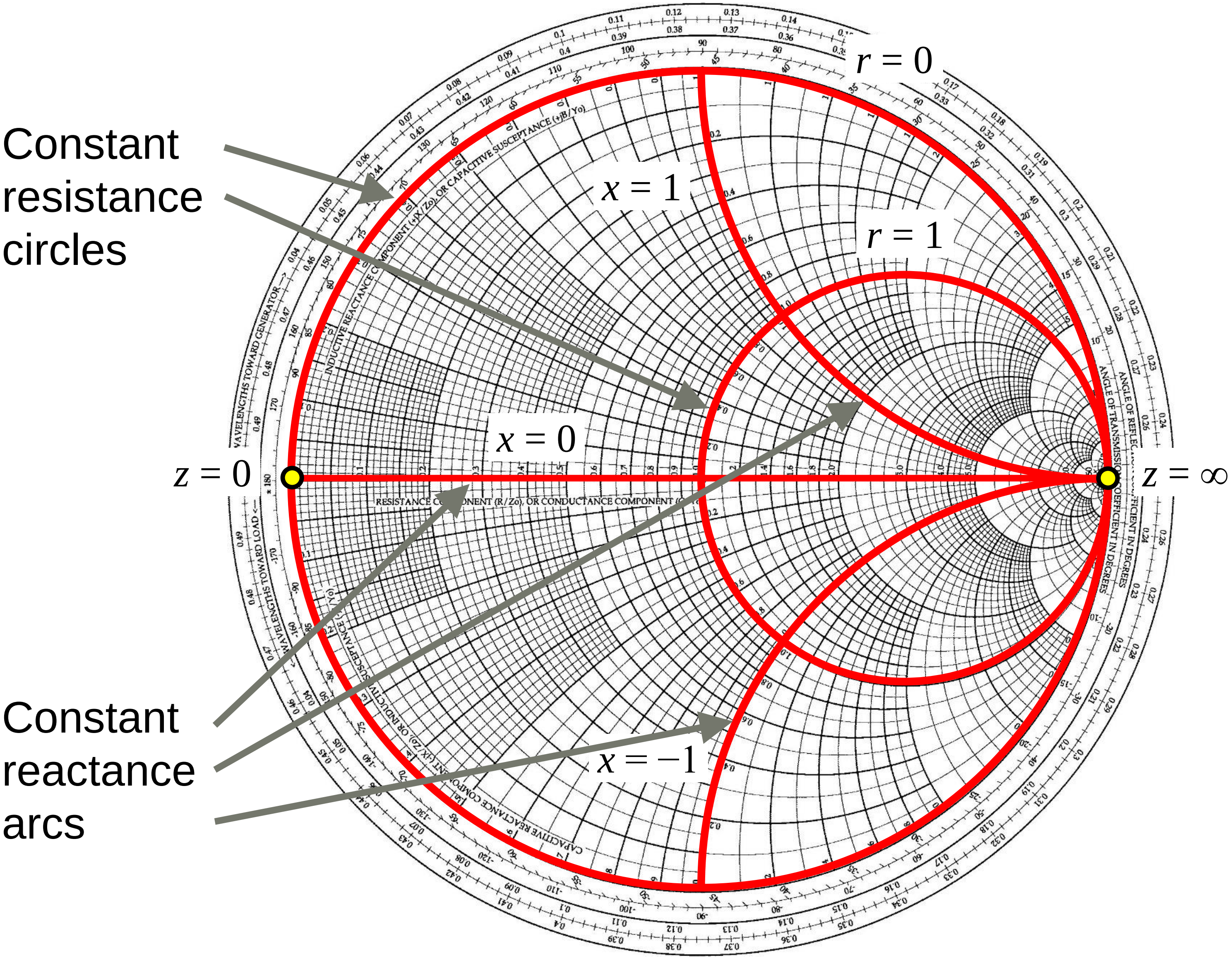


$$\Gamma_r + j\Gamma_i = \frac{(r-1)+jx}{(r+1)+jx}$$

$$\Gamma_r = \frac{r^2+x^2-1}{(r+1)^2+x^2}$$

$$\Gamma_i = \frac{2x}{(r+1)^2+x^2}$$

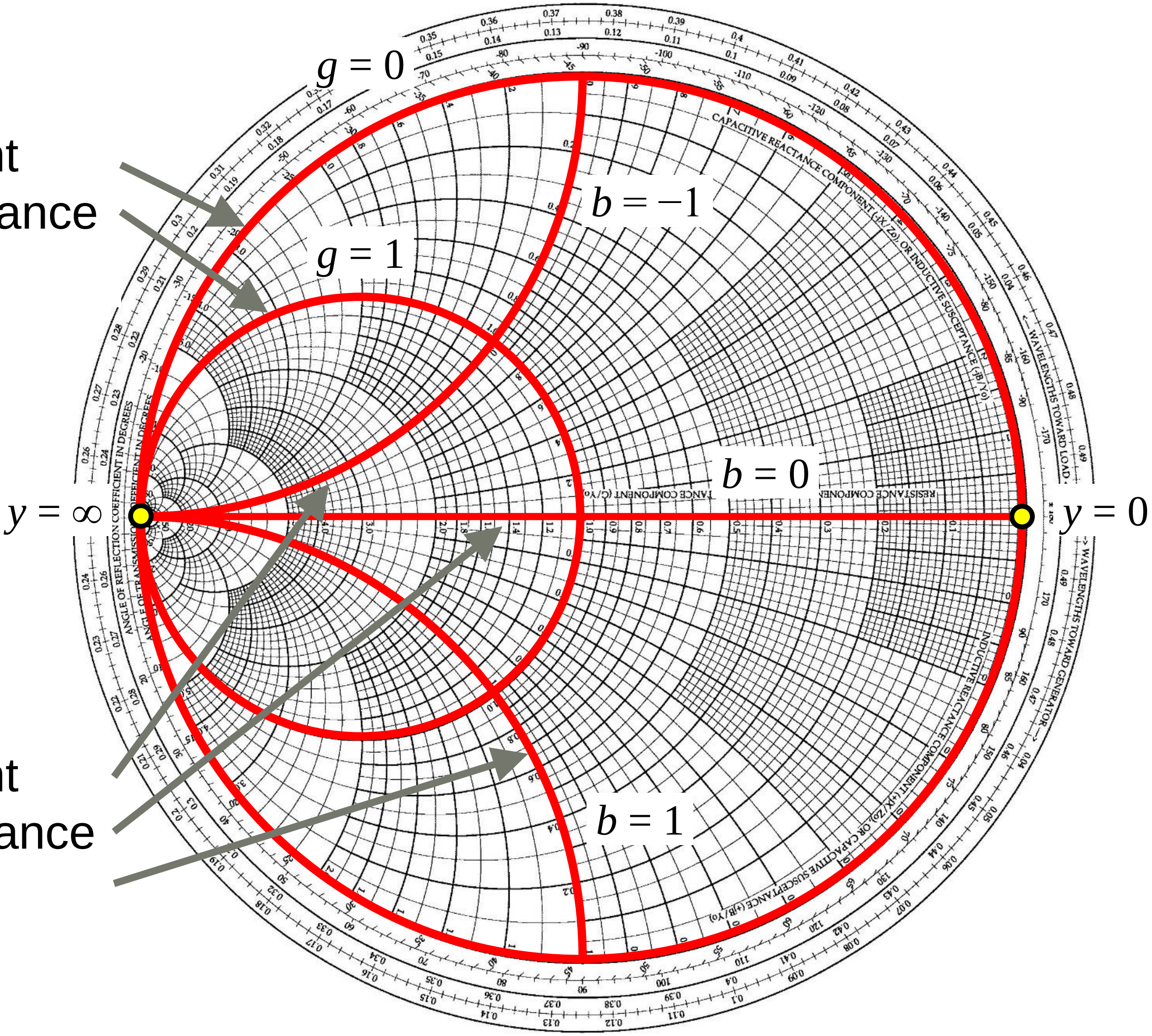
Impedance Coordinates



Constant resistance circles

Constant reactance arcs

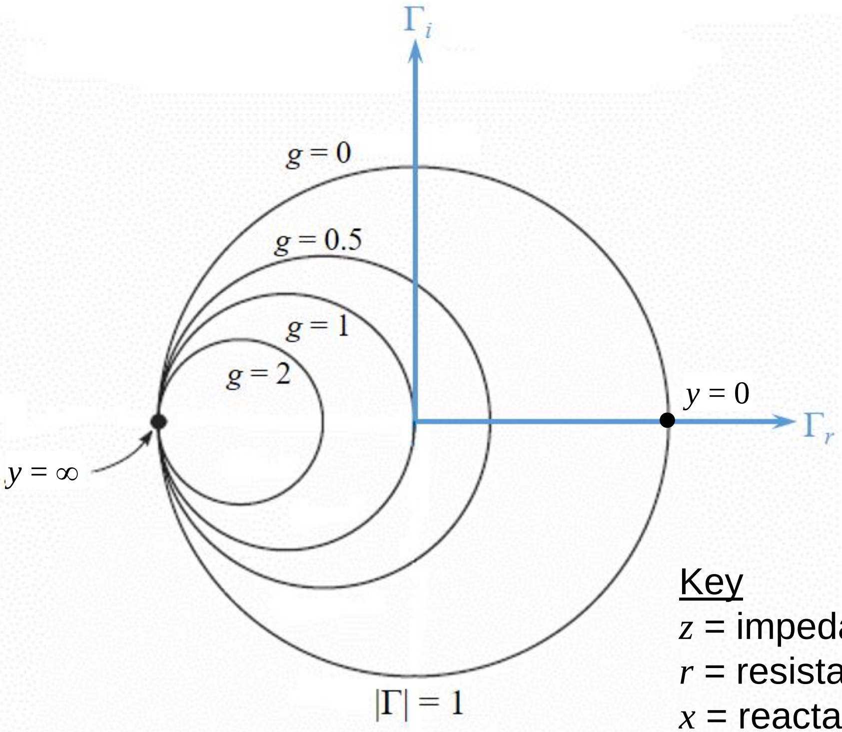
Admittance Coordinates

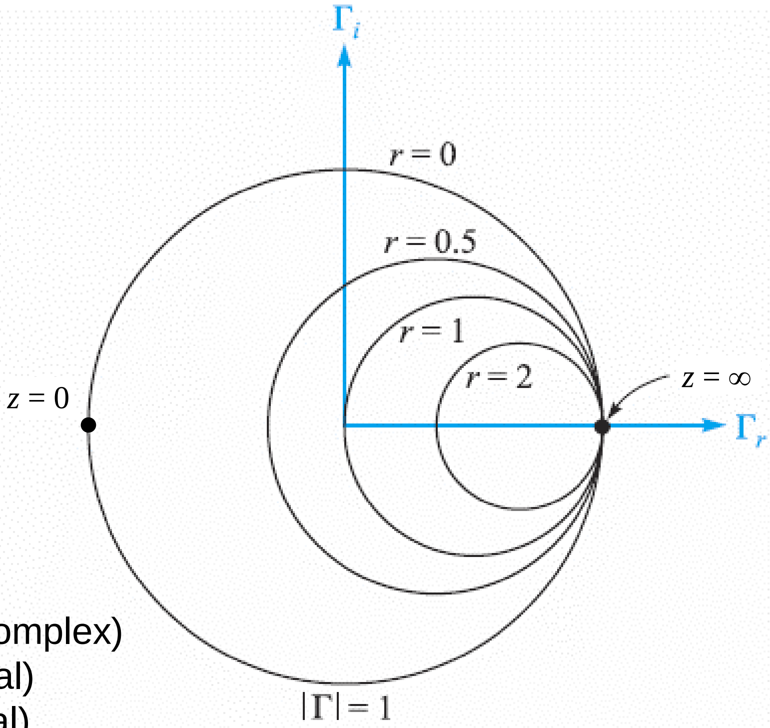


Constant conductance circles

Constant susceptance arcs

Conductance and Resistance Circles
Admittance and Impedance Zero and Infinity Points

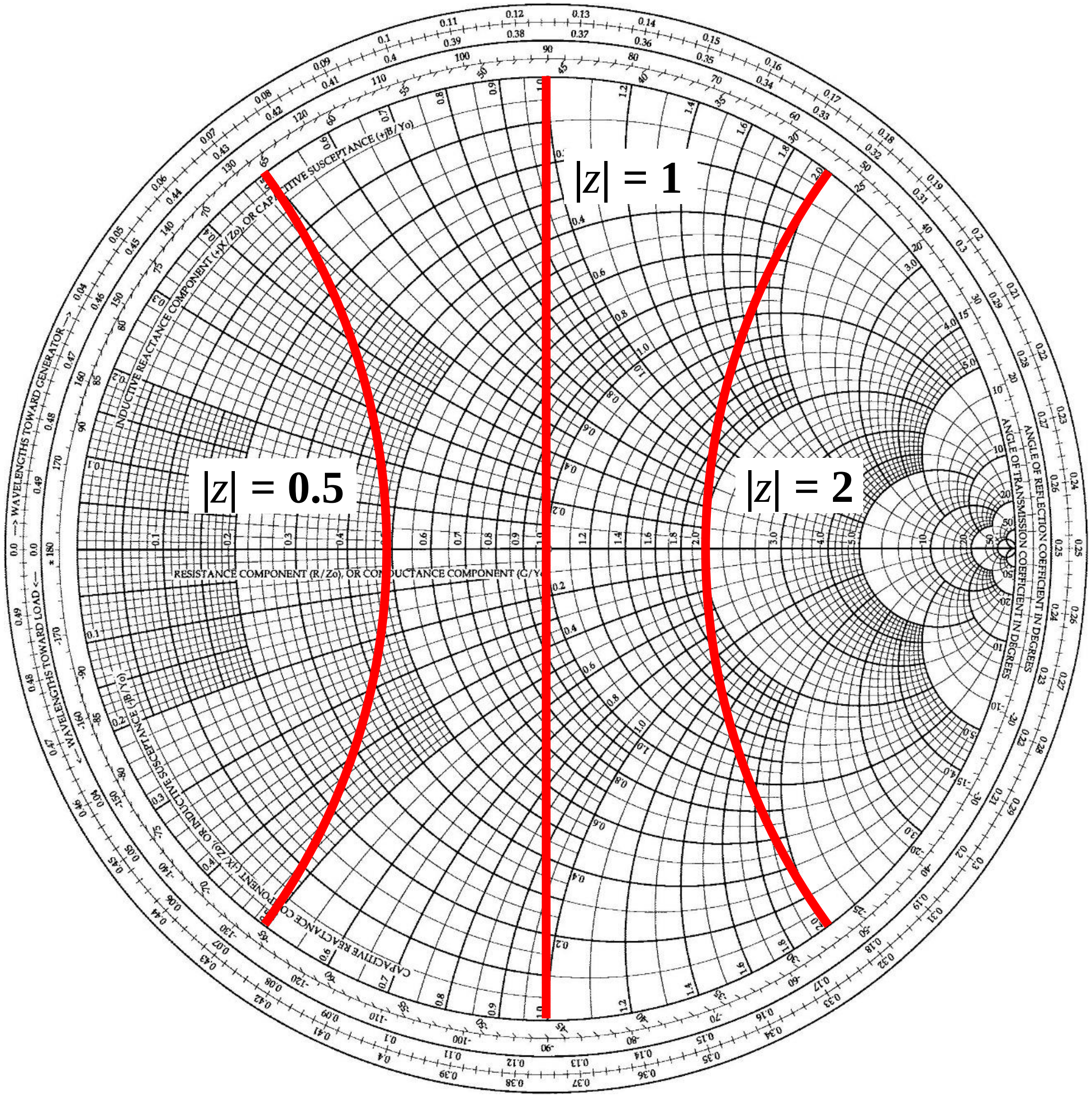




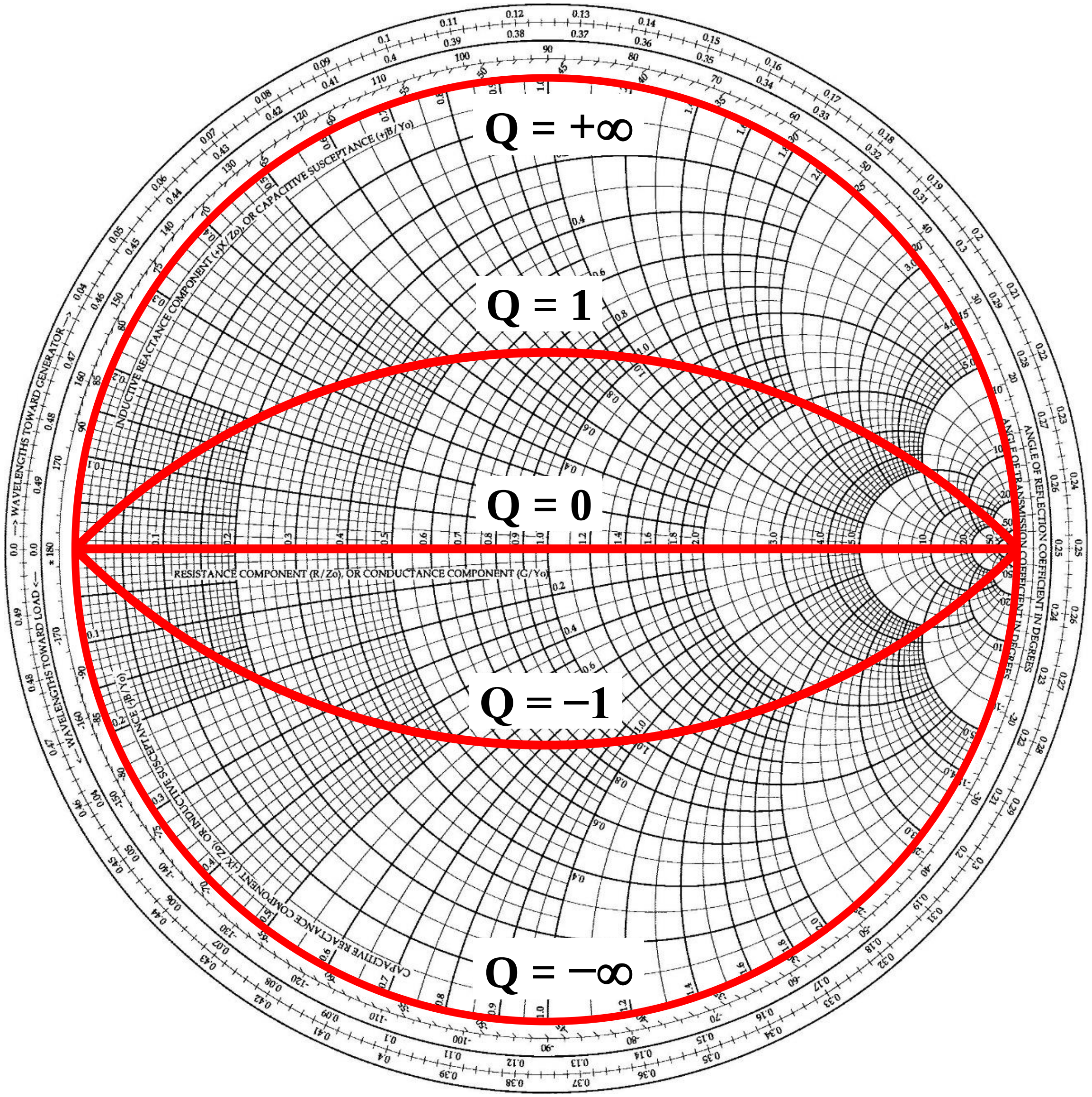
Key

z = impedance (complex)
r = resistance (real)
x = reactance (real)
y = admittance (complex)
g = conductance (real)
b = susceptance (real)
Γ= reflection coefficient (complex)
Γ_r = real part of Γ (real)
Γ_i = imaginary part of Γ (real)
lower case = normalized

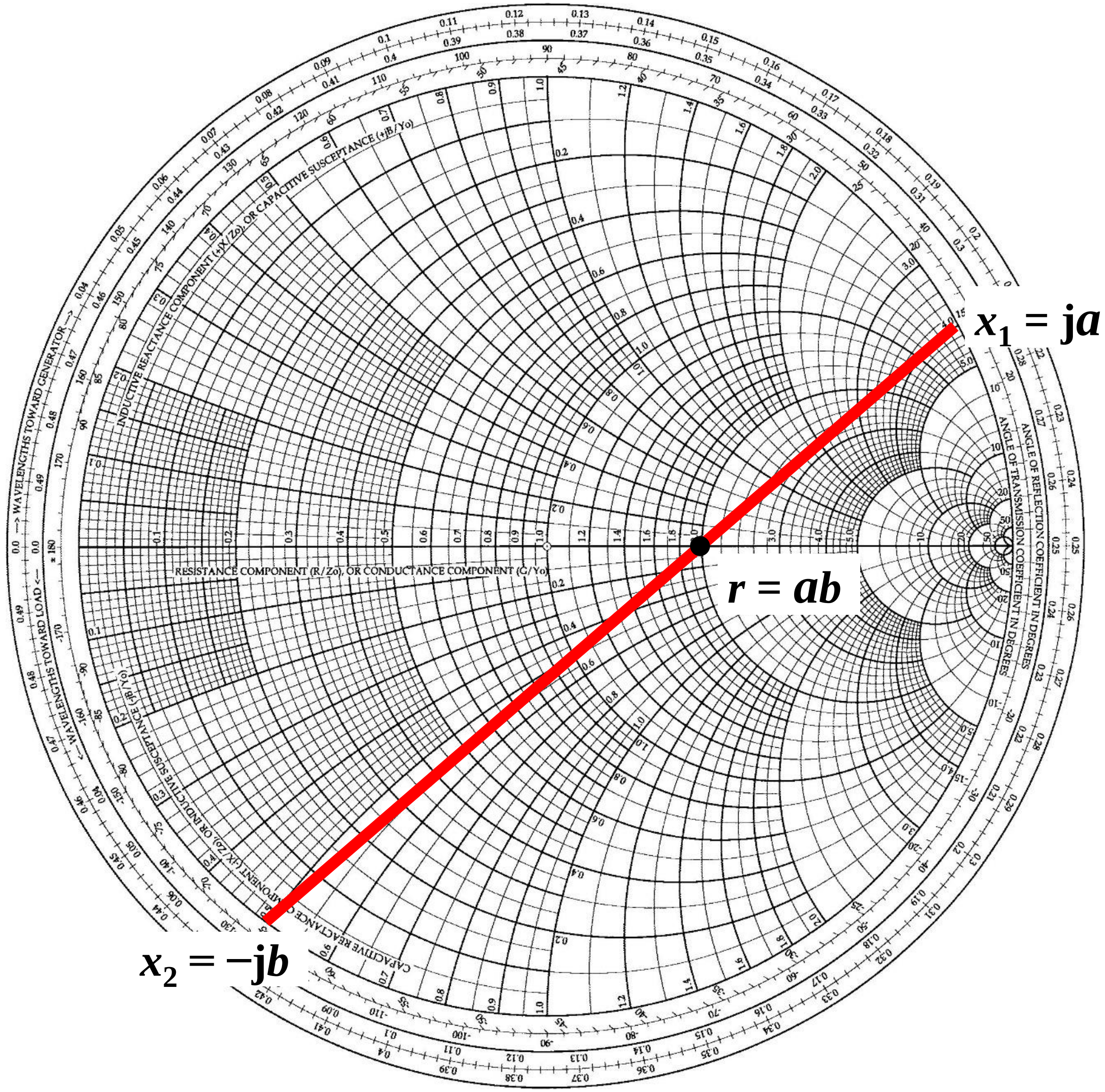
Constant Immittance Magnitude Arcs



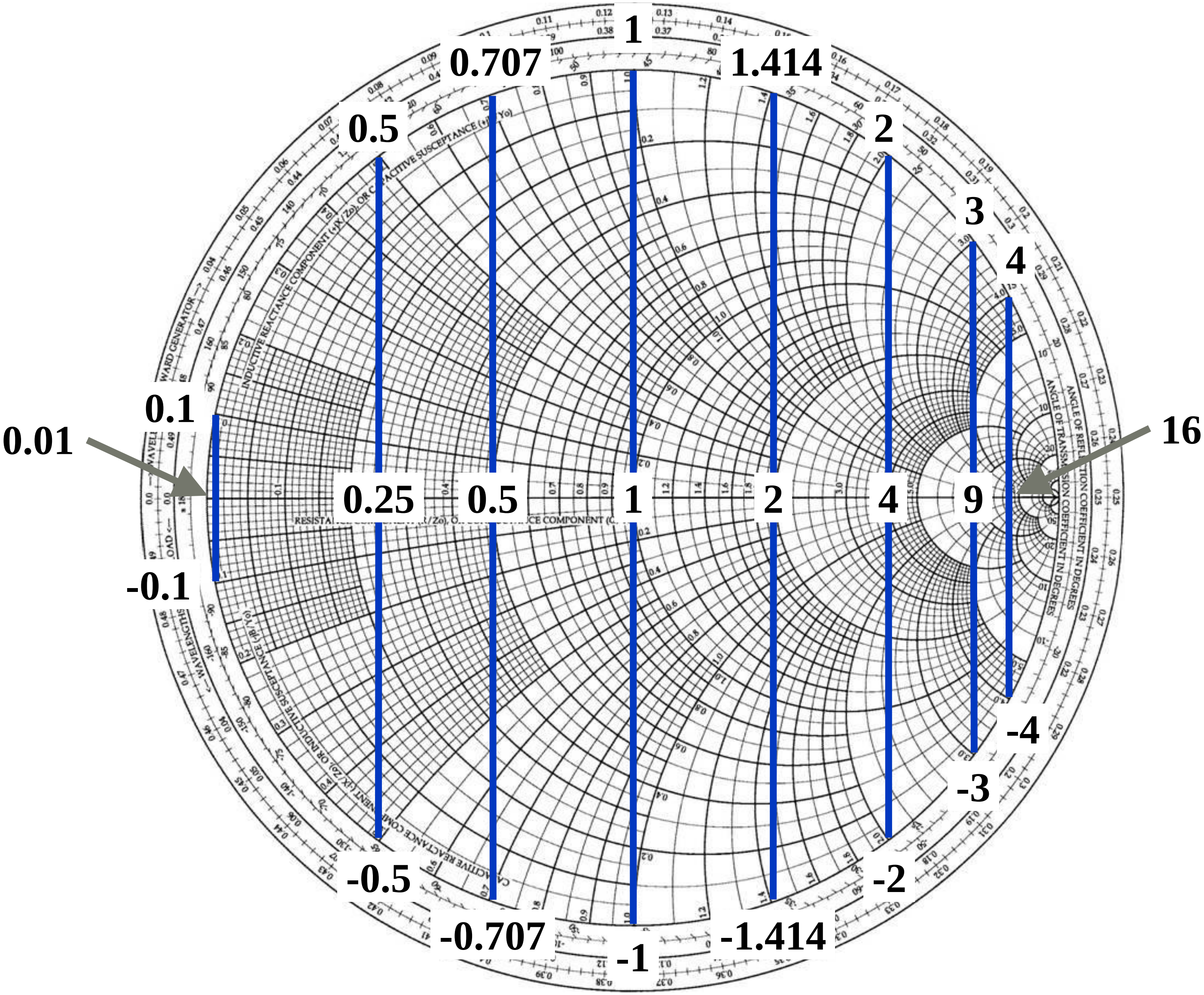
Constant Q (Immittance Phase) Arcs



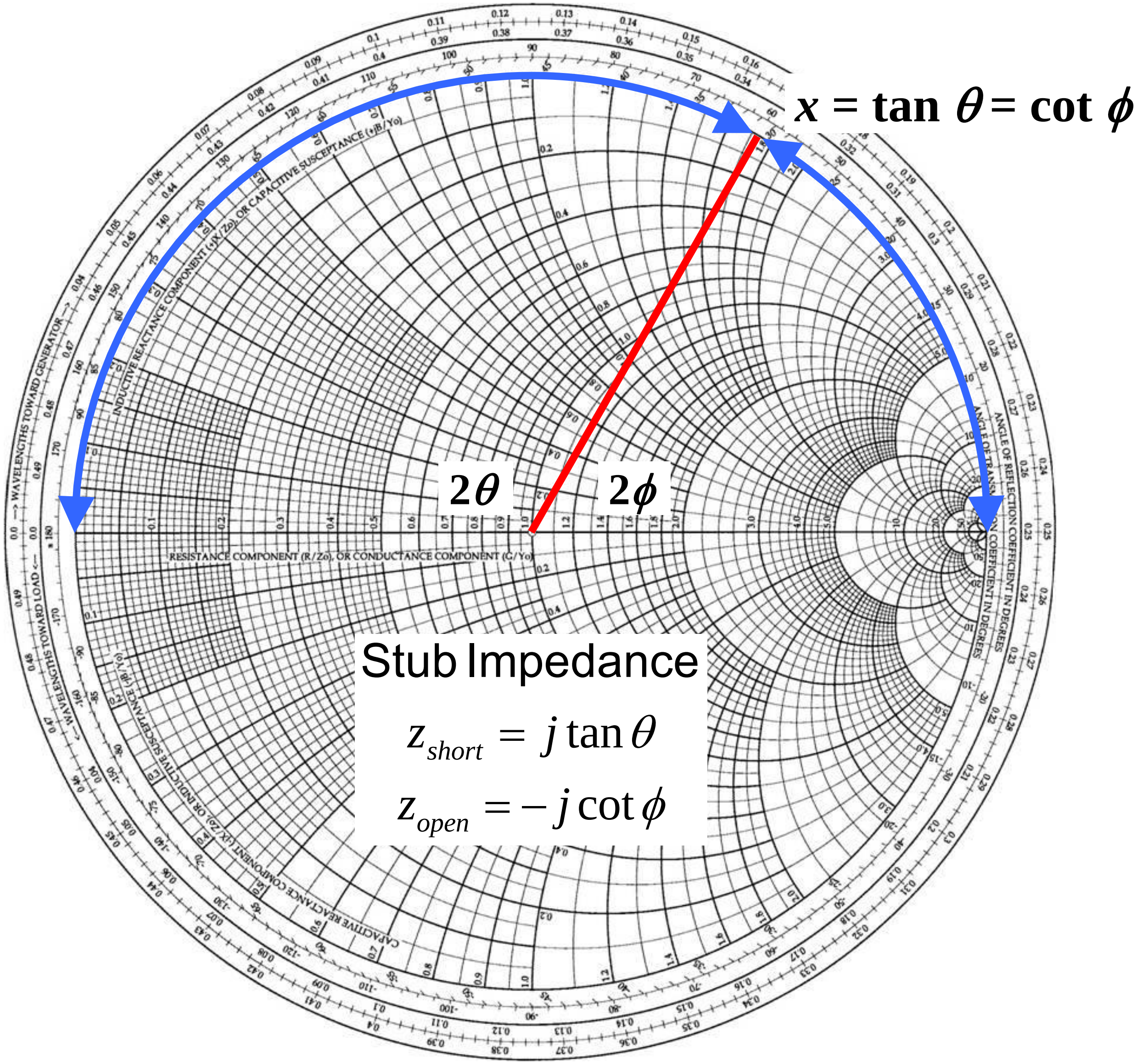
Multiplication and Division



Squares and Square Roots



Tangents and Cotangents

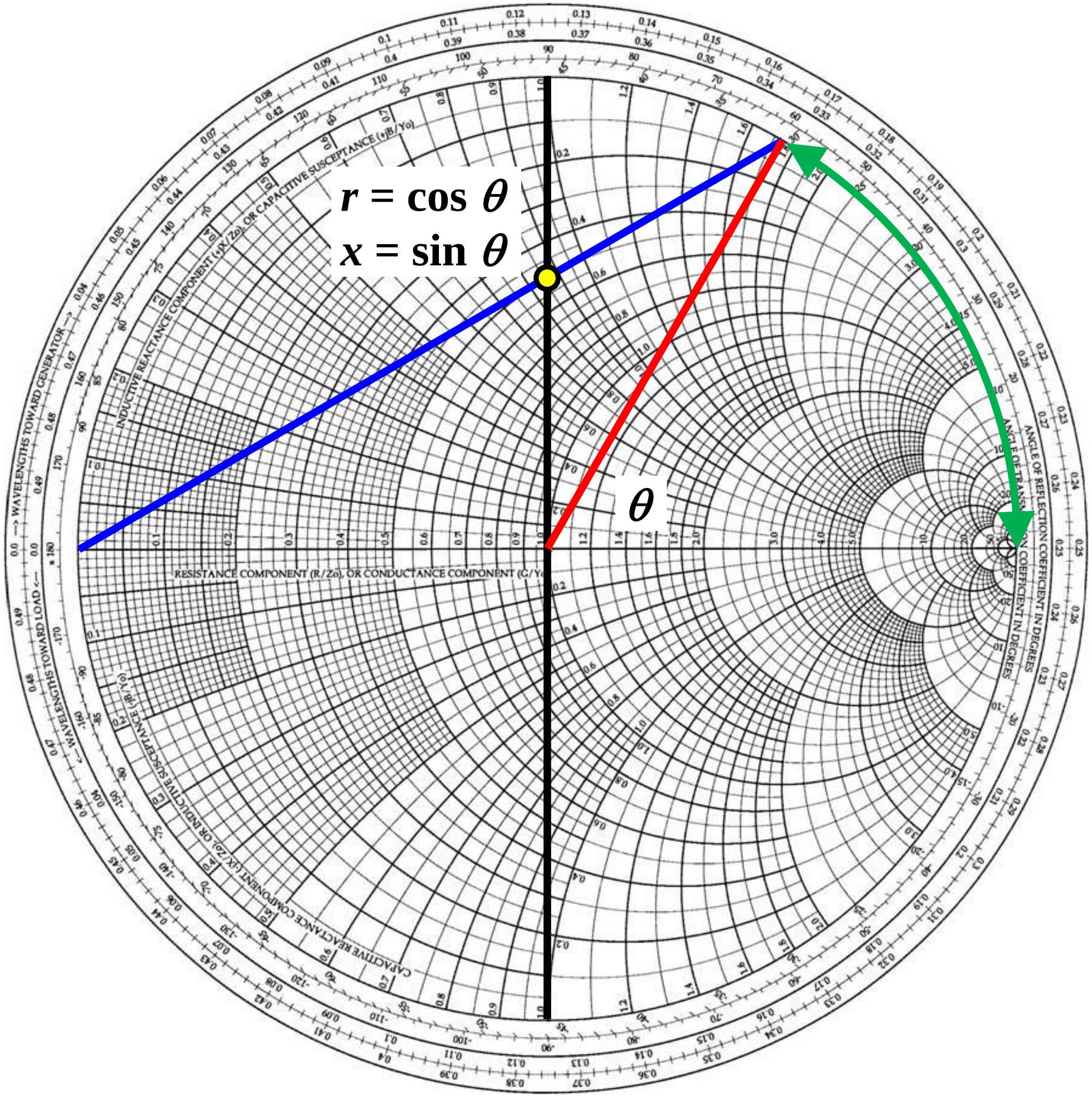


Stub Impedance

$$z_{short} = j\tan\theta$$

$$z_{open} = -j\cot\phi$$

Sines and Cosines



Classical Network Theory

Key Dates in Classical Network Theory

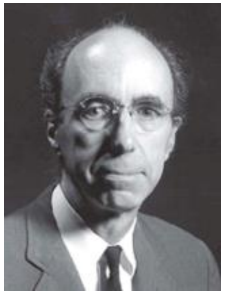
1893	**“Impedance” – A.E. Kennelly**
1893-1905	**AC circuit theory using complex numbers or phasors, “reactance” – C.P. Steinmetz**
1923-1924	**Reactance Theorems – O.J. Zobel, R.M. Foster**
1924	**Lossless impedance synthesis by partial fractions – R.M. Foster**
1926	**Lossless impedance synthesis by continued fractions – W. Cauer**
1931	**Passive impedance synthesis using RLCM – O. Brune**
1937	**Passive impedance synthesis using a single resistor – S. Darlington**
1946	**Passive n-port synthesis using RLCM – Y. Oono**
1949	**Passive synthesis using RLC without transformers – R. Bott & R.J. Duffin**
1957	***Synthesis of Passive Networks* – E.A. Guillemin**
1958	***Network Synthesis* – D.F. Tuttle**
1962	***Linear Active Network Theory* – L. de Pian**
1964	**Singular network elements – H.J. Carlin**
1973	***Network Analysis and Synthesis* – B.D.O. Anderson & S. Vongpanitlerd**
2009	**All singular elements found – A.M. Soliman**
2002-2010	**Realizability, synthesis, and stability of non-Foster active networks – S.E. Sussman-Fort, S.D. Stearns, others**

Pioneers of Electric Network Theory

Ronald Martin Foster
1896-1998

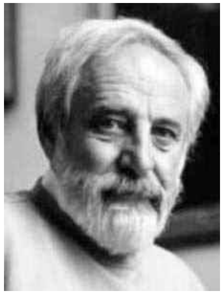
Wilhelm Cauer
1900-1945

Otto Walter Heinrich Oscar Brune
1901-1982



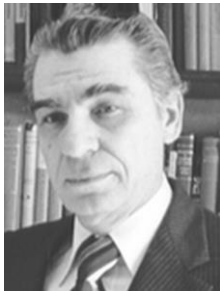
Sidney Darlington
1906-1997

Richard James Duffin
1909-1996



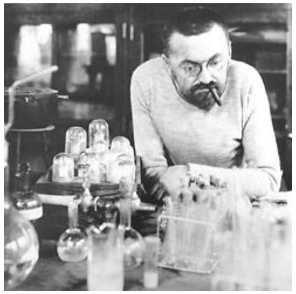
Raoul Bott
1923-2005

Herbert Jacob Carlin
1917-2009



Dante Ciriaco Youla
1925-2021

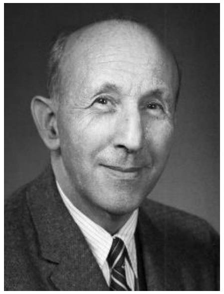
Arthur Edwin Kennelly
1861-1939



Charles Proteus Steinmetz
1865-1923

George Ashley Campbell
1870-1954

Otto Julius Zobel
1887-1970

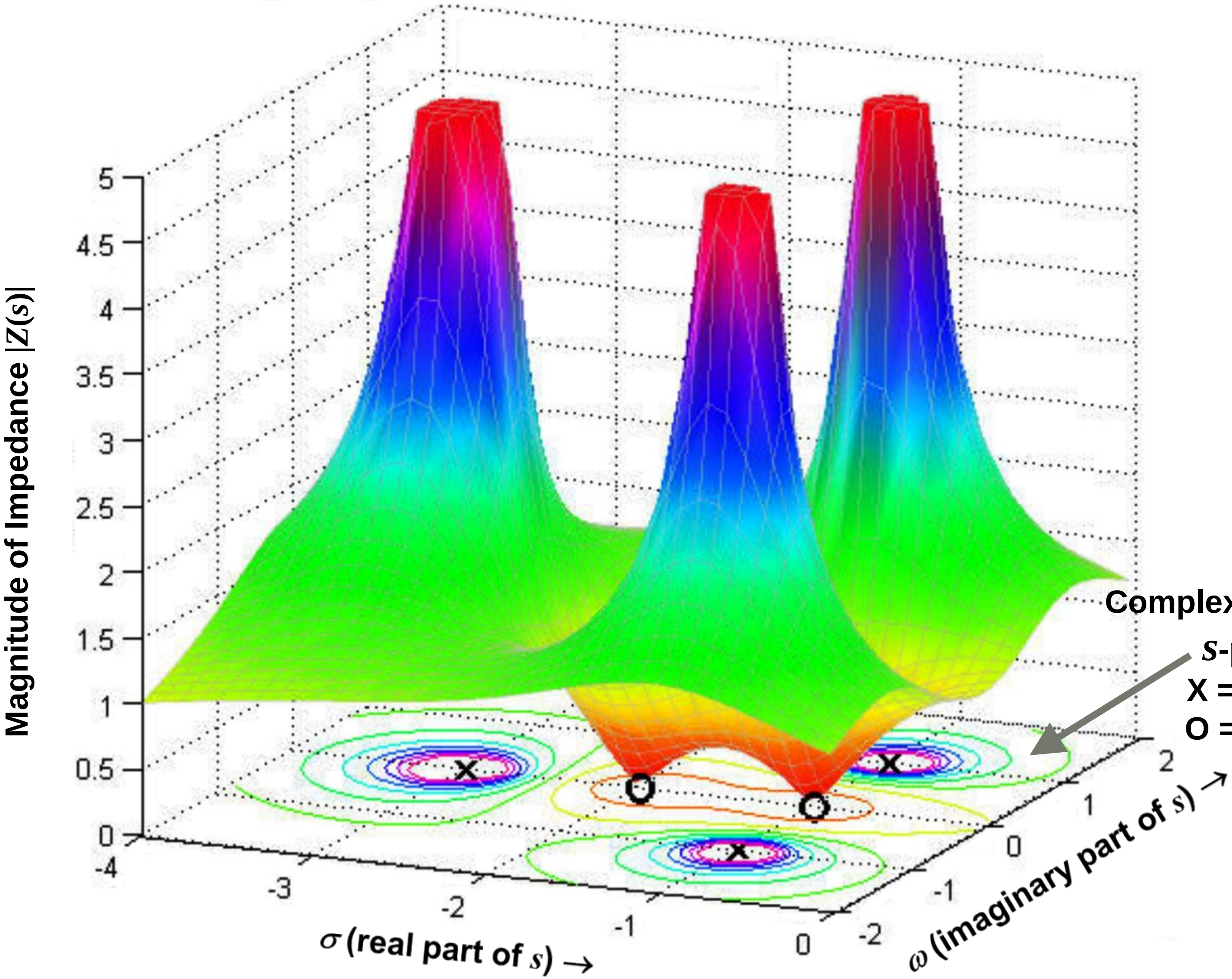


Ernst Adolph Guillemin
1898-1970

Mac Elwyn Van Valkenburg
1921-1997

Norman Balabanian
1922-2009

Poles, Zeros, and Complex Frequency



Complex Frequency

s-plane

X = poles

O = zeros

Immittance (Impedance & Admittance) Functions

- **Immittance functions of passive devices, elements, and networks are "positive-real" functions of complex frequency $s = \sigma + \mathbf{j}\omega$**
- **Analytic in the right half plane**
- **Poles and zeros may exist only on $\mathbf{j}\omega$ axis or in the left half plane**
- **Immittance functions of lumped RLC networks are rational functions**

$$Z(s) = \frac{as^3 + bs^2 + cs + d}{es^2 + fs}$$

  - All coefficients positive, no middle terms missing
  - Degrees of numerator and denominator polynomials differ by at most 1
  - If the degrees are the same, the network has losses
- **The magnitude of an immittance function of a passive element or network cannot increase faster than f (1^{st} power of frequency) nor decrease faster than $1/f$ (inverse frequency)**
- **For lossless elements, devices, and networks, the slopes of reactance and susceptance vs frequency are strictly positive at all frequencies (except discontinuities)**

Reactance Theorems for Lossless Devices and Networks

For lossless elements, devices, and networks, the slopes of reactance and susceptance vs frequency are strictly positive at all frequencies (except discontunities).

- **O.J. Zobel (1923)**

$$\frac{dX(\omega)}{d\omega} > 0$$

- **R.M. Foster (1924)**

$$\frac{\partial X(\omega)}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial B(\omega)}{\partial \omega} > 0$$

- **Consequences**
  - Poles and zeros exist only on the real frequency axis
  - Poles and zeros are simple
  - Poles and zeros have positive real residues
  - Poles alternate with zeros
  - A pole or zero exists at zero and at infinity

O.J. Zobel, “Theory and Design of Uniform and Composite Electric Wave-filters,” *Bell System Technical Journal*, vol. 2, no. 1, pp. 1-46, Jan. 1923.
R.M. Foster, “A Reactance Theorem,” *Bell System Technical Journal*, vol. 3, no. 2, pp. 259-267, Apr. 1924.

An Important Fact about Impedance Functions

The real and imaginary parts of a passive immittance (impedance or admittance) function cannot be specified independently. One determines the other.

- **Real and imaginary parts are related by Poisson/Schwarz integrals**
- **A resistance is generally accompanied by reactance or is not passive**
- **Realizable circuits use only passive components**

Real and Imaginary Parts are Not Independent

- Impedance

$$Z(s) = \frac{as^3 + bs^2 + cs + d}{es^2 + s}$$

- Real part

$$R(j\omega) = \frac{(be - a)\omega^2 + (c - de)}{e^2\omega^2 + 1}$$

- Imaginary part

$$X(j\omega) = \frac{ae\omega^4 + (b - ce)\omega^2 - d}{e^2\omega^3 + \omega}$$

Relation Between Real and Imaginary Parts

- **Poisson/Schwarz integrals**
  - AKA "Hilbert transform" or Kramers-Kronig relations

$$R(\omega) = \frac{2}{\pi}\int_0^\infty \frac{u\,X(u)}{\omega^2 - u^2}\,du + R_0$$

A positive constant

An analytic function

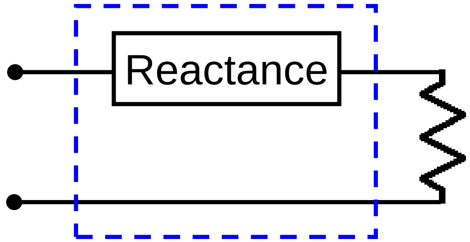
$$X(\omega) = \frac{-2\omega}{\pi}\int_0^\infty \frac{R(u)}{\omega^2 - u^2}\,du + X_0(\omega)$$

- **Conductance and susceptance are related similarly**
- **A consequence**
  - Letting $R(\omega) = R\omega^2$, we find that $X(\omega)$ does not exist
  - Passive square-law resistors are an impossibility

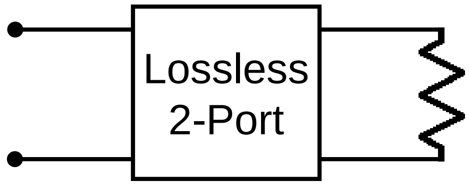
The real and imaginary parts of a passive impedance are not independent.

Darlington Forms (1937, 1939)

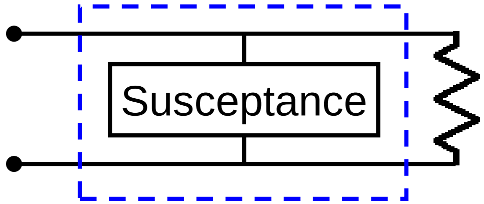
- **Every positive real, rational immittance function can be realized as a reactance 2-port terminated by a resistor**



Not series!



Darlington form



Not parallel!

- **Antennas can be represented by equivalent circuits in Darlington form, over any given band**
- **2-port antenna emulators are based on Darlington forms**

S. Darlington, “Synthesis of Reactance 4-Poles which Produce Prescribed Insertion Loss Characteristics,” *Journal of Mathematics and Physics*, vol. 18, no. 4, pp. 257-353, April 1939.
B.S. Yarman, et al., “An Immitance Based Tool for Modelling Passive One-Port Devices by Means of Darlington Equivalents,” *Int. J. Electron. Commun.*, vol. 55, no. 6, pp. 443-451, Dec 2001.

Every antenna has an equivalent circuit over any given band, in Darlington form.

Antenna Impedance Functions

On the Smith Chart

Dipole Antenna in Free Space


A.C. source

- A dipole antenna is symmetric and center fed
- A dipole's feedpoint impedance is a complex-valued function of frequency, length, and diameter

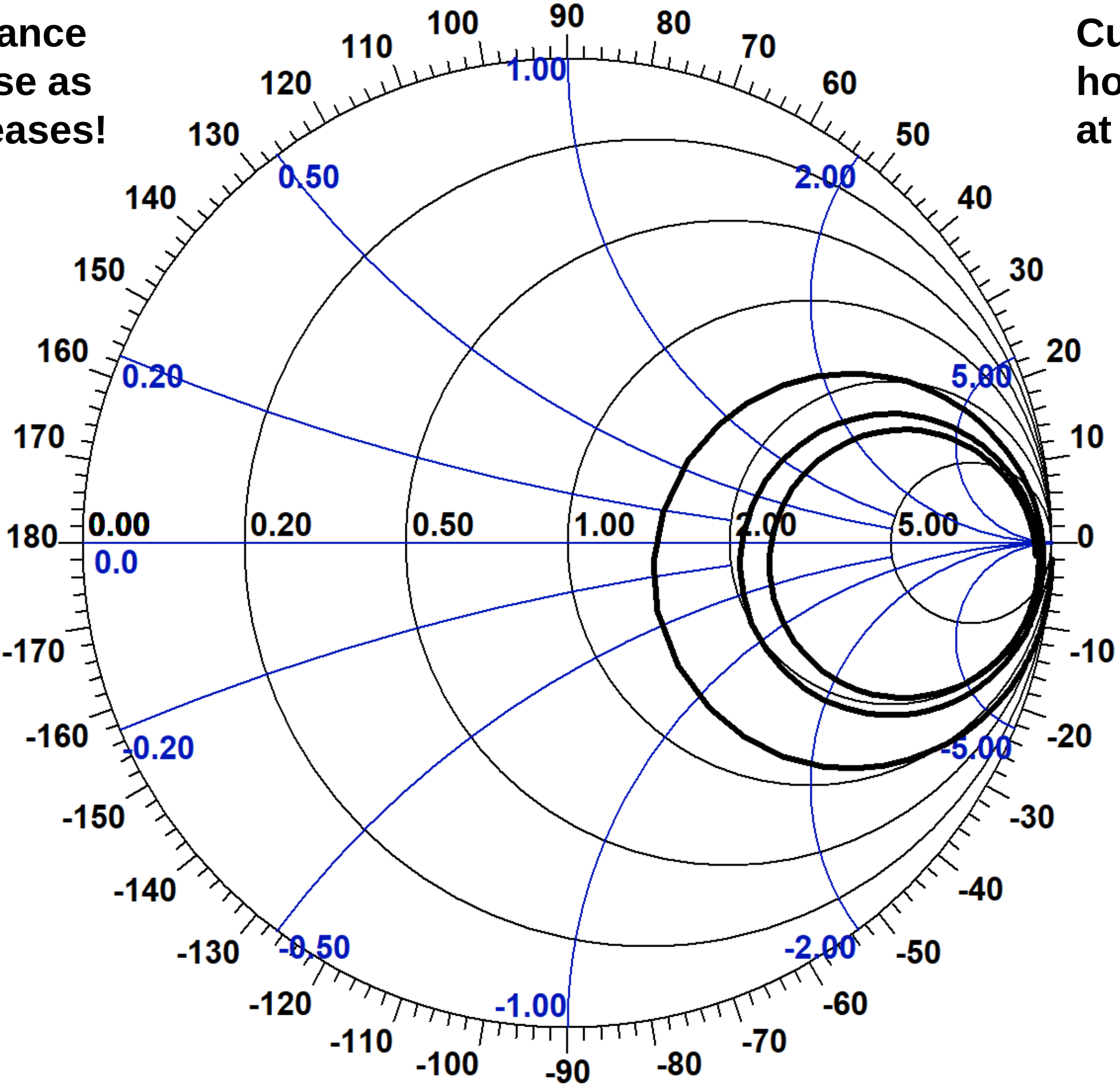
$$Z(f,L,d)=R(f,L,d)+jX(f,L,d)$$

- 1st resonance occurs when length is a little less than a half wavelength

$$L=\frac{K\lambda}{2}=\frac{Kc}{2f}$$

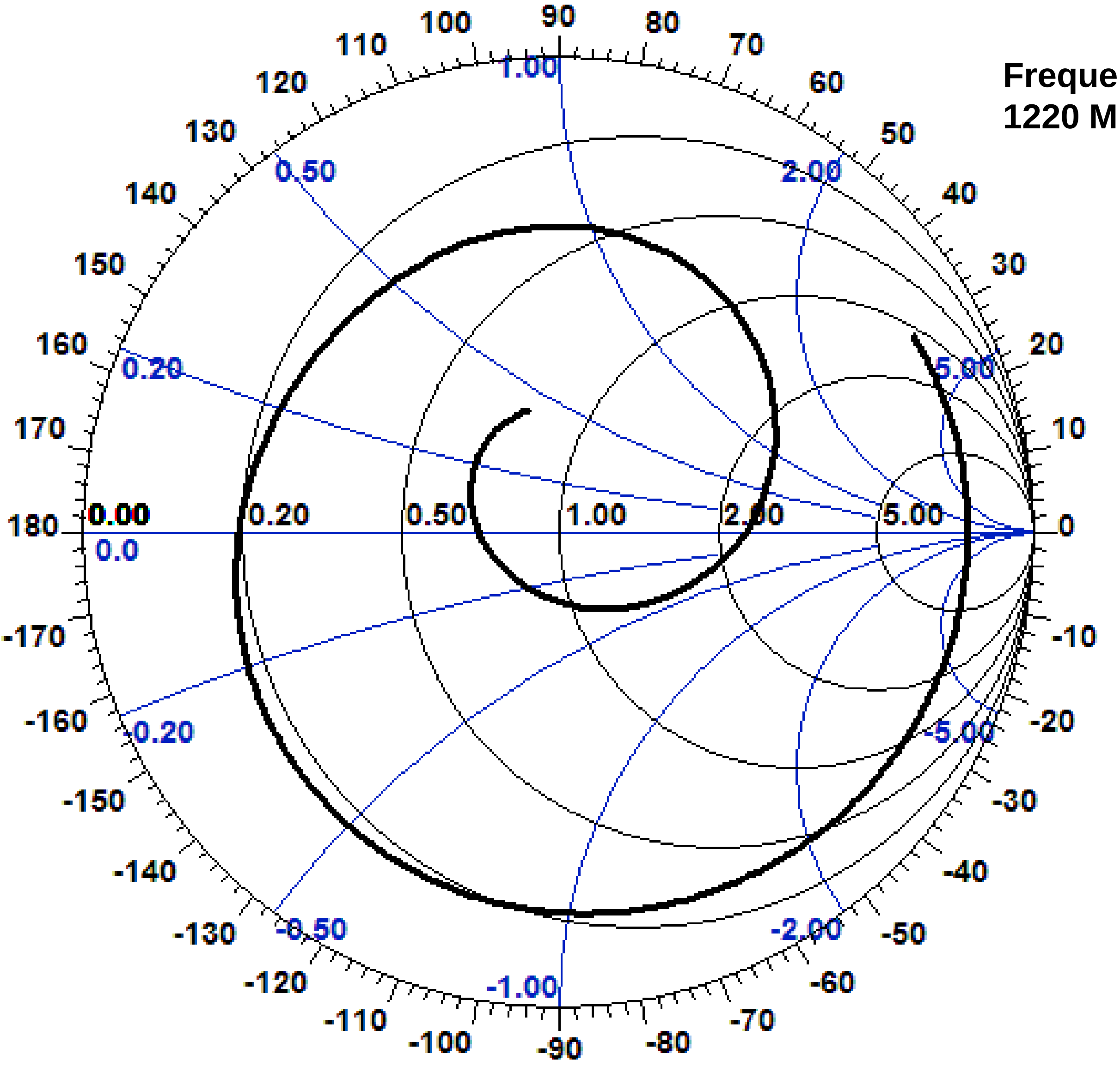
Example 1: 98.4-ft Thin-Wire Dipole (*L*/*d* = 11,200) from 1 MHz to 30 MHz

Antenna impedance curves clockwise as frequency increases!



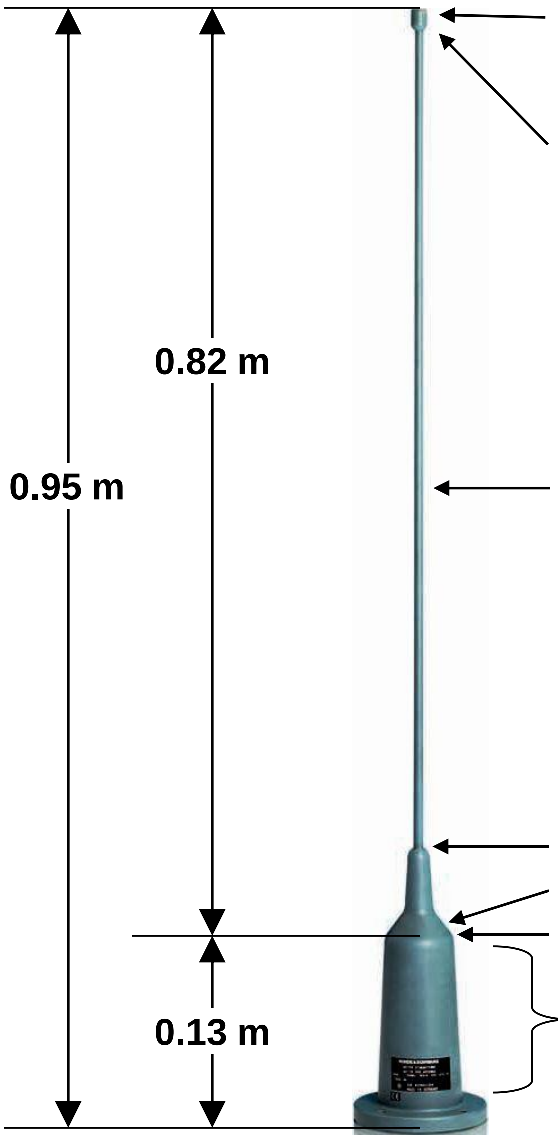
Curve starts on horizontal axis at *f* = 0

Example 2: GPS Antenna



Frequency sweep
1220 MHz to 1585 MHz

Example 3: Rohde & Schwarz HE010 Active Monopole



End cap
length 15 mm
diameter 15.3 mm

Rod diameter 7.55 mm

Tapered rod

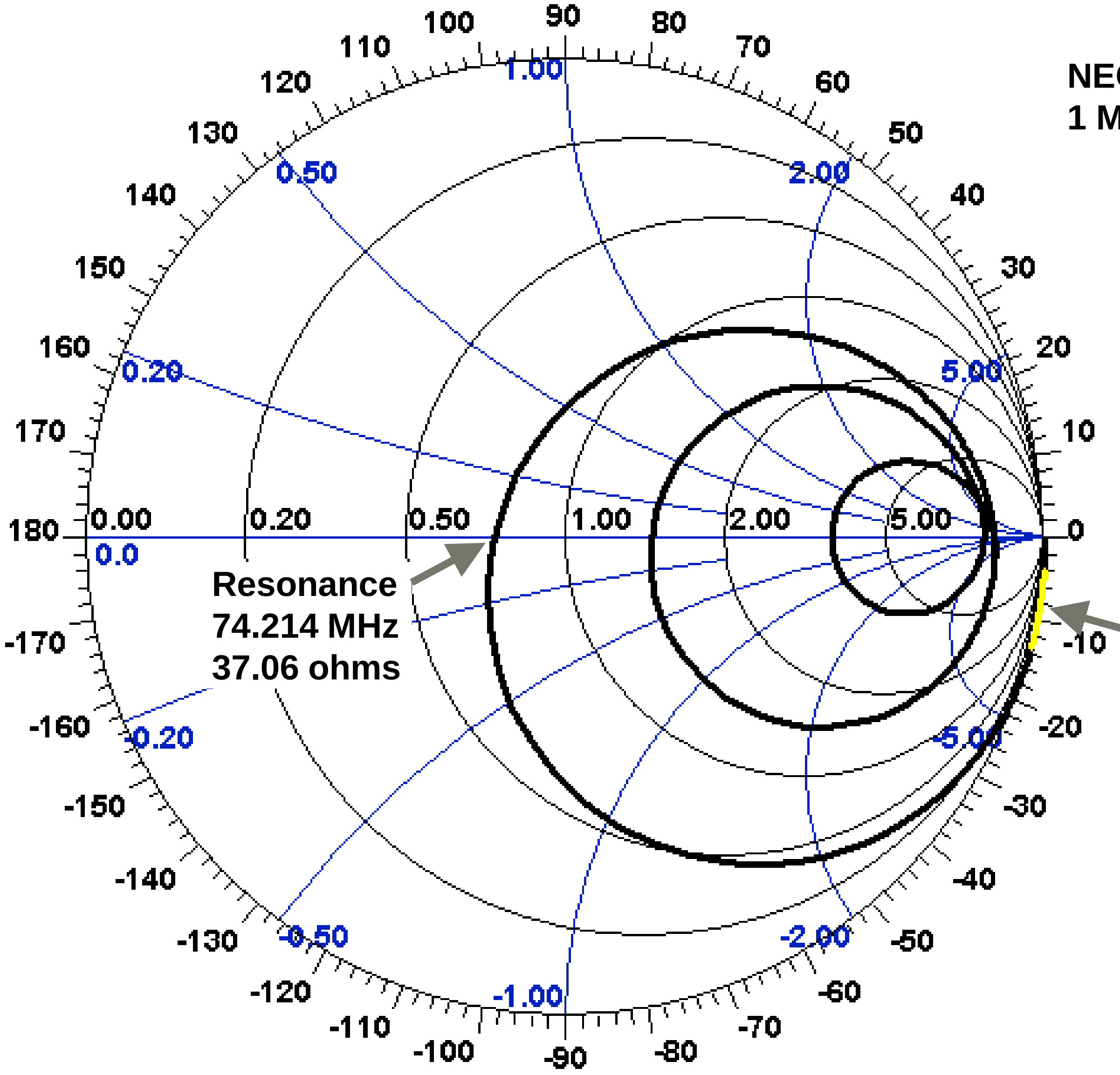
Rod diameter 9.2 mm

Rod feedpoint

Rod diameter 9.4 mm

Preamplifier

Example 3: R&S HE010 Monopole Impedance

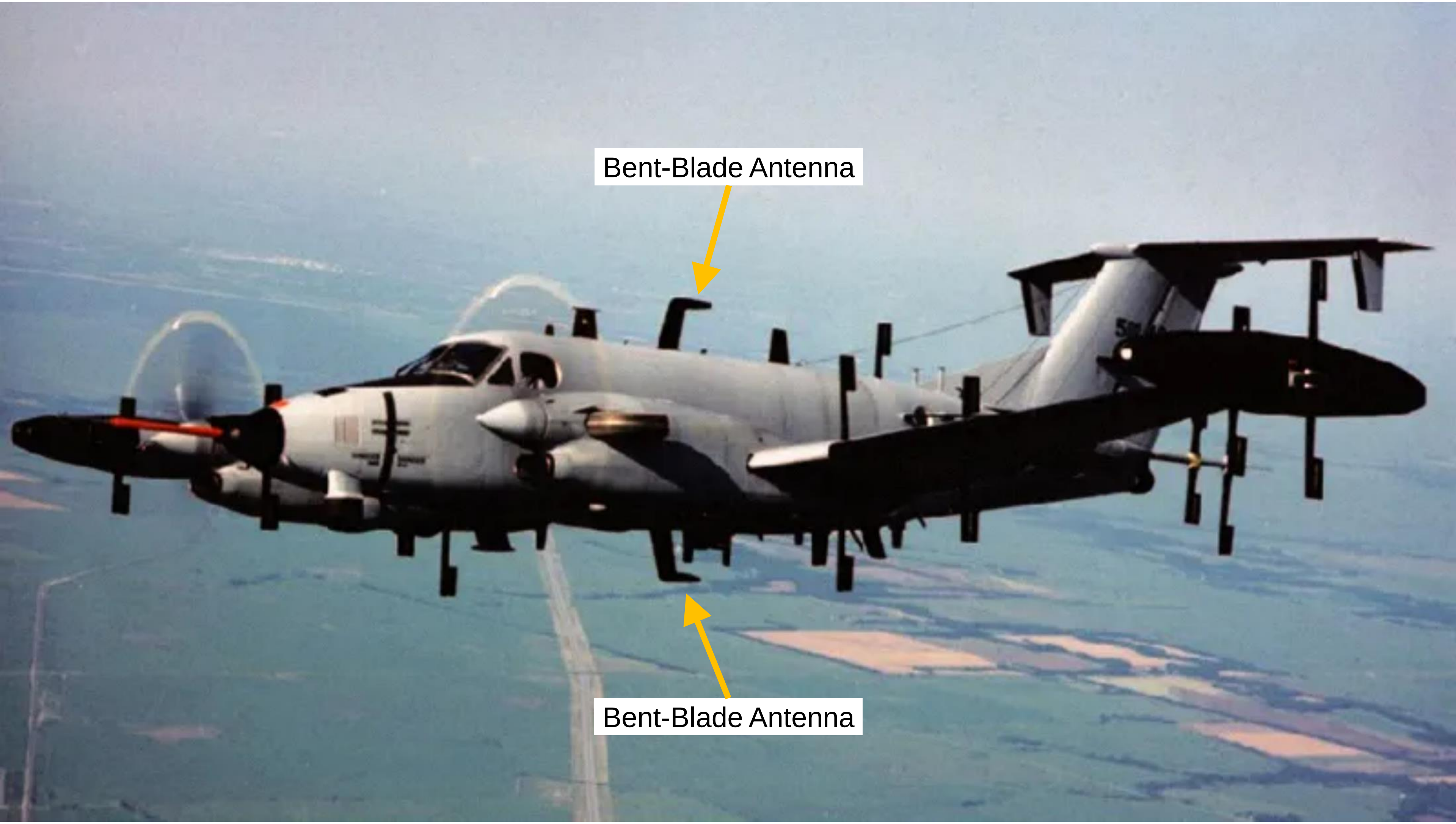


NEC4 Data
1 MHz to 500 MHz

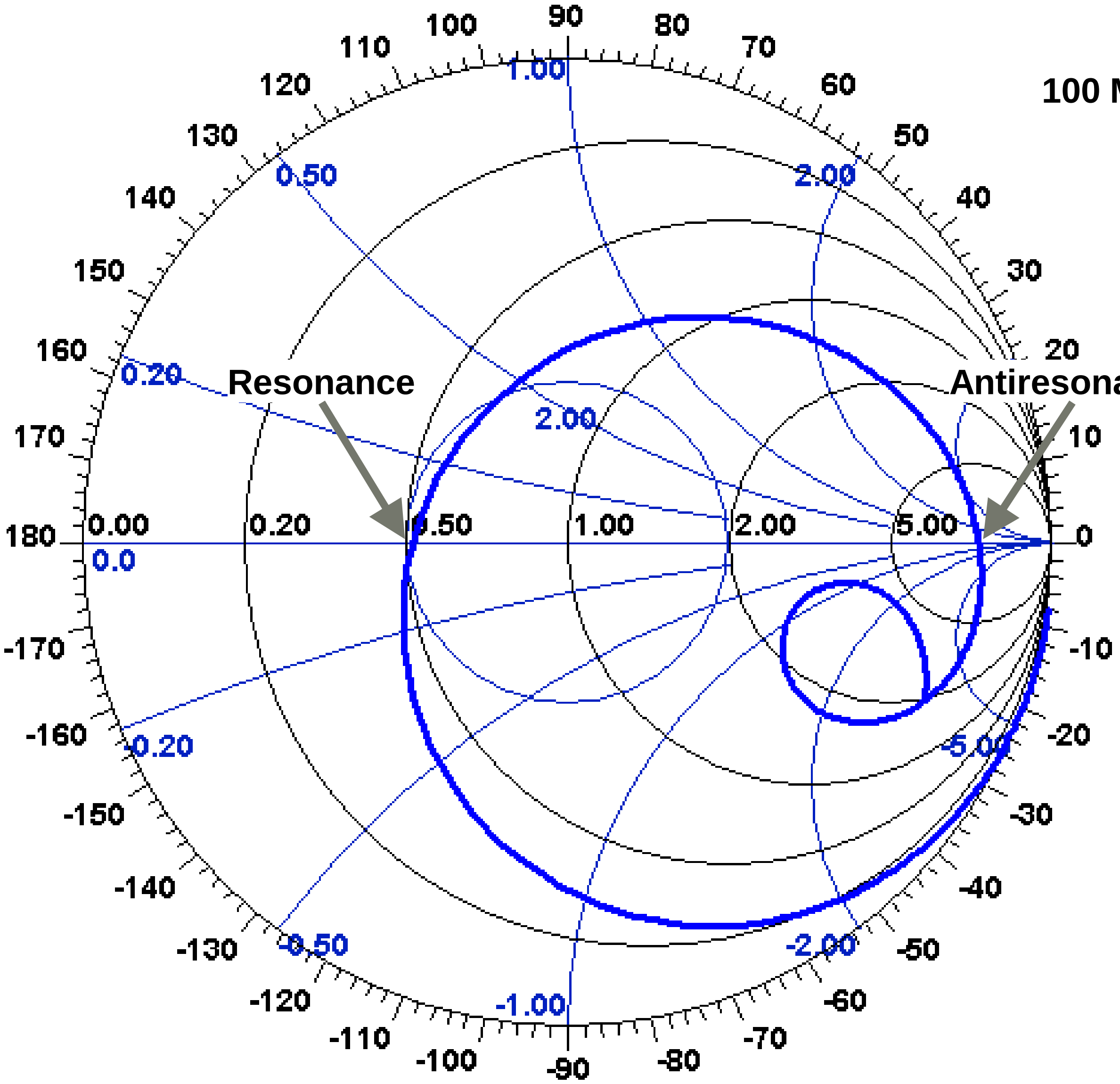
Resonance
74.214 MHz
37.06 ohms

10 MHz to 30 MHz

Typical Spy Plane
Tupolev Tu-16R Badger

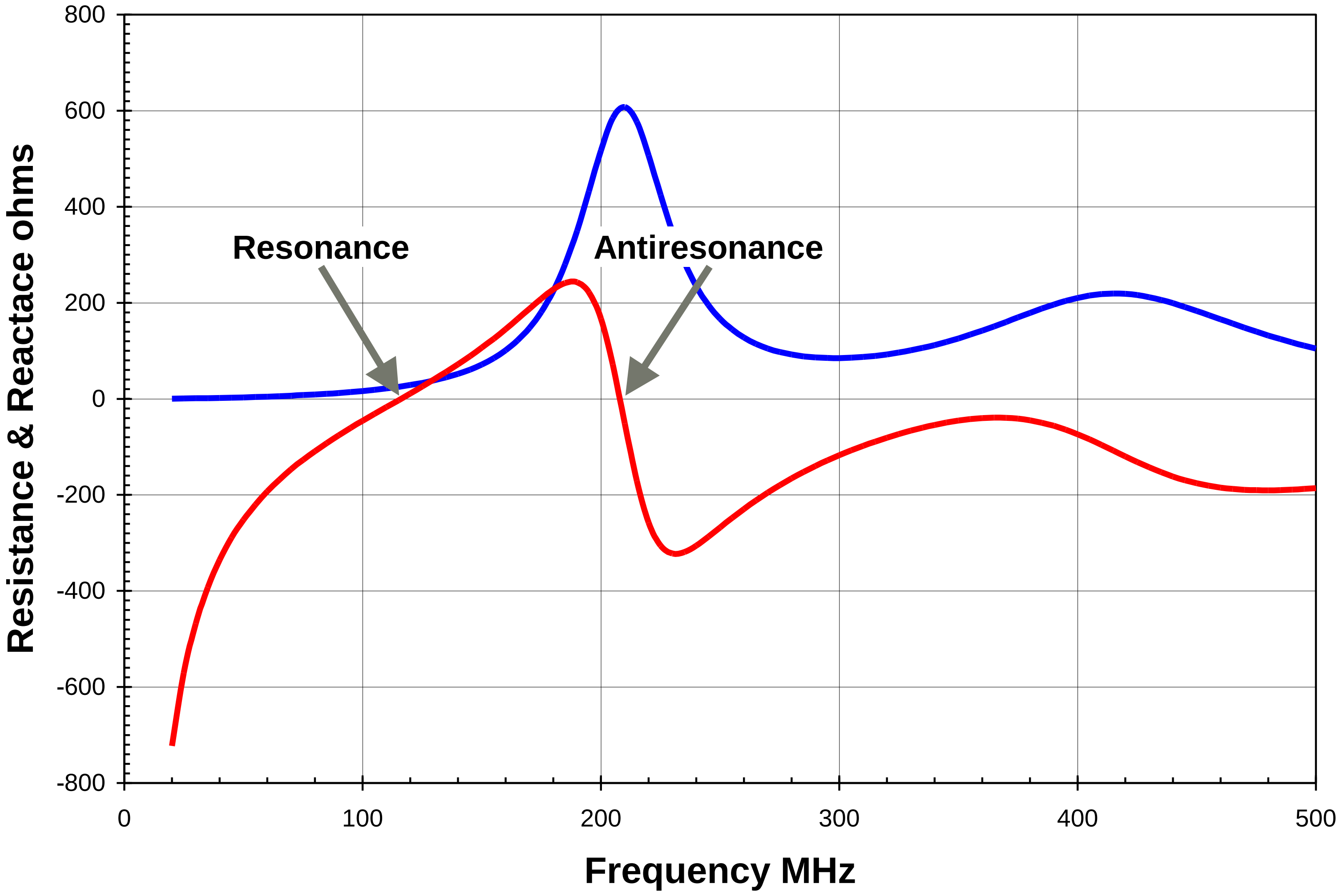


Example 4: Bent-Blade Antenna

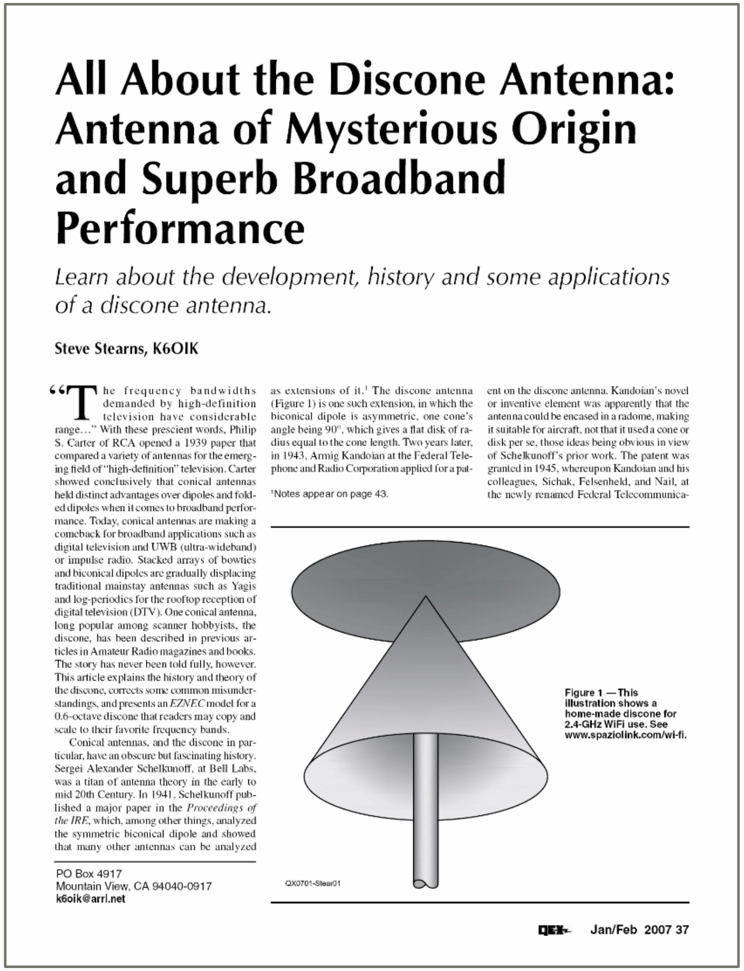


100 MHz to 500 MHz

Example 4: Bent-Blade Antenna Feedpoint Impedance



Example 5: VHF-UHF Discone Antenna



All About the Discone Antenna: Antenna of Mysterious Origin and Superb Broadband Performance

Learn about the development, history and some applications of a discone antenna.

Steve Stearns, K6OIK

"The frequency bandwidths demanded by high-definition television have considerable range…" With these prescient words, Philip S. Carter of RCA opened a 1939 paper that compared a variety of antennas for the emerging field of "high-definition" television. Carter showed conclusively that conical antennas held distinct advantages over dipoles and folded dipoles when it comes to broadband performance. Today, conical antennas are making a comeback for broadband applications such as digital television and UWB (ultra-wideband) or impulse radio. Stacked arrays of bowties and biconical dipoles are gradually displacing traditional mainstay antennas such as Yagis and log-periodics for the rooftop reception of digital television (DTV). One conical antenna, long popular among scanner hobbyists, the discone, has been described in previous articles in Amateur Radio magazines and books. The story has never been told fully, however. This article explains the history and theory of the discone, corrects some common misunderstandings, and presents an *EZNEC* model for a 0.6-octave discone that readers may copy and scale to their favorite frequency bands.

Conical antennas, and the discone in particular, have an obscure but fascinating history. Sergei Alexander Schelkunoff, at Bell Labs, was a titan of antenna theory in the early to mid 20th Century. In 1941, Schelkunoff published a major paper in the *Proceedings of the IRE*, which, among other things, analyzed the symmetric biconical dipole and showed that many other antennas can be analyzed as extensions of it.[1] The discone antenna (Figure 1) is one such extension, in which the biconical dipole is asymmetric, one cone's angle being 90°, which gives a flat disk of radius equal to the cone length. Two years later, in 1943, Armig Kandoian at the Federal Telephone and Radio Corporation applied for a patent on the discone antenna. Kandoian's novel or inventive element was apparently that the antenna could be encased in a radome, making it suitable for aircraft, not that it used a cone or disk per se, those ideas being obvious in view of Schelkunoff's prior work. The patent was granted in 1945, whereupon Kandoian and his colleagues, Sichak, Felsenheld, and Nail, at the newly renamed Federal Telecommunica-

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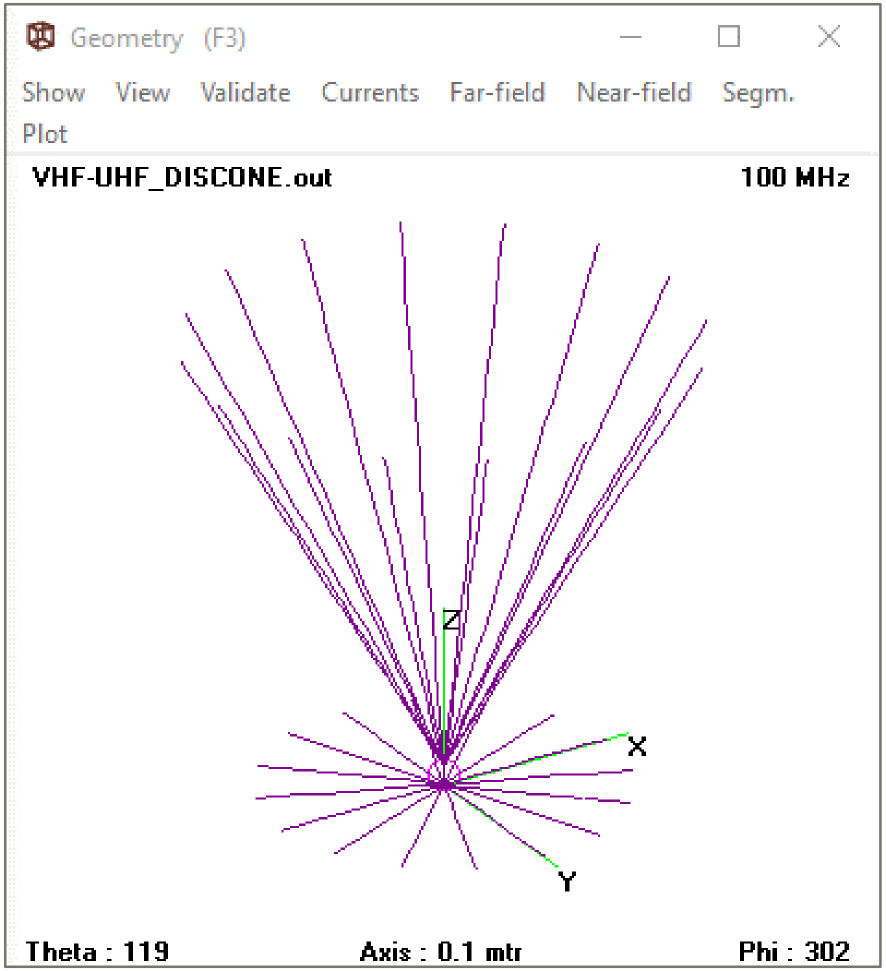
[1] Notes appear on page 43.

QX0701-Stear01

Figure 1 — This illustration shows a home-made discone for 2.4-GHz WiFi use. See www.spaziolink.com/wi-fi.

PO Box 4917
Mountain View, CA 94040-0917
k6oik@arrl.net

QEX– Jan/Feb 2007 37

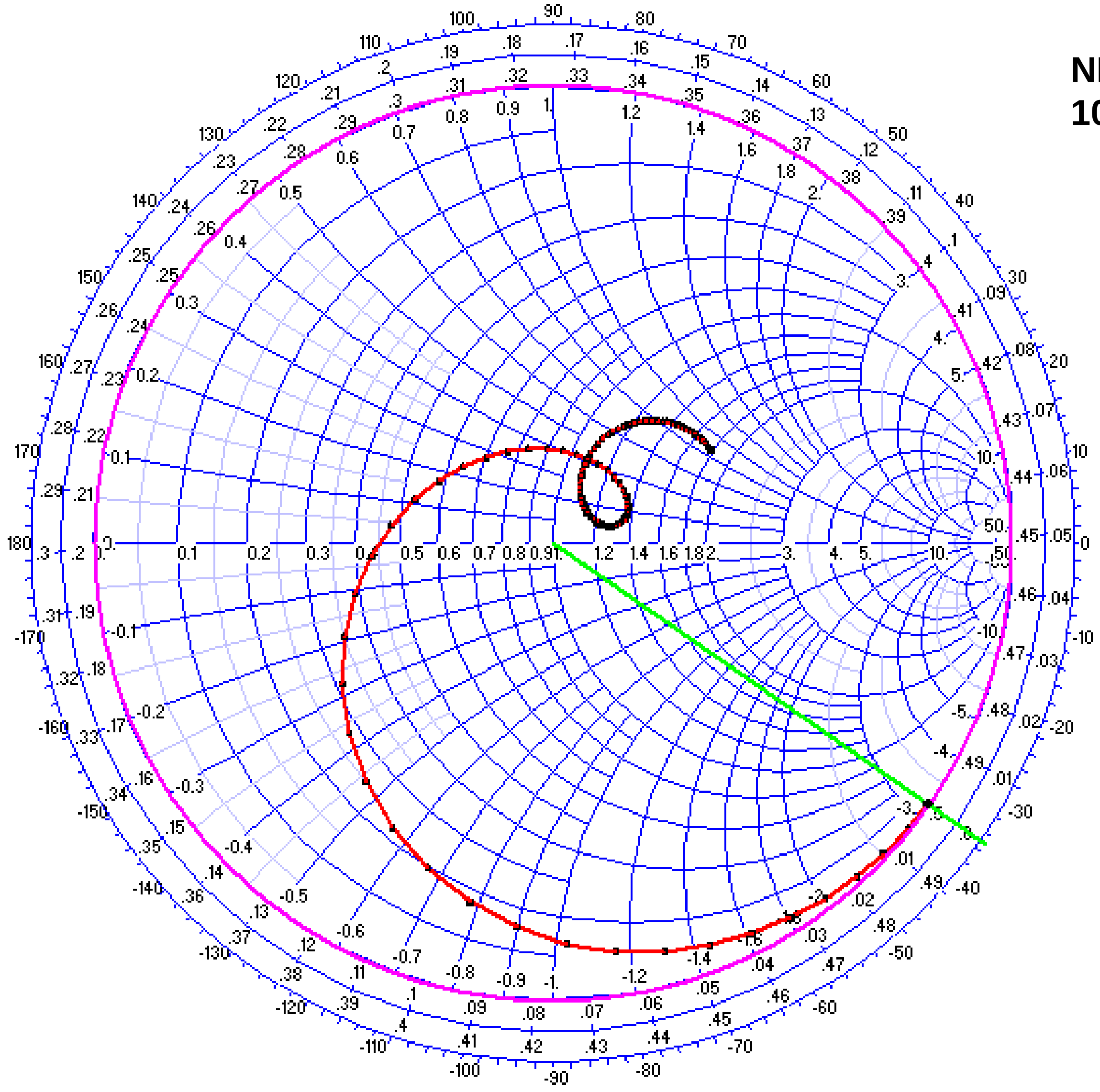


VHF-UHF_DISCONE.out 100 MHz

Theta : 119 Axis : 0.1 mtr Phi : 302

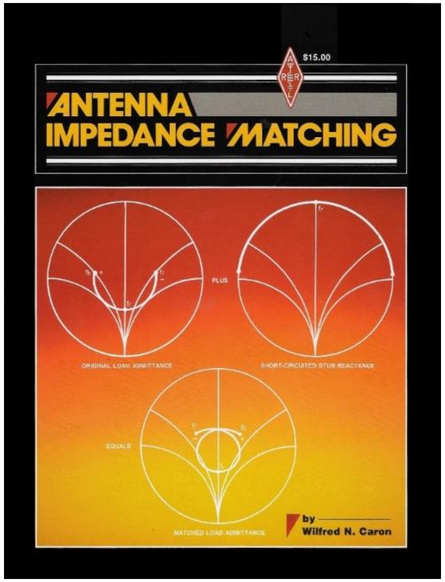
Steve Stearns, K6OIK "All About the Discone Antenna," *QEX*, pp. 37-44, January/February 2007.

Example 5: VHF-UHF Discone Antenna

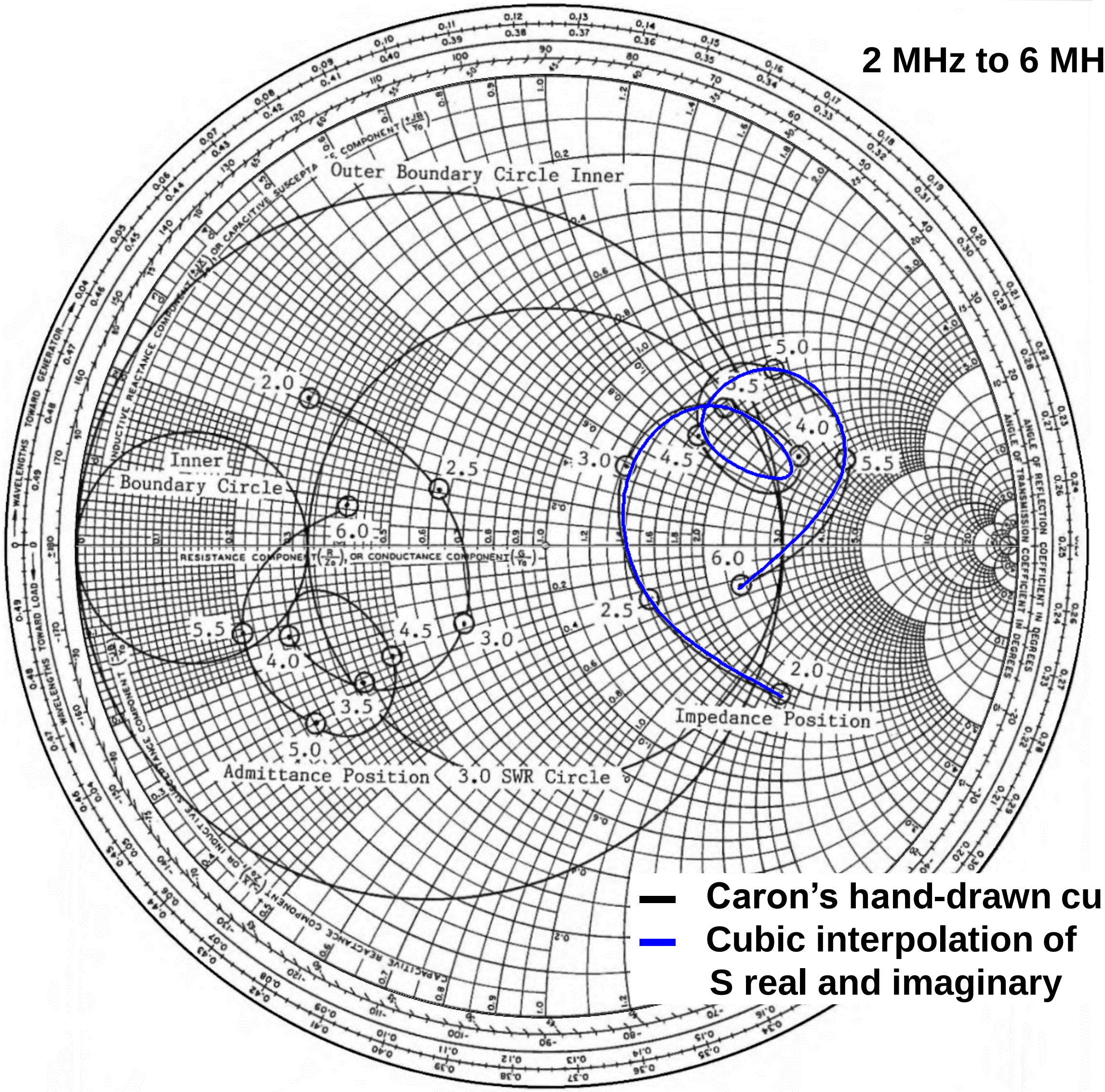


NEC2 Data
100 MHz to 1 GHz

Example 6: Broadband Dipole Antenna



Wilfred N. Caron, *Antenna Impedance Matching*, ARRL, 1989



2 MHz to 6 MHz

▬ Caron’s hand-drawn curve

▬ Cubic interpolation of S real and imaginary

Summary

- **Impedance curves always bend or spiral clockwise as frequency increases**
  - A property of passive impedance functions whether lossy or lossless
  - Unrelated to Foster's Reactance theorem
- **Crossings of the horizontal axis of the Smith chart are called *resonances***
  - Downward crossings are called *anti-resonances*
- **An antenna can have any number of resonances**
  - An antenna can have ***no resonances***. Its impedance curve can lie entirely in the top or bottom half of the Smith chart
  - An antenna can have a ***finite*** number of resonances, e.g. fat dipoles, loop antennas, and bent blade antennas
  - An antenna can have an ***infinite*** number of resonances, e.g. dipole reactance given by the induced emf method

Impedance Model or Equivalent Circuit ?

Impedance Model vs Equivalent Circuit

$$R_{in} = \frac{\eta}{4\pi \sin^2(k\,l)} \left\{ 2\gamma + 2\ln(2k\,l) - 2\mathrm{Ci}(2k\,l) \;+\; \sin(2k\,l)\left[\mathrm{Si}(4k\,l) - 2\mathrm{Si}(2k\,l)\right] \;+\; \cos(2k\,l)\left[\mathrm{Ci}(4k\,l) - 2\mathrm{Ci}(2k\,l) + \gamma + \ln(k\,l)\right]\right\}$$

$$X_{in} = \frac{\eta}{4\pi \sin^2(k\,l)} \left\{ 2\mathrm{Si}(2k\,l) \;-\; \cos(2k\,l)\left[\mathrm{Si}(4k\,l) - 2\mathrm{Si}(2k\,l)\right] \;+\; \sin(2k\,l)\left[\mathrm{Ci}(4k\,l) - 2\mathrm{Ci}(2k\,l) + \gamma + 2\ln(k\,a) - \ln(k\,l)\right]\right\}$$

- **Impedance Models**
  - A mathematical formula or algorithm for calculating impedance
  - Need not represent a physical equivalent
  - Can be any kind of function from simple to special
    - Polynomials
    - Rational functions
    - Sine and cosine integrals
    - Ad hoc formulas
  - A compact compressed representation of impedance behavior
  - Formula approximates given impedance behavior to a specified accuracy over a specified bandwidth
  - Good for computation, interpolation, and SWR prediction, but not physical testing and measurement

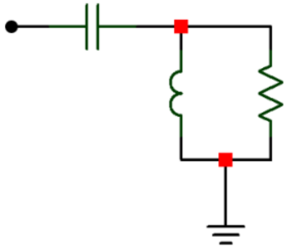
- **Equivalent Circuits**
  - A special kind of model – an electric circuit or network
  - Physically realizable
  - Made of *passive* elements
    - Resistors
    - Inductors
    - Capacitors
    - Mutual inductance
    - Ideal transformers
  - Impedance specified by a *positive real* rational function
  - Circuit approximates given impedance behavior to a specified accuracy over a specified bandwidth
  - Can build to make antenna dummies and emulators for reflection and transmission tests and measurements

Impedance Model vs Equivalent Circuit

$$R_{in} = \frac{\eta}{4\pi \sin^2\left(k\,l\right)} \left\{\; 2\gamma + 2\ln(2k\,l) - 2\mathrm{Ci}(2k\,l) \;+\; \sin(2k\,l) \right.$$

$$X_{in} = \frac{\eta}{4\pi \sin^2\left(k\,l\right)} \left\{\; 2\mathrm{Si}(2k\,l) \;-\; \cos(2k\,l)\left[\; \mathrm{Si}(4k\,l) - 2\mathrm{Si}(\right.\right.$$

• • •



- **Impedance Models**
    - A mathematical formula or algorithm for calculating impedance
    - Need not represent any physical device
    - Can be any kind of function from simple to special
        - Polynomials
        - Rational functions
        - Sine and cosine integrals
        - Ad hoc formulas
    - A compact compressed representation of impedance behavior
    - Formula approximates given impedance behavior to a specified accuracy over a specified bandwidth
    - Good for computation, interpolation, and SWR prediction, but not physical testing and measurement

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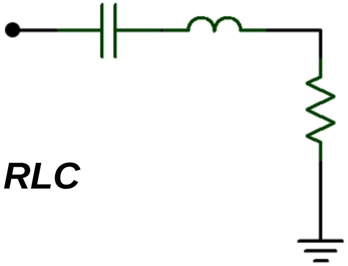
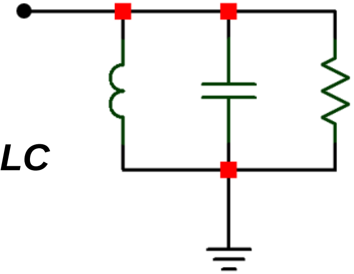
Low-Order Equivalent Circuits

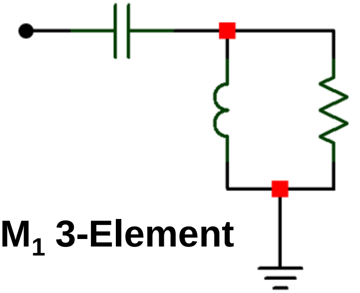
For narrowband modeling

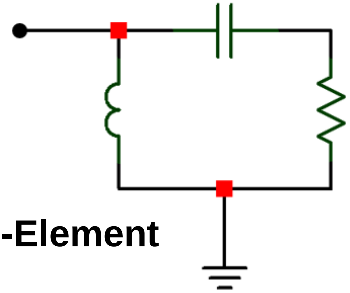
Low-Order Equivalent Circuits for Narrow Bands Near 1st Resonance

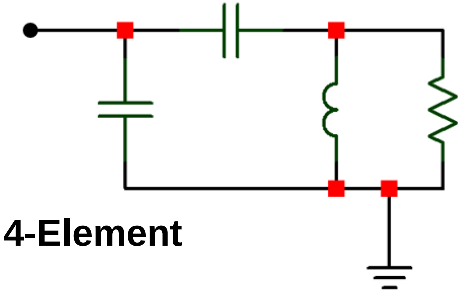
Circuits for antennas near resonance

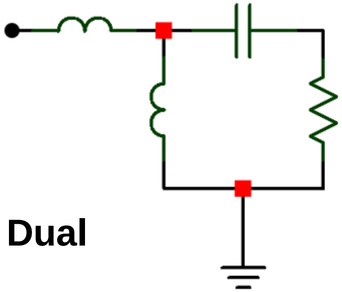

Series *RLC*


Parallel *RLC*


Chu TM_1 3-Element


Chu TE_1 3-Element


4-Element


4-Element Dual

Circuits for antennas near antiresonance

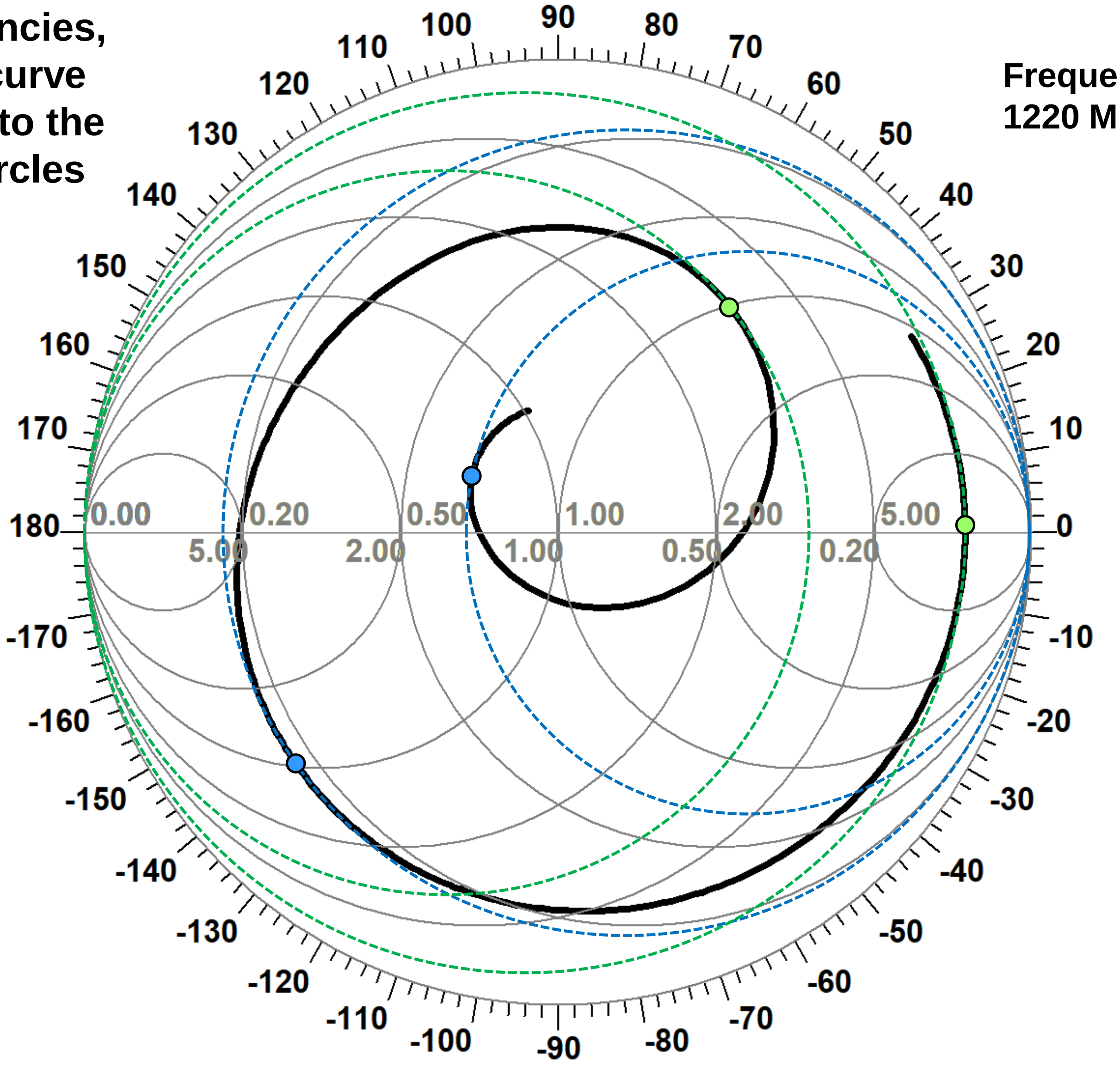
Series and Parallel RLC Equivalent Circuits

- **As an antenna impedance curve spirals around the Smith chart, there may be points of tangency with constant conductance circles or constant resistance circles**
  - "Tangency" here means location and direction, i.e. curvature sense
- **Each tangency corresponds to a 3-element RLC equivalent circuit**
  - Resistance circle tangency $\Rightarrow \dfrac{dR}{df} = 0 \Rightarrow$ a series RLC circuit
  - Conductance circle tangency $\Rightarrow \dfrac{dG}{df} = 0 \Rightarrow$ a parallel RLC circuit
- **An antenna can have many narrowband RLC equivalent circuits at different frequencies, one for each tangency**
- **R-circle and G-circle tangencies may alternate but don't have to**

The resonant frequencies of such equivalent circuits don't equal, or imply the existence of, antenna resonant frequencies!

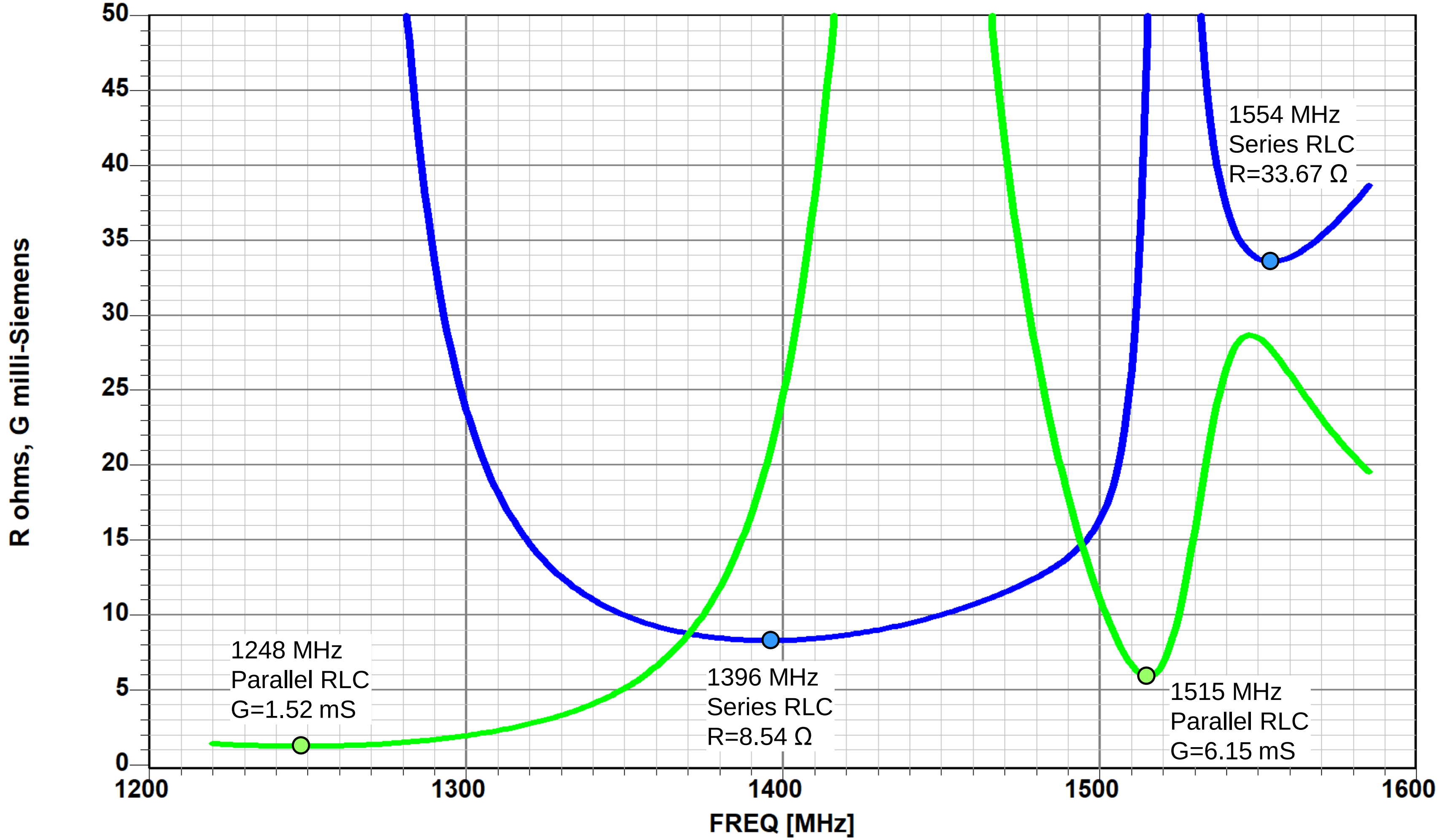
Impedance Tangencies of GPS Antenna

At some frequencies, an impedance curve can be tangent to the *r*-circles or *g*-circles



Frequency sweep 1220 MHz to 1585 MHz

Best Series/Parallel RLC Equivalent Circuits



The best series/parallel RLC approximations occur at *r*-circle or *g*-circle tangencies, which occur at *R* and *G* minima, not at resonances.

Conditions for Reactance and Susceptance Minima

- **From the Poisson/Schwarz integrals**

$$\frac{dR(\omega)}{d\omega}=0 \qquad \Leftrightarrow \qquad \int_0^{\infty}\frac{\omega^2}{u^2-\omega^2}\left(\frac{dX(u)}{du}\right)du = X(0)$$

and

Susceptance, not decibel

$$\frac{dG(\omega)}{d\omega}=0 \qquad \Leftrightarrow \qquad \int_0^{\infty}\frac{\omega^2}{u^2-\omega^2}\left(\frac{dB(u)}{du}\right)du = B(0)$$

Defective Equivalent Circuits

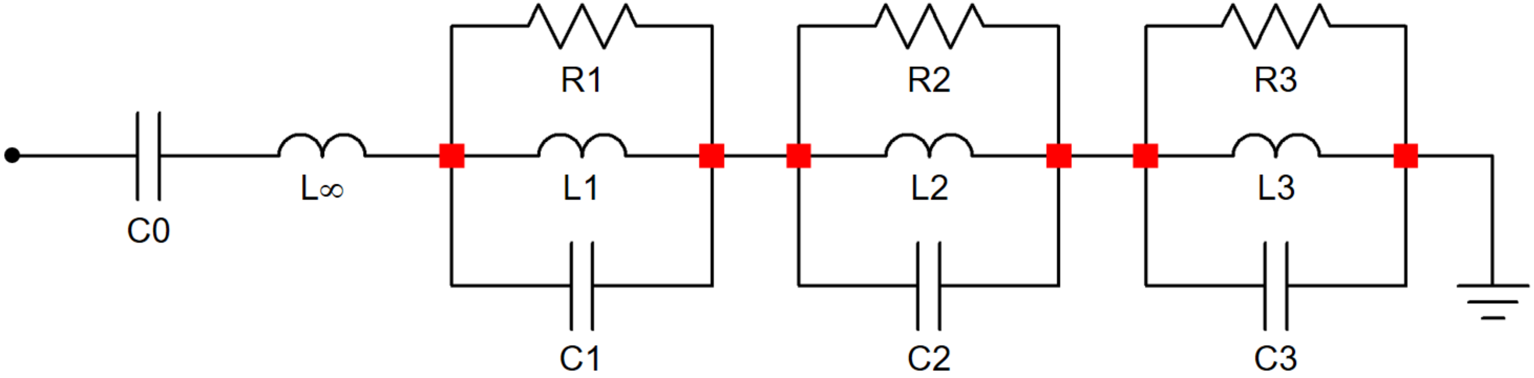
Models vs equivalent circuits

Ad hoc

Unrealizable

Problematic stabililty

Foster's 1st Form, Modified for Small Losses



C0 = 43.9 pF
L∞ = 4.49 mH

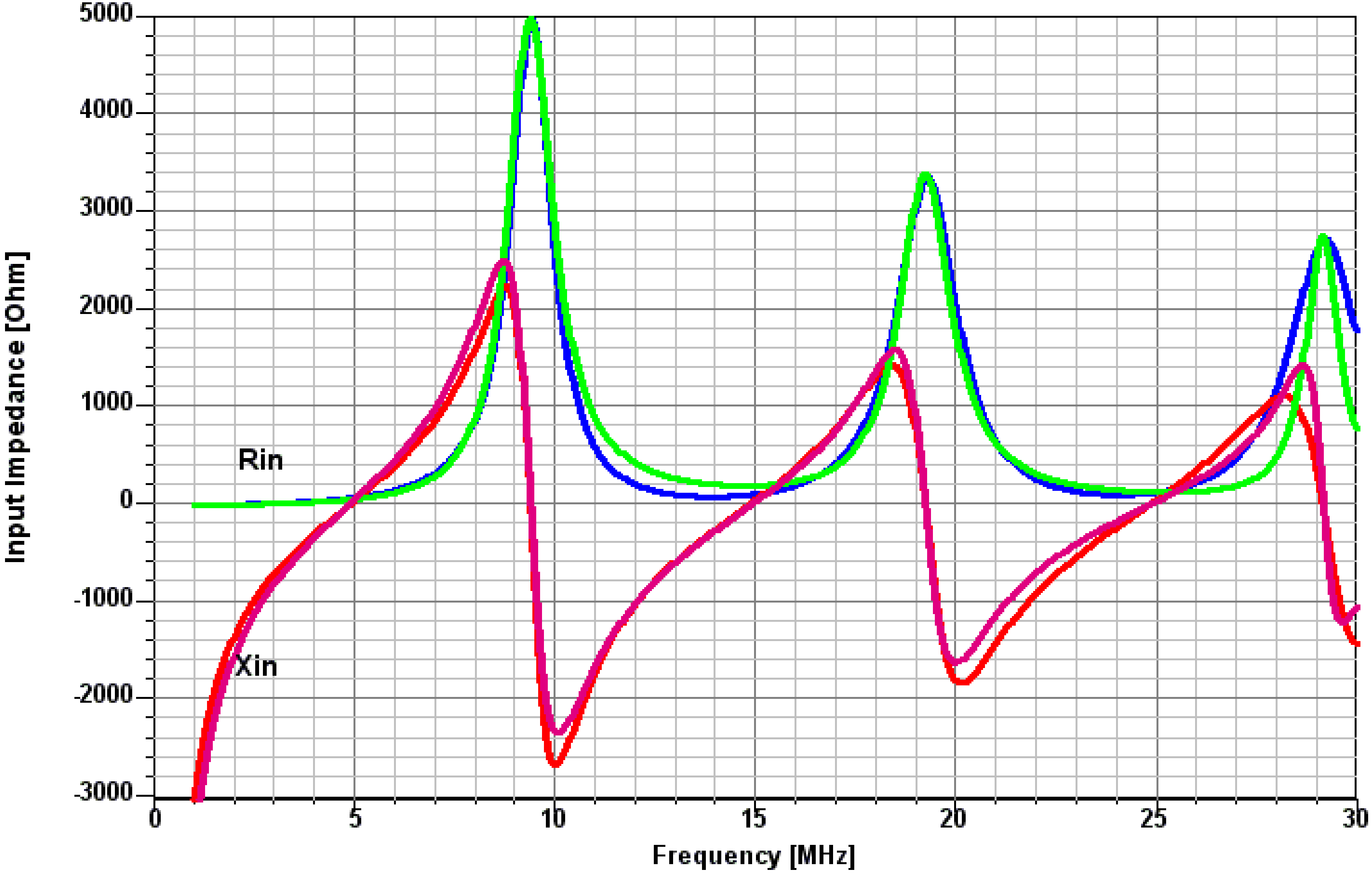
C1 = 22.9 pF
L1 = 12.5 mH
R1 = 4,970 Ω

C2 = 30.3 pF
L2 = 2.26 mH
R2 = 3,338 Ω

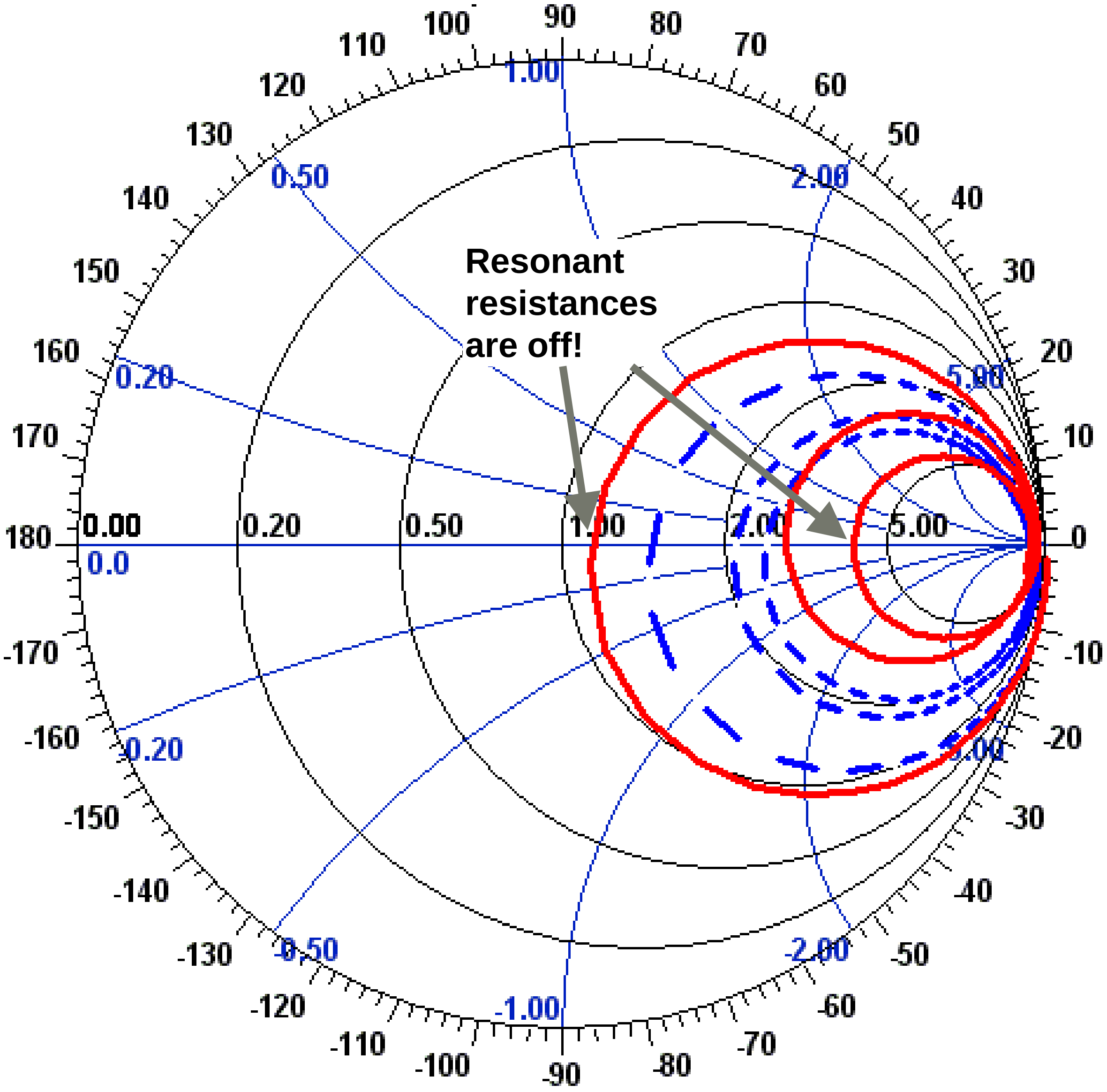
C3 = 57.1 pF
L3 = 522 nH
R3 = 2,702 Ω

- **Ramo, Whinnery, and Van Duzer, *Fields and Waves in Communication Electronics*, Wiley, 1965. Section 11.13**
  - Tang-Tieng-Gunn (1993)
  - Hamid-Hamid (1997)
  - Rambabu-Ramesh-Kalghatgi (1999)
- **Fits dipole impedance better near antiresonances, worse near resonances**

Accuracy of Hamid & Hamid’s Equivalent Circuit

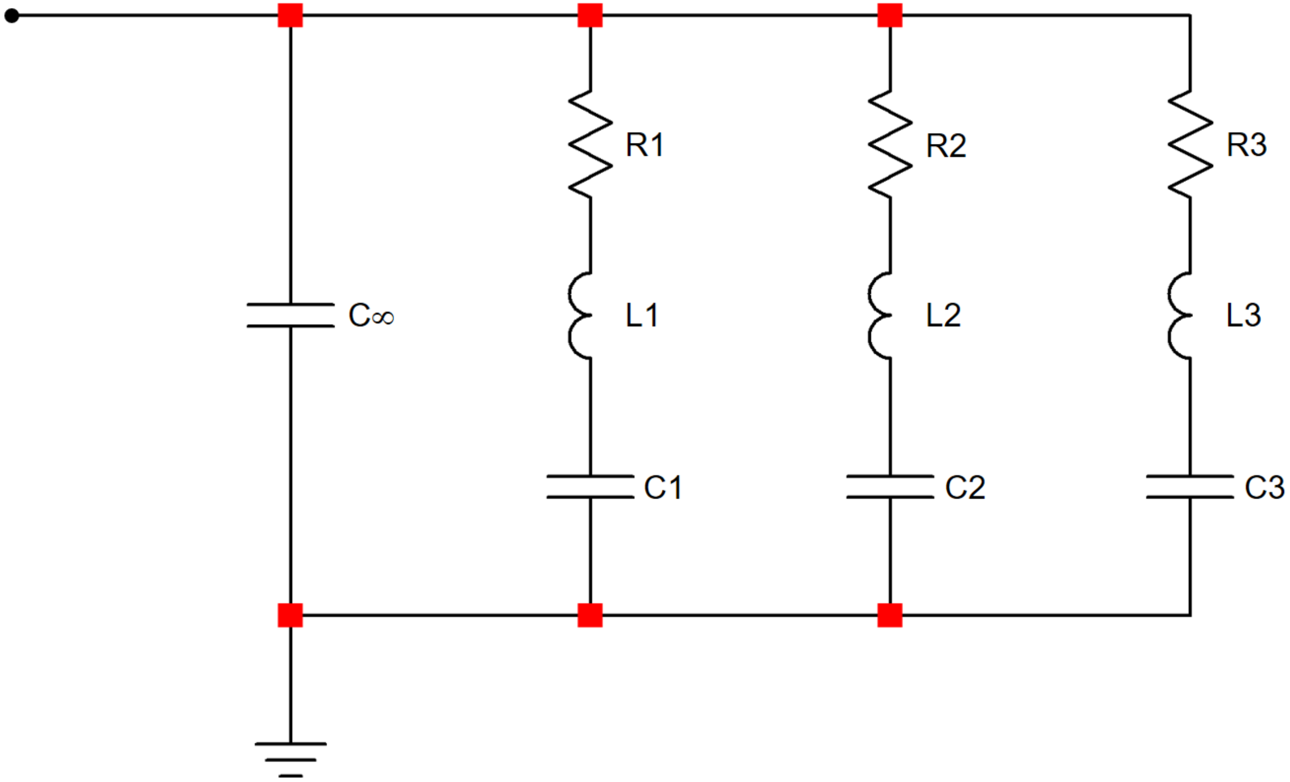


Accuracy of Hamid & Hamid’s Equivalent Circuit



Resonant resistances are off!

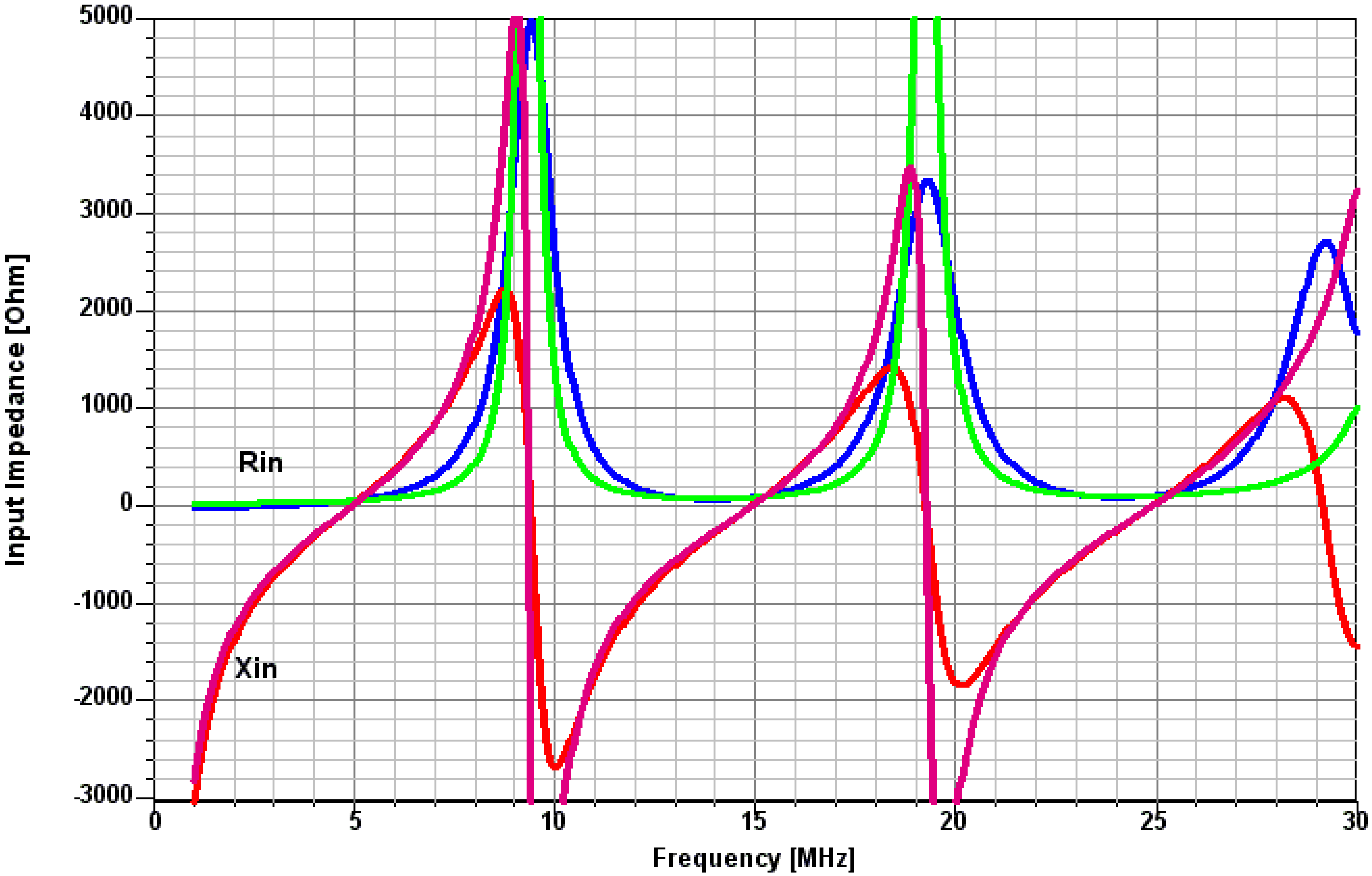
Foster's 2nd Form, Modified for Small Losses



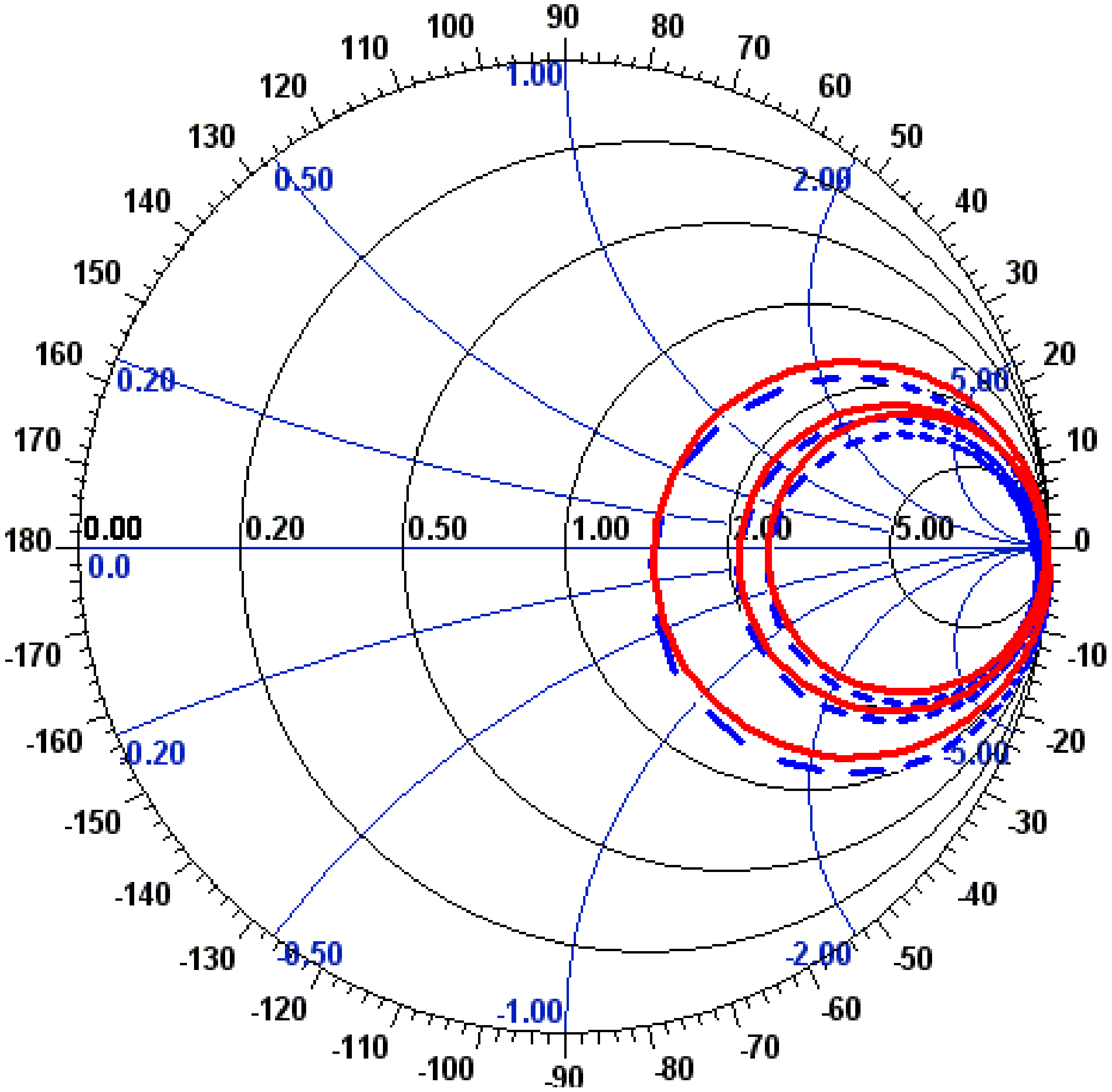
C∞ = 5.44 pF
C1 = 42.9 pF
C2 = 5.05 pF
C3 = 1.92 pF
L0 = ∞
L1 = 24.9 mH
L2 = 22.8 mH
L3 = 21.4 mH
R1 = 72.2 Ω
R2 = 106 Ω
R3 = 122 Ω

- **Fits dipole impedance better near resonances, worse near antiresonances**
- **McKinley *et al*. (2012) used similar topology for circular loop antennas**

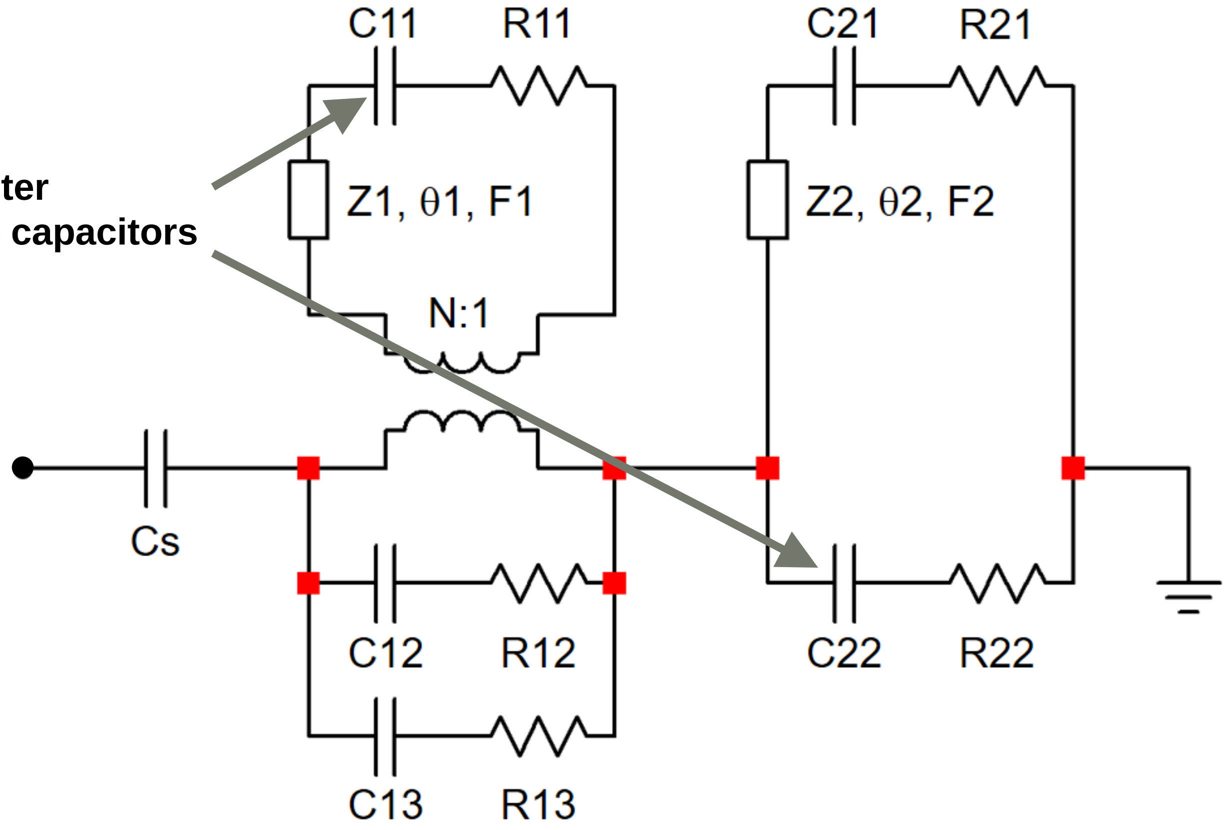
Accuracy of Foster's 2nd Form With Small Losses



Accuracy of Foster's 2nd Form With Small Losses



Long, Werner, & Werner's Broadband Model (2000) Frequency Scaled to f_0 = 5 MHz, Ω' = 7.8



Non-Foster negative capacitors

Cs = 150 pF
Z1 = 215 Ω
Z2 = 195 Ω
N = 1

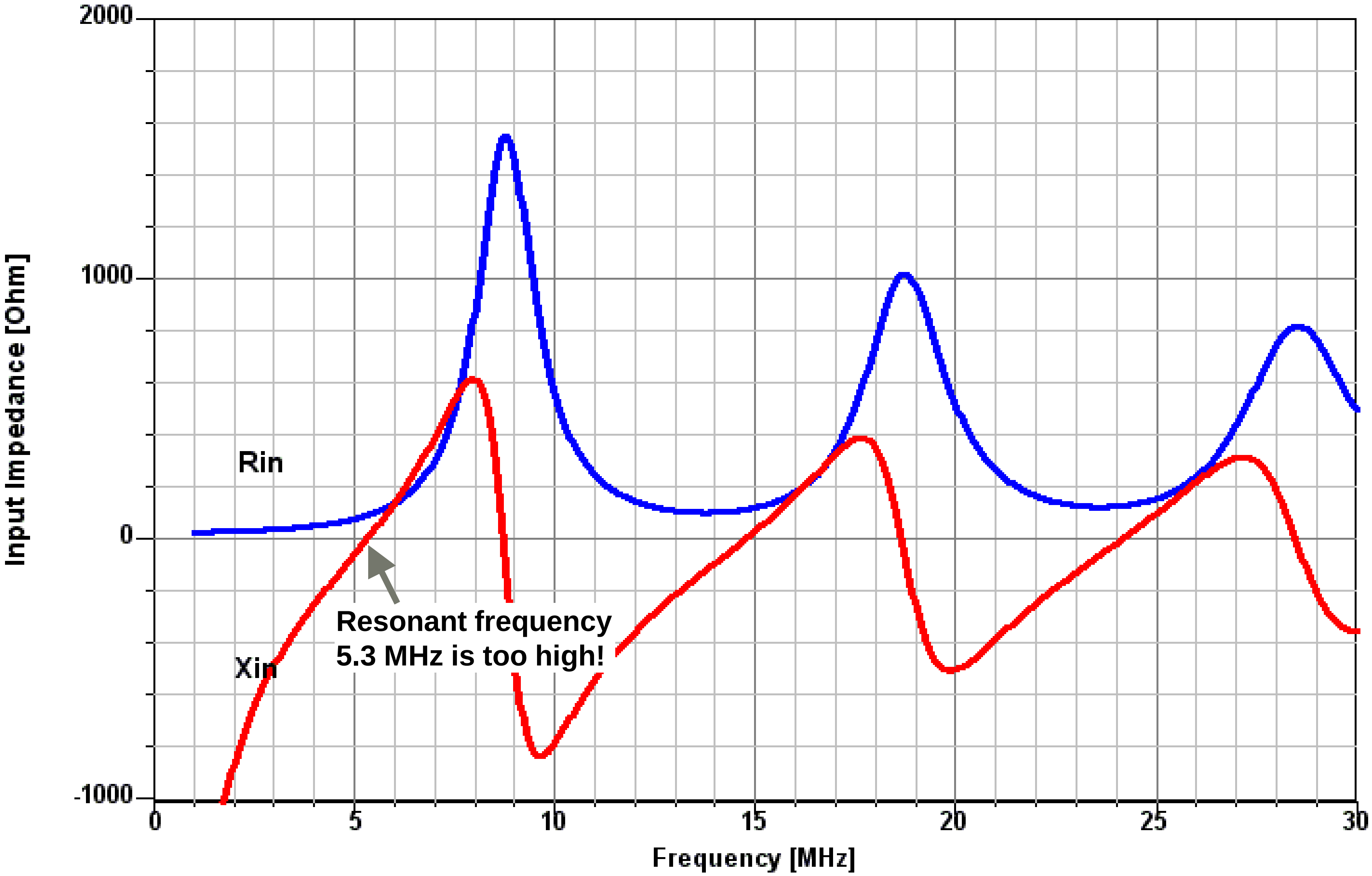
C11 = −975 pF
θ1 = 44.9°
C12 = 24.0 pF
C13 = 8.33 pF

R11 = 13.1 Ω
F1 = 5 MHz
R12 = 3,600 Ω
R13 = 500 Ω

C21 = 17.6 pF
θ2 = 46.9°
C22 = −3.00 pF

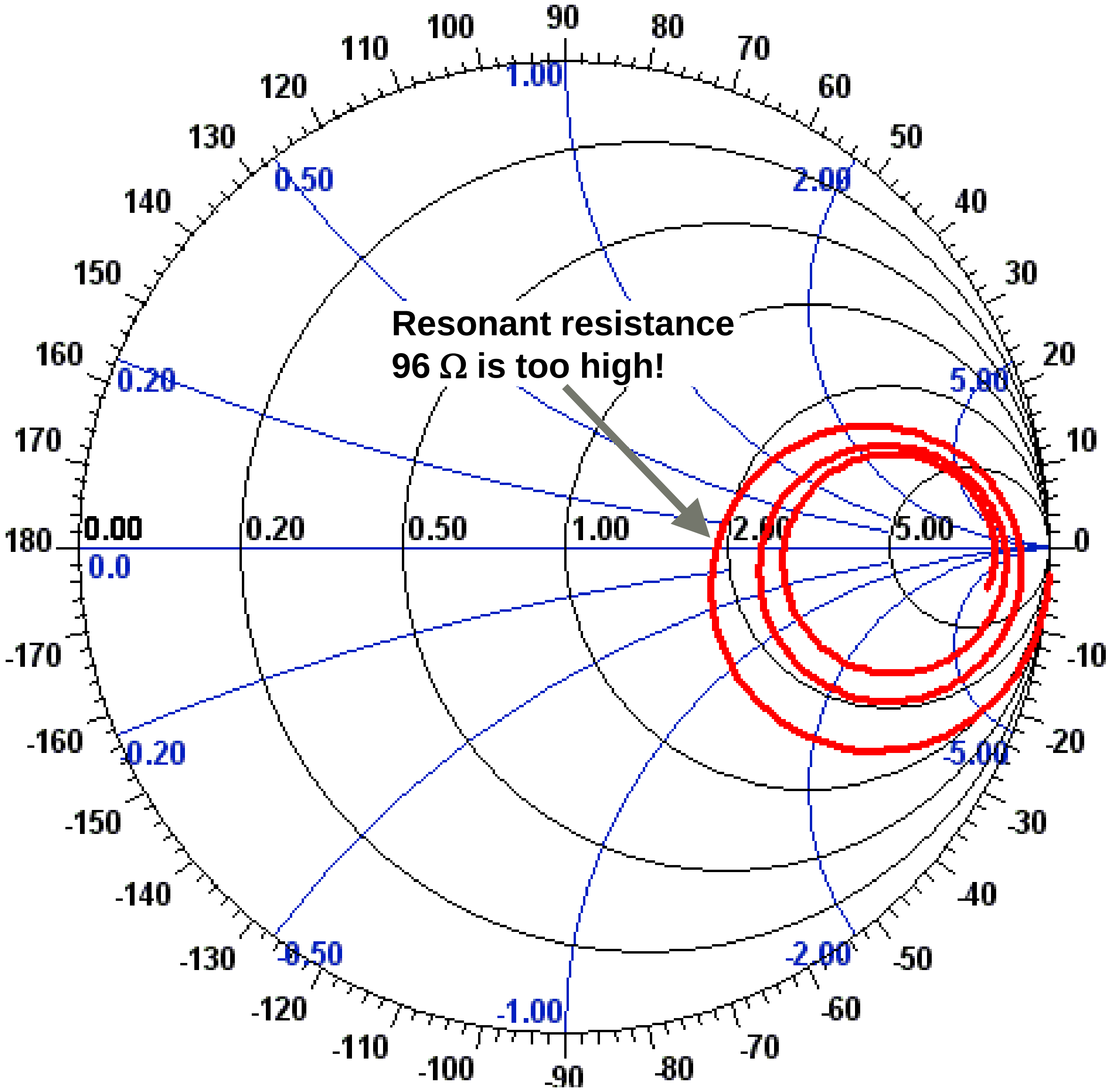
R21 = 700 Ω
F2 = 5 MHz
R22 = 295 Ω

Accuracy of Long, Werner, & Werner's Model



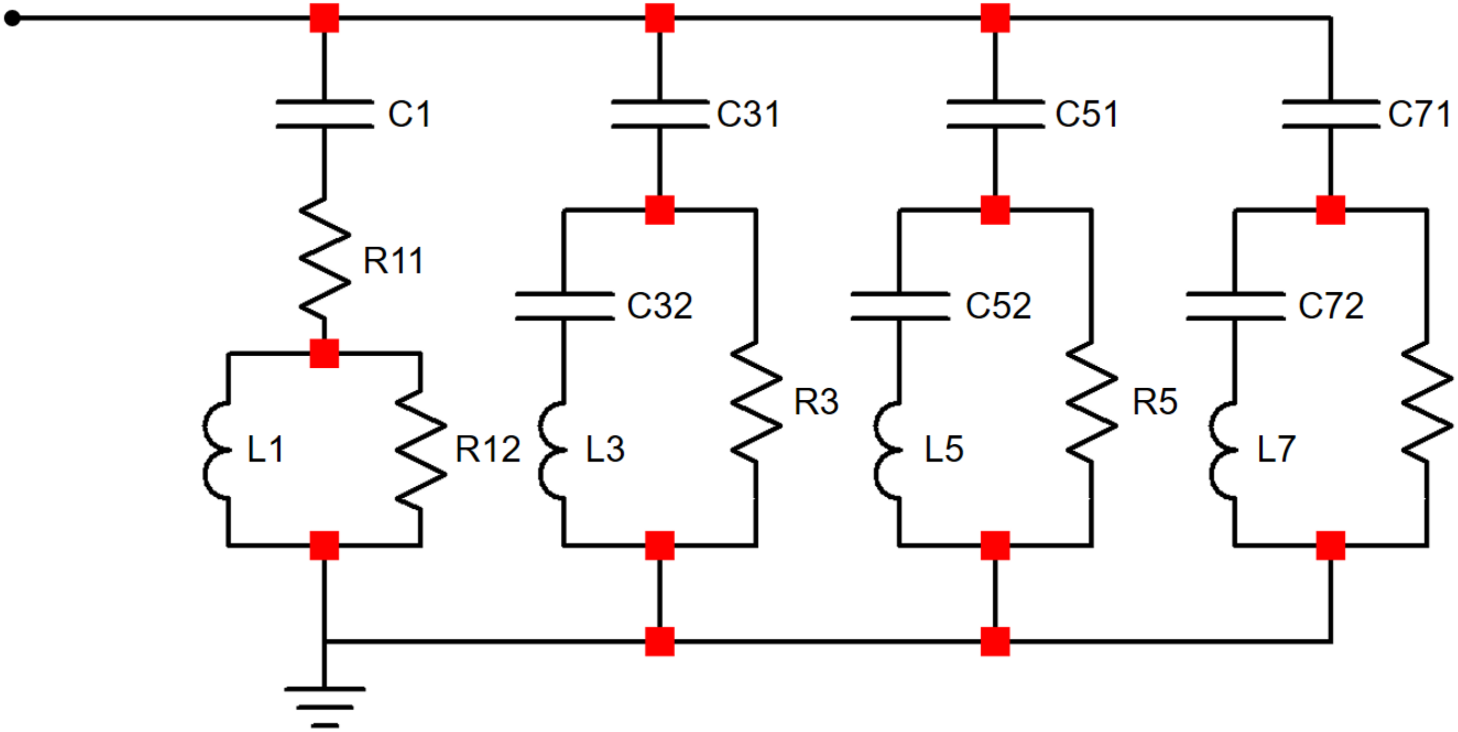
Resonant frequency
5.3 MHz is too high!

Accuracy of Long, Werner, & Werner's Model



Resonant resistance
96 Ω is too high!

Streable & Pearson's Broadband Equivalent Circuit Frequency Scaled to $f_0 = 5$ MHz, $\Omega' = 10.6$



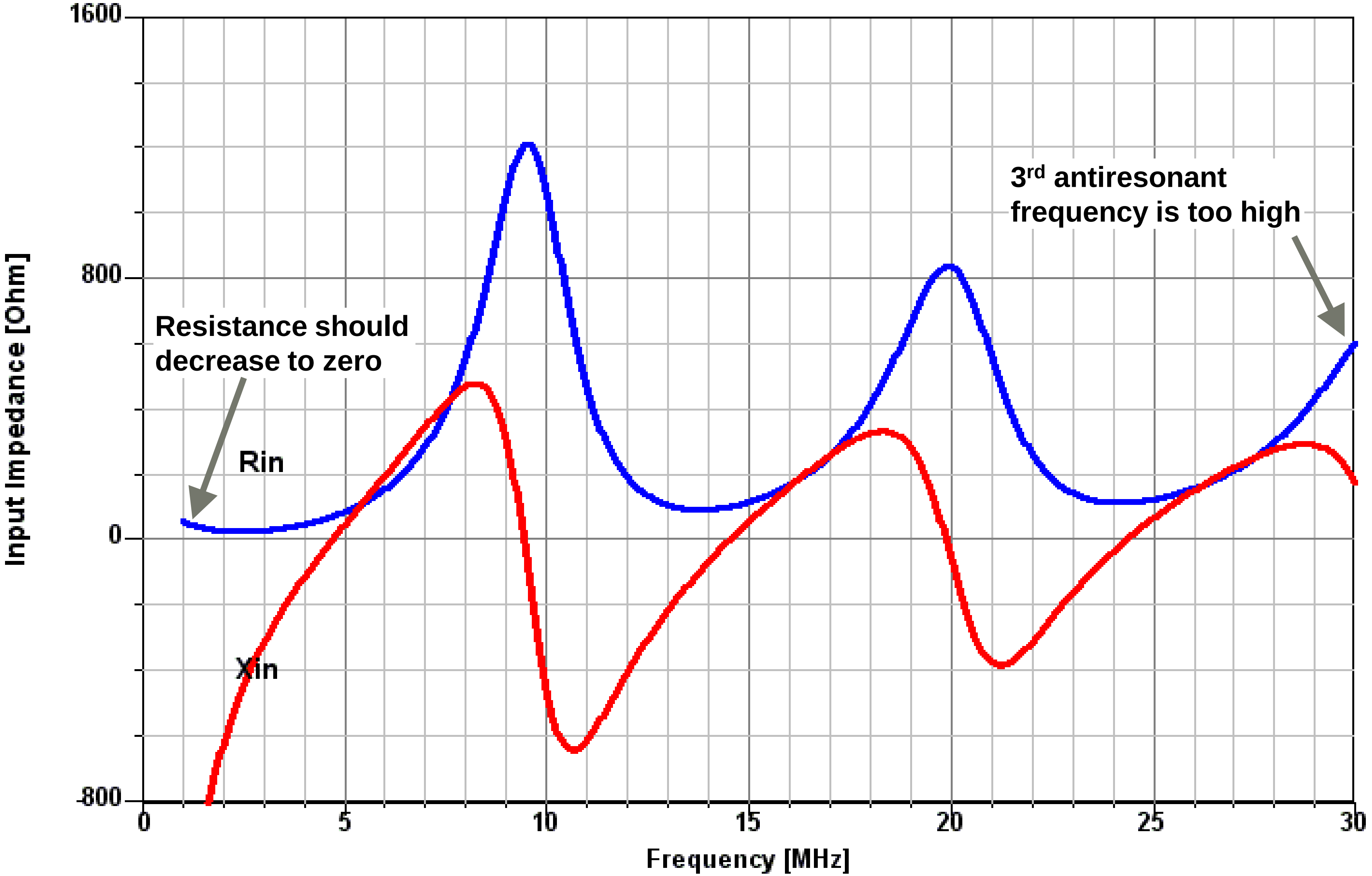
C1 = 86.6 pF
L1 = 13.8 mH
R11 = 0.663 Ω
R12 = 2,201 Ω

C31 = 15.0 pF
C32 = 33.8 pF
L3 = 11.7 mH
R3 = 4,959 Ω

C51 = 7.17 pF
C52 = 8.87 pF
L5 = 10.9 mH
R5 = 6,514 Ω

C71 = 4.51 pF
C72 = 3.98 pF
L7 = 10.3 mH
R7 = 7,542 Ω

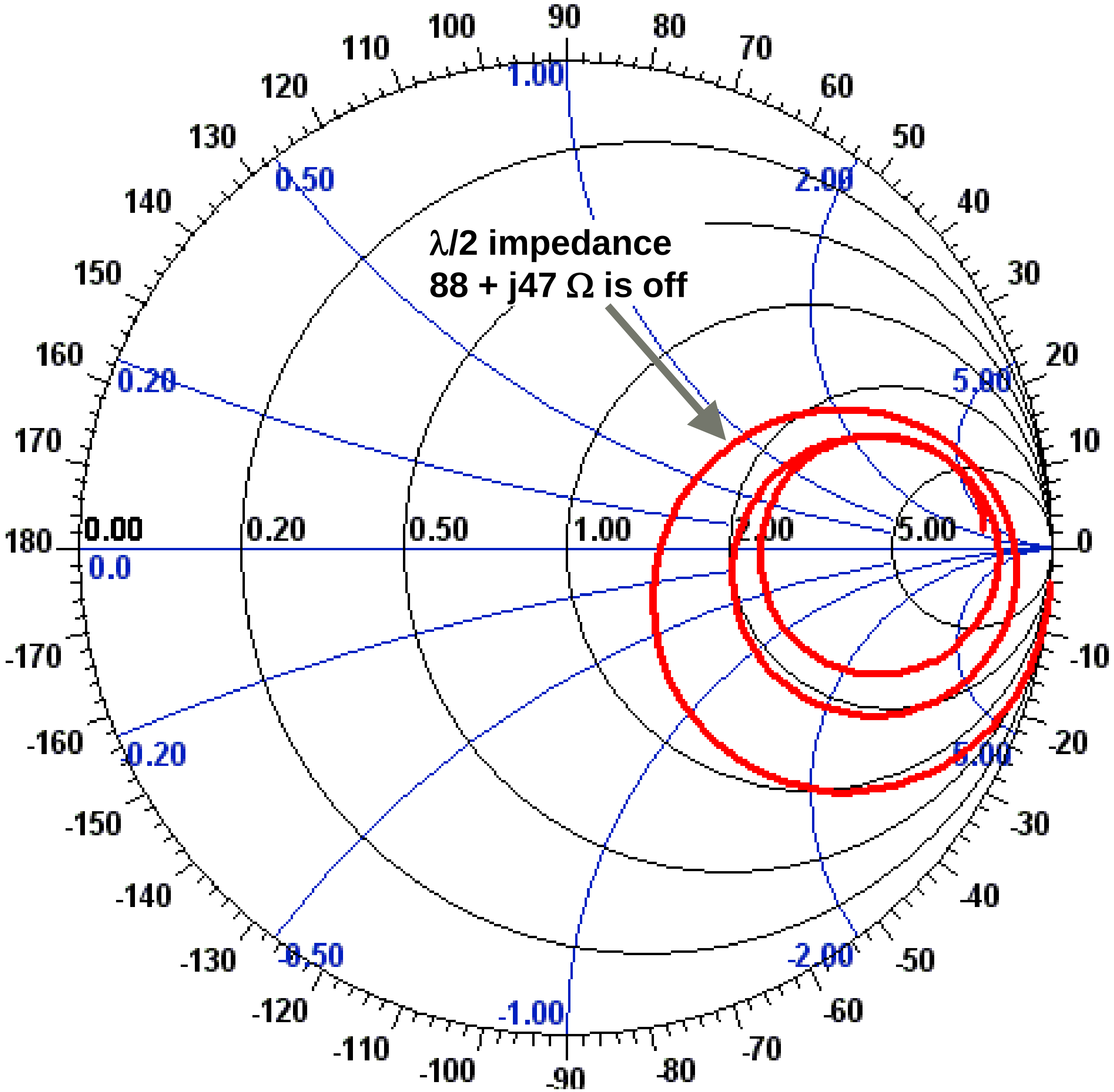
Accuracy of Streable & Pearson’s Equivalent Circuit



Resistance should decrease to zero

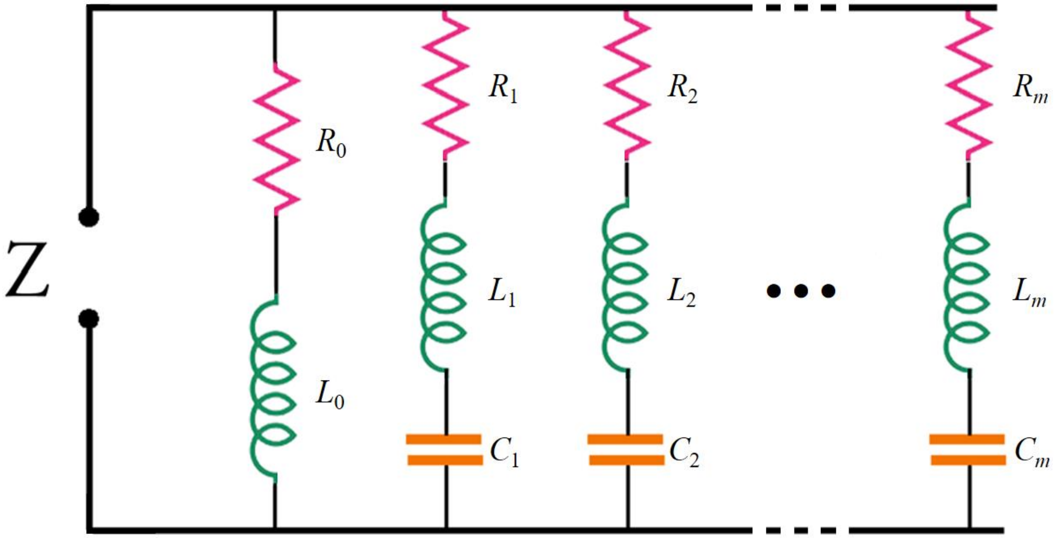
3rd antiresonant frequency is too high

Accuracy of Streable & Pearson’s Equivalent Circuit



λ/2 impedance
88 + j47 Ω is off

McKinley *et al*. Circular Loop Antenna Model



- **McKinley *et al*. (2012) broadband model for large circular loop antennas derived from analyses of Storer (1955) and Wu (1962)**
- **Circuit topology similar to Foster's 2nd form with loss**
- **Absence of capacitor in 1st stage puts admittance pole near origin and antiresonance occurs below resonance**
- **Model is unrealizable because elements are functions of frequency, not constants**
- **Assumed incorrectly that each current expansion function corresponds to a single system mode and natural frequency, thereby confusing expansion functions with modes**
- **Moreover, the modes of circular loop antennas were already solved by R.F. Blackburn (1976)**

A.F. McKinley, *et al*., "The Analytical Basis for the Resonances and Anti-Resonances of Loop Antennas and Meta-Material Ring Resonators," *J. Applied Physics*, vol. 112, no. 9, Nov. 2012.
J.E. Storer, "Impedance of Thin-Wire Loop Antennas," *Transactions of the AIEE*, vol. 75, no. 5, pp. 606-619, Nov. 1956.
T.T. Wu, Theory of the Thin Circular Loop Antenna, *J. of Mathematical Physics*, vol. 3, no. 6, pp. 1301-1304, Nov-Dec. 1962.
R.F. Blackburn, *Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method*, Ph.D. dissertation, U. Mississippi, ADA033089, USAF, Nov. 1976.

Problematic “Equivalent Circuits”

- **Antenna models that use non-passive elements, such as square-law resistors, $R(f) = Rf^2$, are active and therefore unrealizable and/or problematic with respect to stability**
  - Witt (1995)
  - Long, Werner, and Werner (2000)
  - Rudish and Sussman-Fort (2002)
  - Aberle and Romak (2007)
  - Karawas and Collin (2008)
  - McKinley, White, Maksymov, and Catchpole (2012)
- **Such models are mathematical representations of impedance but should not be called “equivalent circuits”**
- **The term “equivalent circuit” is reserved for passive, realizable circuit models of impedance**

Comparison of Antenna Equivalent Circuits by the author, Pacificon 2003, 2007

Antenna Impedance Model	Approximation Accuracy	Realizable Equivalent Circuit	Darlington Form	Element Types	Maximum Frequency Range
Series RLC	fair	yes	yes	*R, L, C*	0.94 f_0 to 1.05 f_0
Witt model	fair	no	no	*R*(*f*) and TL stub	0.6 f_0 to 1.2 f_0
Chu 3-Element	good	yes	yes	*R, L, C*	0.90 f_0 to 1.08 f_0
Tang-Tien-Gunn 4-Element	excellent	yes	yes	*R, L, C*	DC to 1.4 f_0
Schelkunoff 4-Element	excellent	yes	yes	*R,L,C*,TL	DC to 1.4 f_0
Author's 5-Element	excellent	yes	yes	*R, L, C*	DC to 1.4 f_0
Fosters 1st Form with small losses	poor, best near antiresonances	yes	no	*R, L, C*	no limit
Fosters 2nd Form with small losses	poor, best near resonances	yes	no	*R, L, C*	no limit
Long-Werner-Werner	fair	no	no	*R, C*, TL	5 octaves
Streable-Pearson	good	yes	no	*R, L, C*	no limit
Author's Broadband	excellent	yes	no	*R, L, C*	no limit
Schelkunoff TL cascade	fair	yes	yes	*R, L, C, TL*	limited
Spherical TE-TM modes	excellent	yes	no	*R, L, C*	no limit

Vibration Theory of Bells

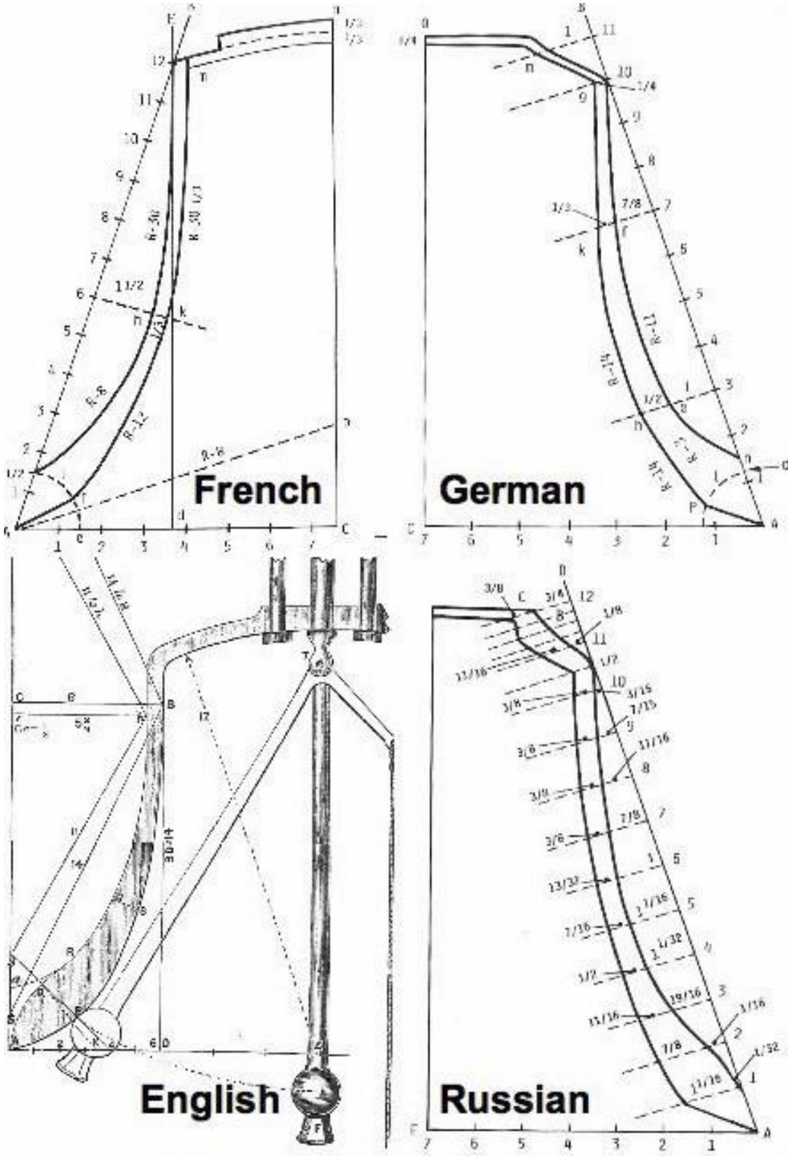
A Brief Interlude

Bells

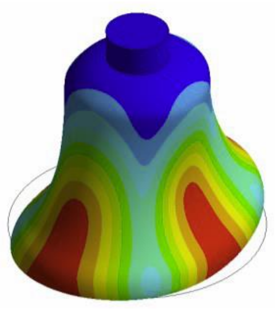
- Large bells are found in churches, cathedrals, and monasteries, and public buildings
- Weights range from 300 lbs to 202 (possibly 300) tons

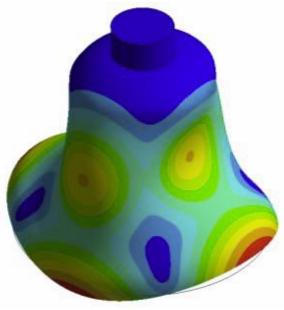
Tsarsky Kolokol III (Royal Bell 3), Kremlin

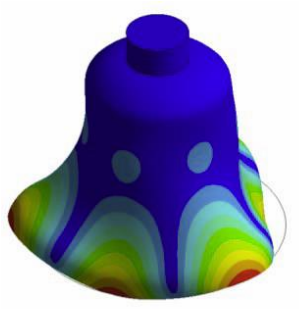
Bell Shapes Vary by Foundry and Country

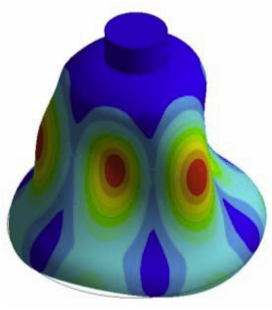


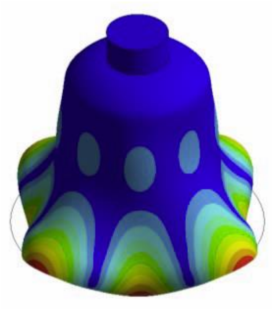
Vibration Modes Computed by FEM (ANSYS)

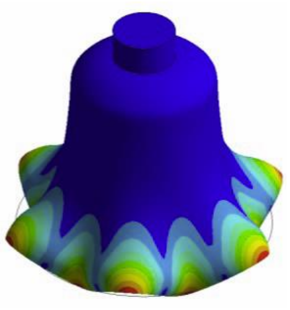

Hum


Prime


Tierce


Quint


Nominal

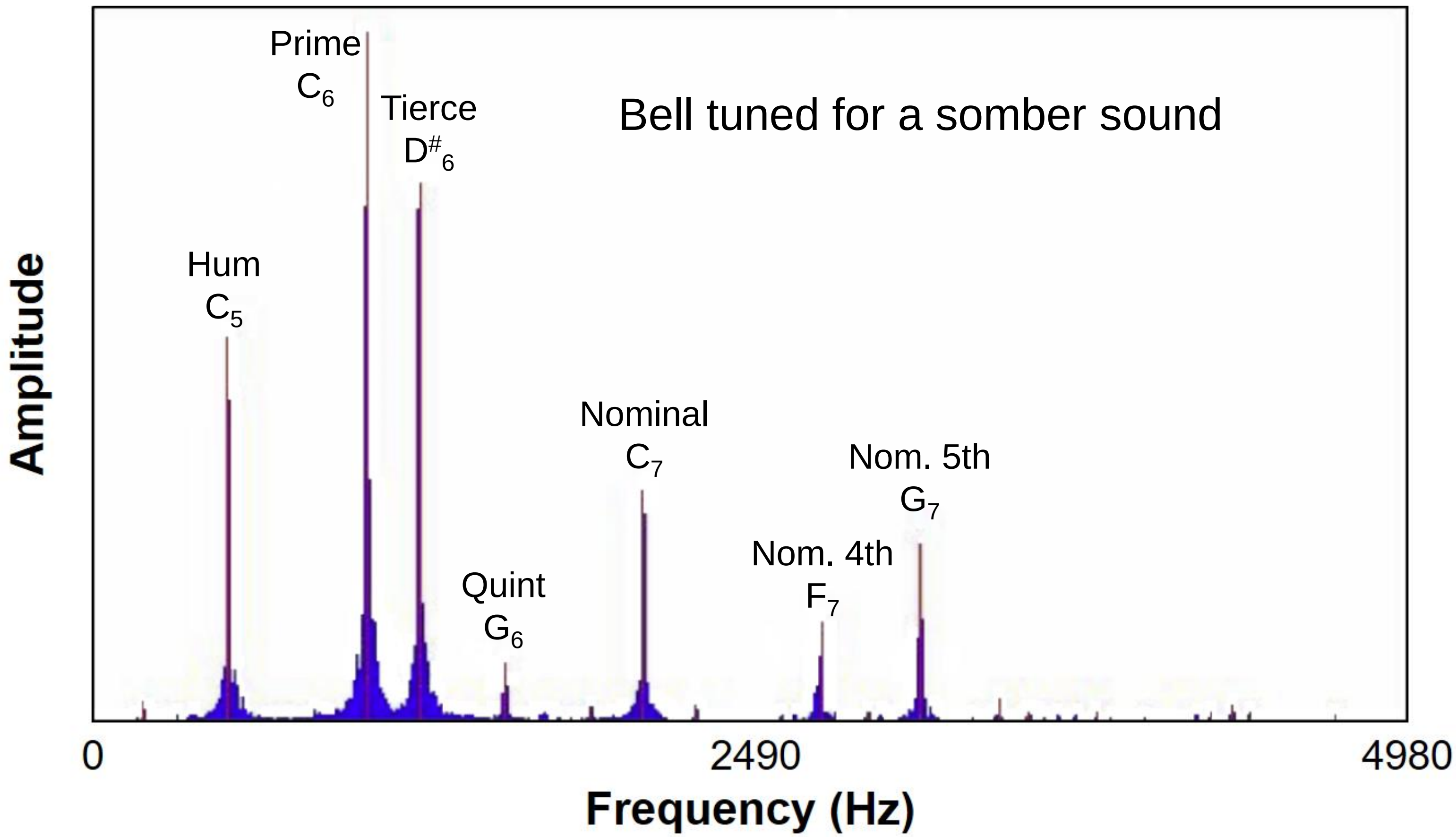

Octave

- **A bell’s sound is not simple; transient response has early and late parts**
  - Early response is rough, discordant
  - Late response has many tones (modes), each with different decay rate
    - https://www.gregniemeyer.com/tsarsky
  - Bells have at least five audible “partial” tones
    - Hum: The lowest partial, one octave below Prime
    - Prime: The main partial, one octave above Hum
    - Tierce: A minor third (6:5) above Prime
    - Quint: A perfect fifth (3:2) above Prime
    - Nominal: One octave (2:1) above Prime
  - The final late tone, after decay, is pure, sinusoidal steady state

Bells are Tuned at the Foundry

- Formerly done by hammer and chisel, using ear and tuning fork
- Now done by vertical lathe and spectrum analyzer

Example Spectrum After Tuning



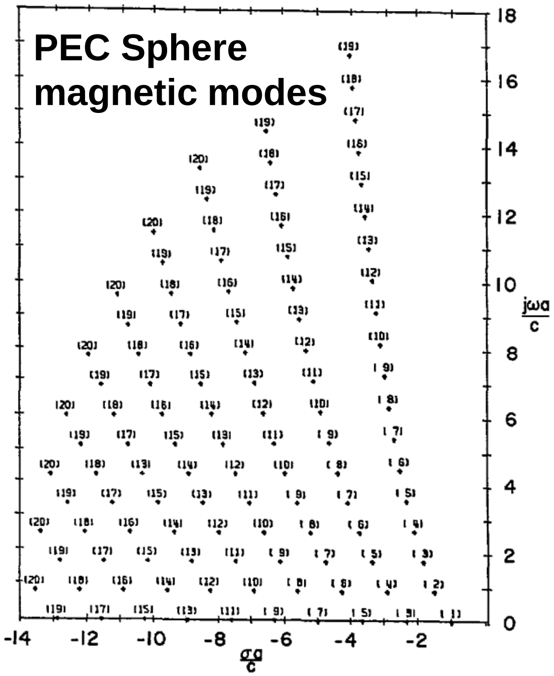
D. Bartocha and C. Baron, "… Bells' Tone," *Archives of Foundry Engineering*, Oct 2016.

Universal Equivalent Circuits

For all antennas

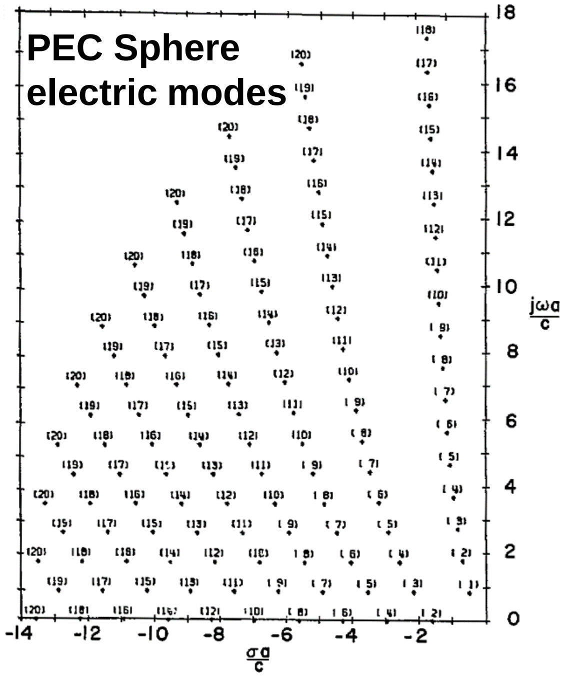
Over any bandwidth

Antennas Ring at Many Frequencies (Like Bells)



Real frequency $\omega \rightarrow$

Real part σ of complex frequency $s \rightarrow$

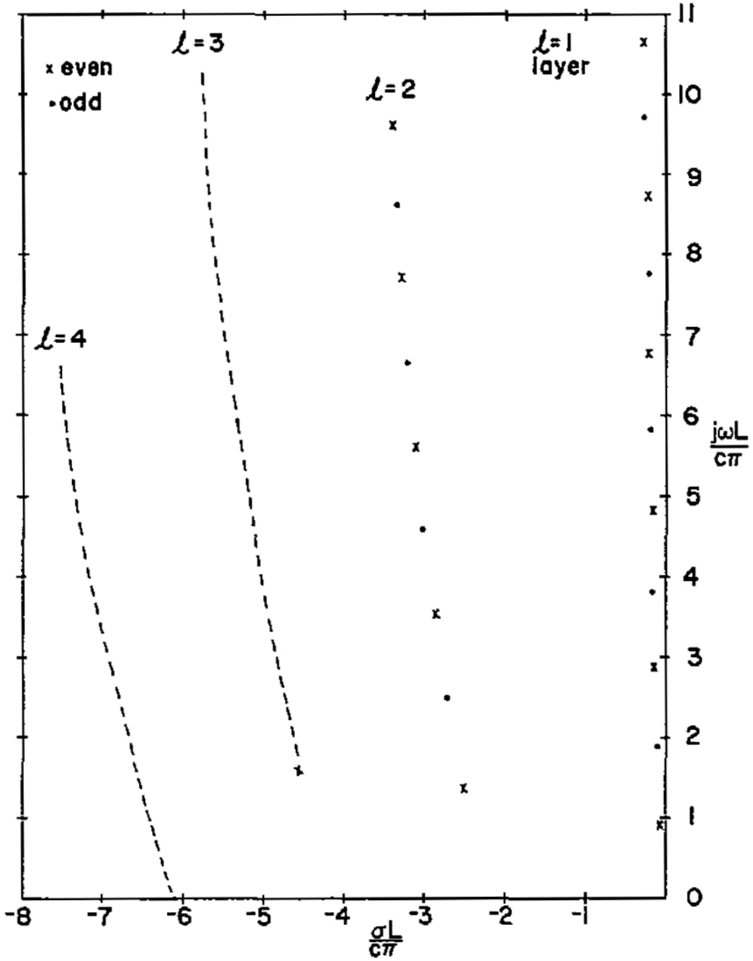


Real frequency $\omega \rightarrow$

Real part σ of complex frequency $s \rightarrow$

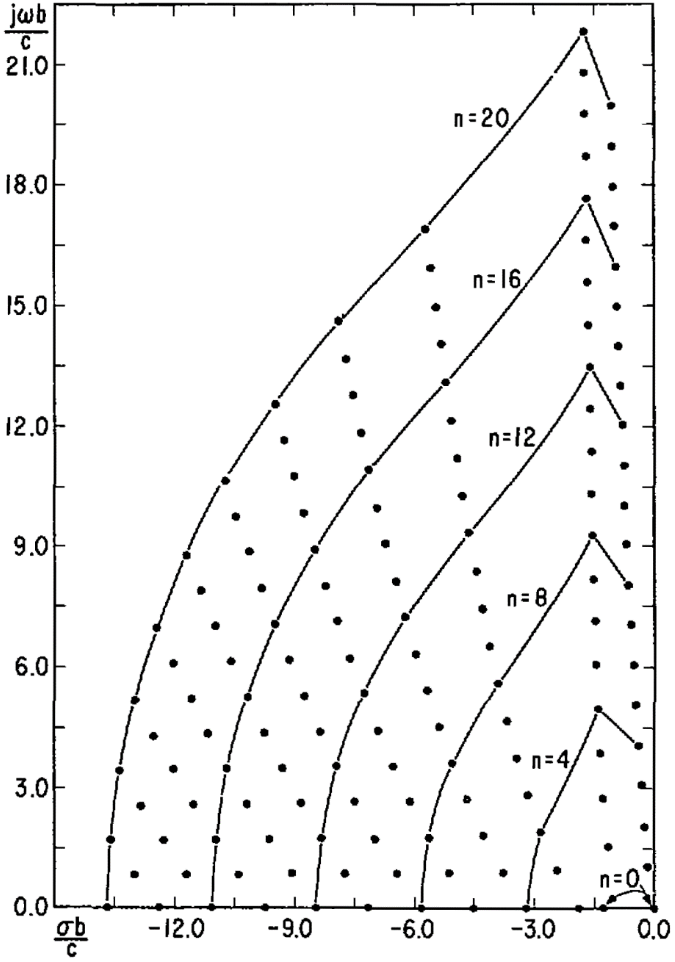
- **Continuous electromagnetic structures (antennas) have early and late responses like bells**
  - An impulse of voltage or current applied at a point on the structure causes currents to build up everywhere on the structure
  - Impedance resonances are determined by how modal currents sum

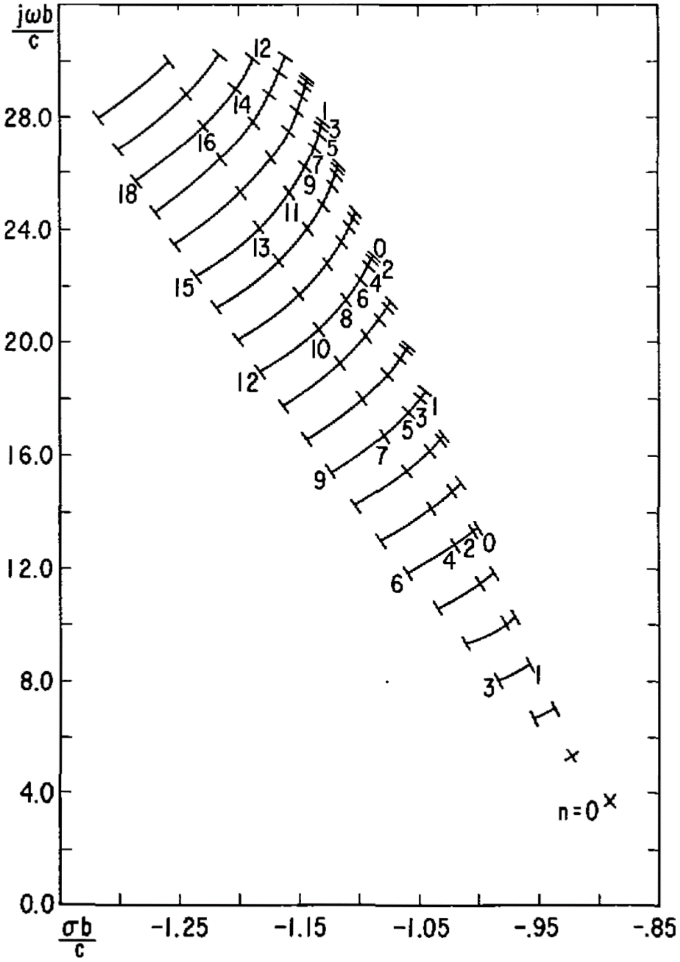
Dipole and Monopole Natural Frequencies (Admittance Poles)



F.M. Tesche, “On the Analysis of Scattering and Antenna Problems Using the Singularity Expansion Technique,” *IEEE Transactions on Antennas and Propagation*, vol. 21, no. 1, pp. 53-62, January 1973.

Circular Loop Antenna Natural Frequencies (Admittance Poles)





R.F. Blackburn, *Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method*, Ph.D. dissertation, U. Mississippi, May 1976.
R.F. Blackburn and D.R. Wilton, “Analysis and Synthesis of an Impedance-Loaded Loop Antenna Using the Singularity Expansion Method,” *IEEE Transactions on Antennas and Propagation*, vol. 26, no. 1, pp. 136-140, January 1978.

Meromorphic Functions

- **A function *f* (*s*) is meromorphic if**
  - It can be expressed as a ratio of two analytic functions
  - Singularities are ordinary poles (infinity excepted)
  - Poles are isolated (poles have no accumulation/cluster points)
- **Consider meromorphic immittance functions that satisfy**
  - All poles lie in the left half of the complex s-plane
  - $f^*(s) = f(s^*)$
  - $\mathrm{Re}\{ f(j\omega)\}$ is non-negative
  - All poles of $f(s)$ are simple
- **Examples**
  - P.r. rational functions: $P(s)/Q(s)$
  - Complex exponentials: exp(s), exp(-s)
  - Trigonometric functions: sin(s), cos(s), tan(s)
  - Hyperbolic functions: sinh(s), cosh(s), tanh(s)
  - Many special functions of mathematical physics

Mittag-Leffler Theorem

- **Mittag-Leffler states a convergent series for $f\,(s)$**

$$f\,(s) = f\,(0) + \lim_{N\to\infty} \sum_{n=-N}^{N} \left(\frac{A_n}{s - s_n} - \frac{A_n}{0 - s_n} \right)$$

$$= f\,(0) + \lim_{N\to\infty} \left(P_N(s) - P_N(0) \right)$$

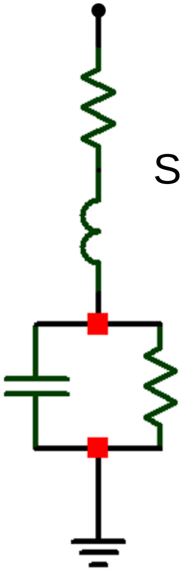
where

$$P_N(s) = \sum_{n=-N}^{N} \frac{A_n}{s - s_n}$$

- **Yields a design recipe for a network that realizes $f\,(s)$**
  - Step 1: Determine the poles of s_n of $f\,(s)$
  - Step 2: Determine the residues A_n

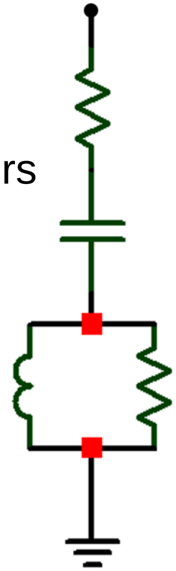
$$A_n = \lim_{s\to s_n}(s - s_n)\,f\,(s)$$

Universal Equivalent Circuits – Four Kinds of Stages

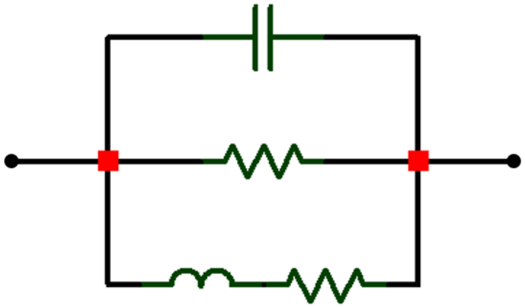


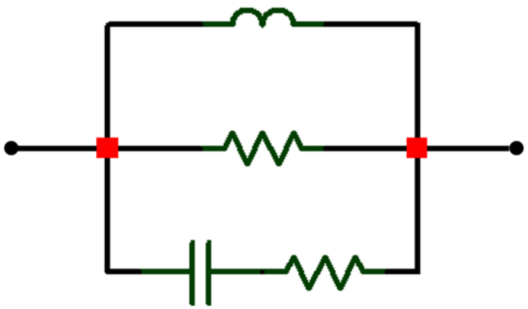
Stages for Type 2 parallel ladders

- Stages must have four degrees of freedom
- Needed to set complex pole location and complex residue



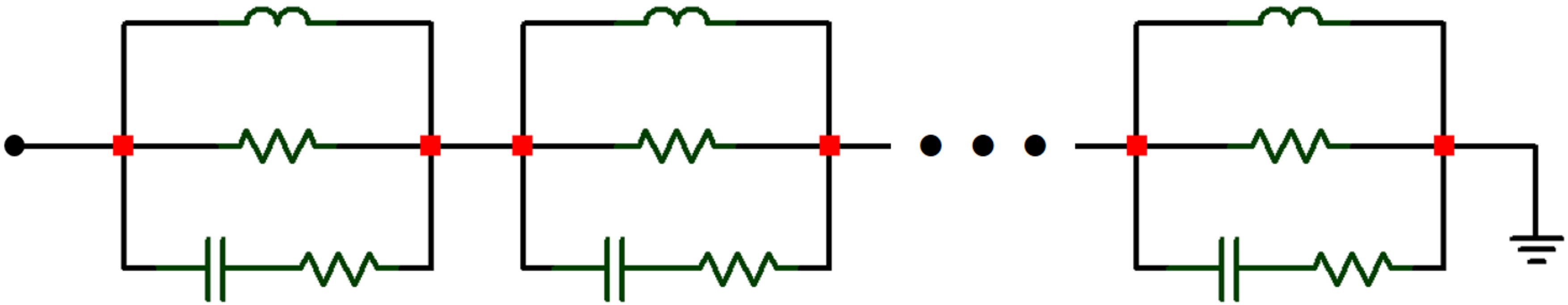
Stages for Type 1 series ladders





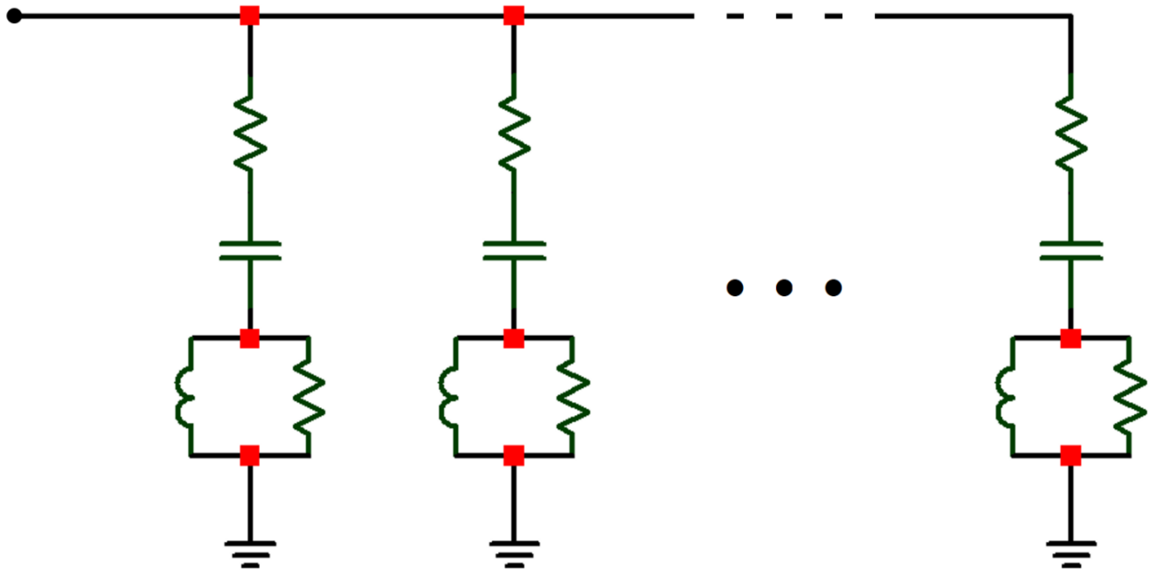
M. K. Zinn, “Network Representation of Transcendental Impedance Functions,” *Bell System Technical Journal*, vol. 31, no. 2, pp. 378-404, March 1952.
S. A. Schelkunoff, “Representation of Impedance Functions in Terms of Resonant Frequencies,” *Proceedings of the IRE*, vol. 32, no. 2, pp. 83-90, February 1944.

Universal Equivalent Circuit – Type 1b



- Type 1b ladder based on impedance poles (admittance zeros)
- Similar to Foster's 1st form but with two resistors per stage
- Each stage gives a conjugate pair of poles of impedance
- Ladder truncation to finite length justified by Mittag-Leffler
- Chu's TE_1 equivalent circuit is a special case (one stage)
- Circuit can be converted to single-resistor form by Darlington synthesis

Universal Equivalent Circuit – Type 2b



- Type 2b ladder based on admittance poles (impedance zeros)
- Similar to Foster’s 2nd form but with two resistors per stage
- Each stage gives a conjugate pair of poles of admittance
- Ladder truncation to finite length justified by Mittag-Leffler
- Chu’s TM_1 equivalent circuit is a special case (one stage)
- Circuit can be converted to single-resistor form by Darlington synthesis

General Applicability to Antennas and Devices

- **Universal Equivalent Circuits given here represent <u>all</u> immittance functions that are meromorphic**
- **Claim: UECs represent the impedance functions of <u>all</u> passive linear antennas and microwave devices**
- **Reasons**
  - Characterization by analysis
    - Impedances are expressed in terms of special functions
    - Almost all special functions of mathematical physics are meromorphic
    - Meromorphic functions are closed under sums, products, quotients, and powers
  - Characterization by numerical CEM or measurement
    - Impedance is determined by signal processing algorithms applied to impulse response or transfer function data
    - Prony and matrix pencil methods determine poles and residues successfully

The existence of exceptions is not ruled out, but none are known.

Mathematical Physics: Entire & Meromorphic Functions

Function	Symbol	Function	Symbol
Airy functions	$\mathrm{Ai}\,(z)$, $\mathrm{Bi}\,(z)$	haversine	$\mathrm{hav}\,(z)$
Airy function derivatives	$\mathrm{Ai}'\,(z)$, $\mathrm{Bi}'\,(z)$	hyperbolic cosine	$\cosh\,(z)$
Anger function	$\boldsymbol{J}_n\,(z)$	hyperbolic sine	$\sinh\,(z)$
Barnes G-function	$G\,(z)$	Jacobi elliptic functions	$\mathrm{cd}\,(u, k)$, $\mathrm{cn}\,(u, k)$, $\mathrm{cs}\,(u, k)$, $\mathrm{dc}\,(u, k)$, $\mathrm{dn}\,(u, k)$, $\mathrm{ds}\,(u, k)$, $\mathrm{nd}\,(u, k)$, $\mathrm{nc}\,(u, k)$, $\mathrm{ns}\,(u, k)$, $\mathrm{sd}\,(u, k)$, $\mathrm{sc}\,(u, k)$, $\mathrm{sn}\,(u, k)$
bei	$\mathrm{bei}_n\,(z)$	Jacobi theta functions	$\vartheta_n\,(z, q)$
ber	$\mathrm{ber}_n\,(z)$	Jacobi theta function derivatives	$\vartheta'_n\,(z, q)$
Bessel function of the first kind	$J_n\,(z)$	Mittag-Leffler function	$E_\alpha\,(z)$
Bessel function of the second kind	$Y_n\,(z)$	modified Struve function	$\mathcal{L}_n\,(z)$
Beurling’s function	$B\,(z)$	Neville theta functions	$\vartheta_c\,(u)$, $\vartheta_d\,(u)$, $\vartheta_n\,(u)$, $\vartheta_s\,(u)$
cosine	$\cos\,(z)$	Shi	$\mathrm{Shi}\,(z)$
coversine	$\mathrm{covers}\,(z)$	sine	$\sin\,(z)$
Dawson's integral	$F\,(z)$	sine integral	$\mathrm{Si}\,(z)$
erf	$\mathrm{erf}\,(z)$	spherical Bessel function of the first kind	$j_n\,(z)$
erfc	$\mathrm{erfc}\,(z)$	Struve function	$\boldsymbol{H}_n\,(z)$
erfi	$\mathrm{erfi}\,(z)$	versine	$\mathrm{vers}\,(z)$
exponential function	$e^z = \exp\,(z)$	Weber functions	$\boldsymbol{E}_n\,(z)$
Fresnel integrals	$C\,(z)$, $S\,(z)$	Wright function	$\phi\,(\rho, \beta; z)$
gamma function reciprocal	$1 / \Gamma\,(z)$	Landau’s xi-function	$\Xi\,(z)$ related to Rieman’s zeta function $\zeta\,(z)$
generalized hypergeometric function	${}_pF_q\,(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$		

Miscellaneous Examples Demonstrating UEC Theory

Open stub

Shorted stub

Thin-wire dipole

Small tuned loop

Large untuned loop (2 circuits)

VHF-UHF discone antenna

Fat VHF dipole

Examples 1 & 2: Transmission Line Devices

Open and shorted stubs

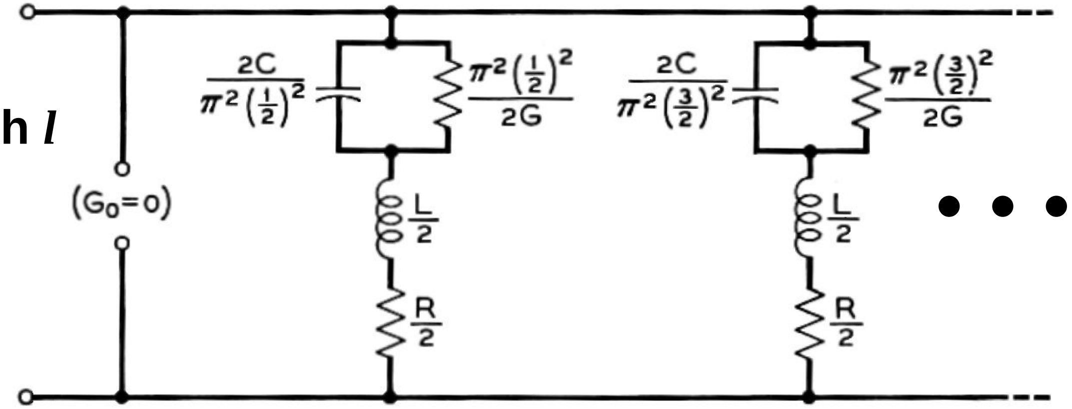
Open Stub Equivalent Circuits – Types 1a and 2a

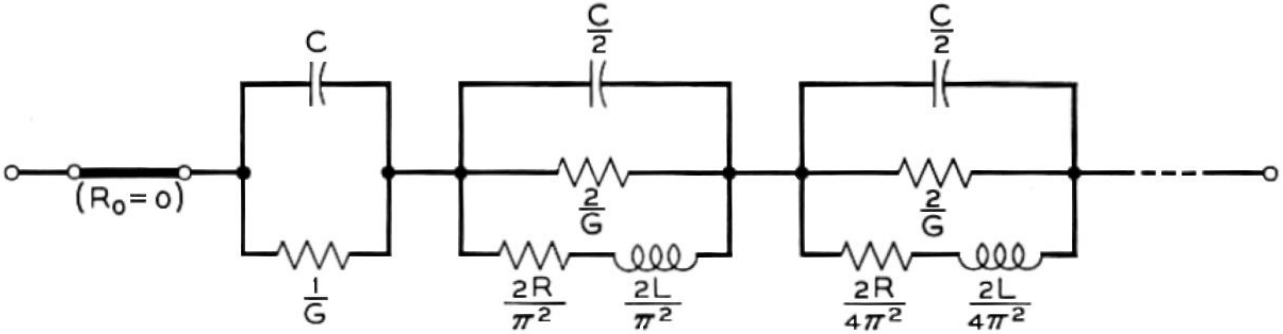
$$Z_{in} = Z_0 \coth \gamma l = Z_0 \coth(\alpha l + j\beta l)$$

- **Loss is frequency-independent**
- ***R*, *L*, *G*, *C* are for total line length *l***

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

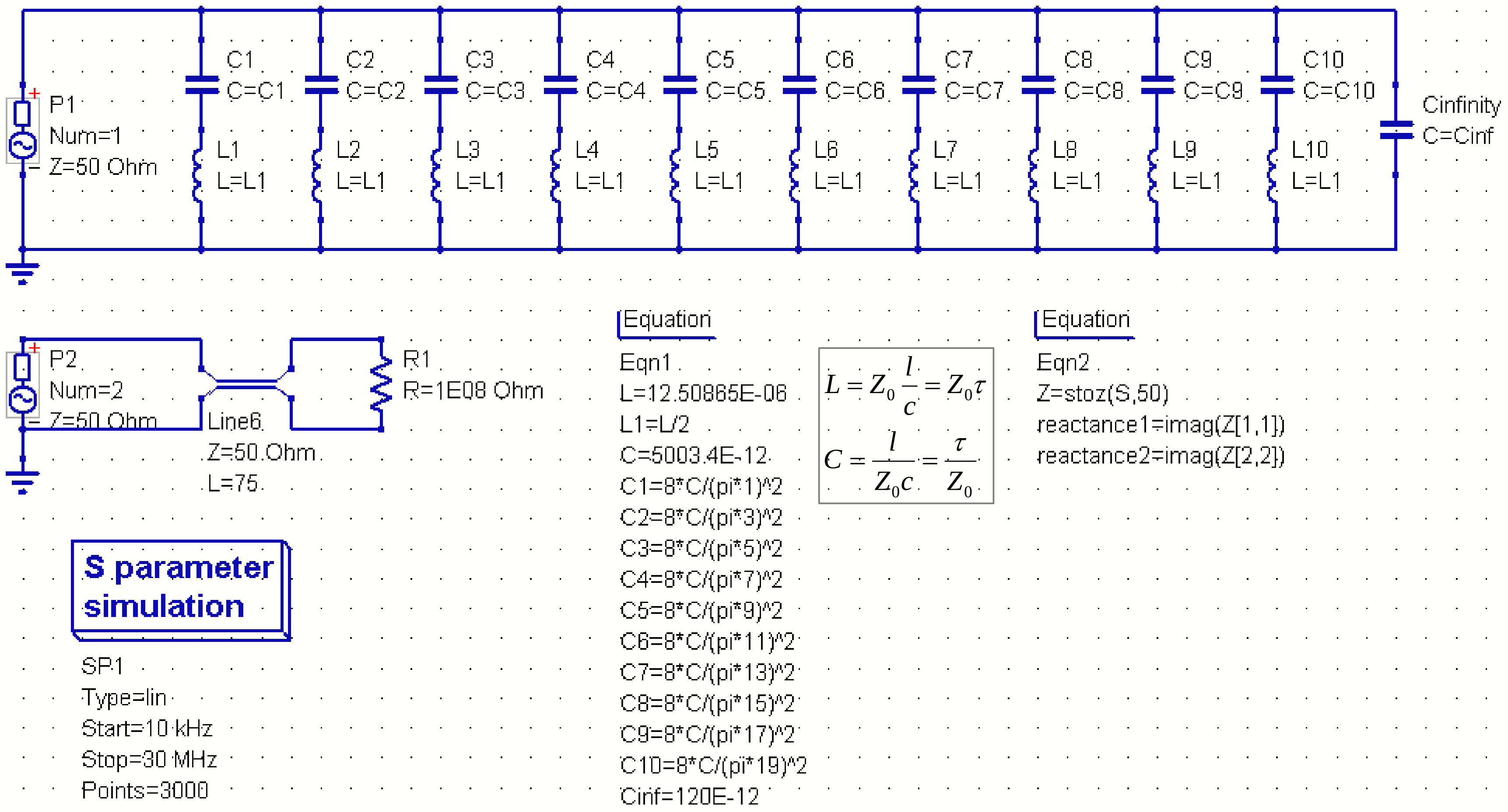
$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$





Numerical Evaluation Open Stub Equivalent Circuit

50 Ω open stub λ/4 @ 1 MHz



SP1
Type=lin
Start=10 kHz
Stop=30 MHz
Points=3000

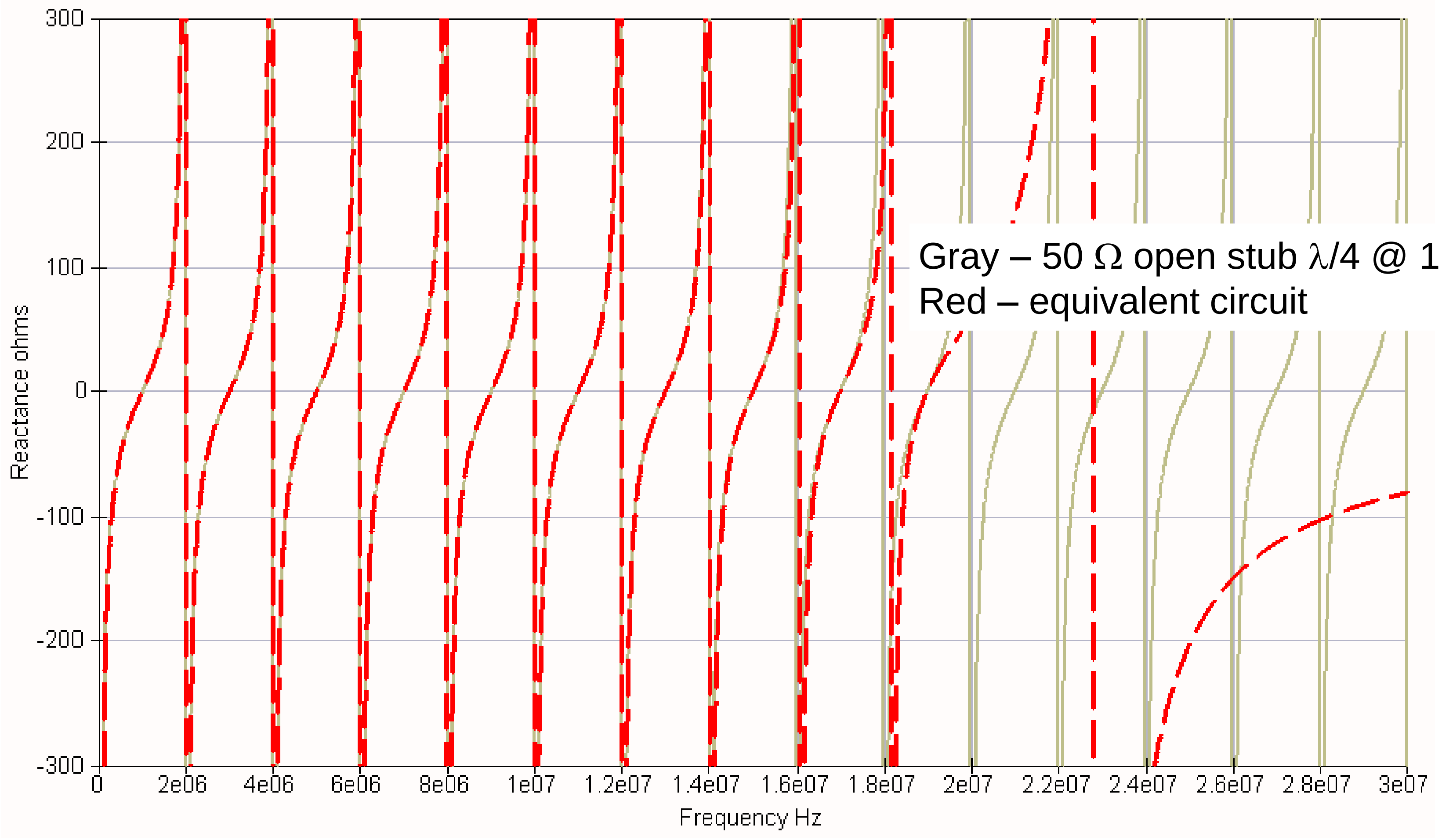
Equation
Eqn1
L=12.50865E-06
L1=L/2
C=5003.4E-12
C1=8*C/(pi*1)^2
C2=8*C/(pi*3)^2
C3=8*C/(pi*5)^2
C4=8*C/(pi*7)^2
C5=8*C/(pi*9)^2
C6=8*C/(pi*11)^2
C7=8*C/(pi*13)^2
C8=8*C/(pi*15)^2
C9=8*C/(pi*17)^2
C10=8*C/(pi*19)^2
Cinf=120E-12

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$

Equation
Eqn2
Z=stoz(S,50)
reactance1=imag(Z[1,1])
reactance2=imag(Z[2,2])

Open Stub Equivalent Circuit from 0 to 19 MHz



Gray – 50 Ω open stub λ/4 @ 1 MHz
Red – equivalent circuit

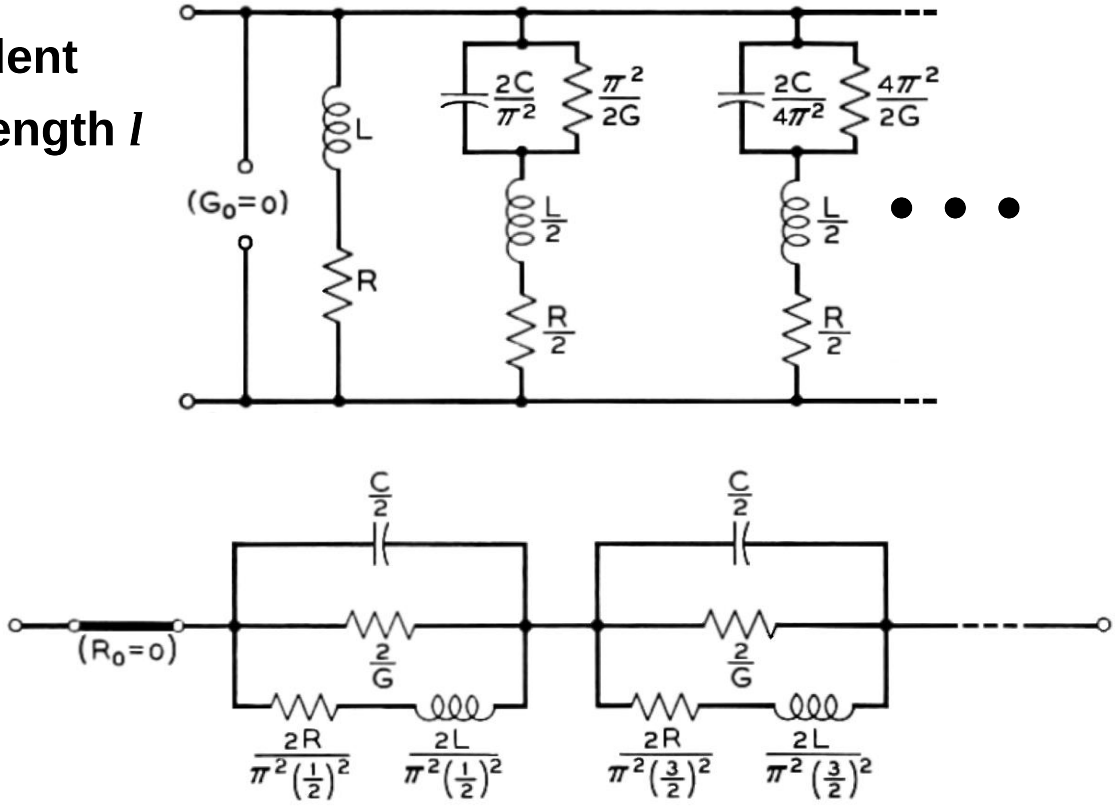
Shorted Stub Equivalent Circuits – Types 1a and 2a

$$Z_{in} = Z_0 \tanh \gamma l = Z_0 \tanh (\alpha l + j\beta l)$$

- **Loss is frequency-independent**
- ***R*, *L*, *G*, *C* are for total line length *l***

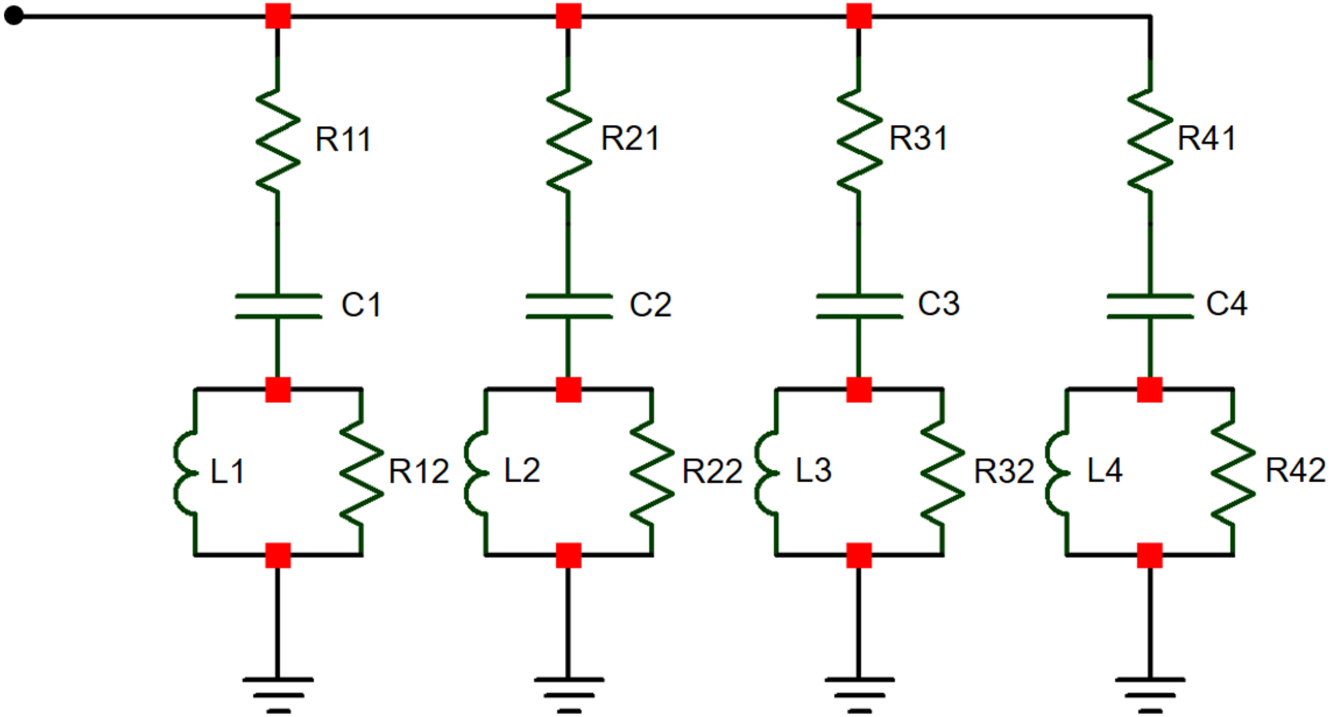
$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$



Example 3: Thin Wire Dipole

98.4-ft Dipole (*L*/*d* = 11,200)
Type 2b Equivalent Circuit for Zero to 30 MHz



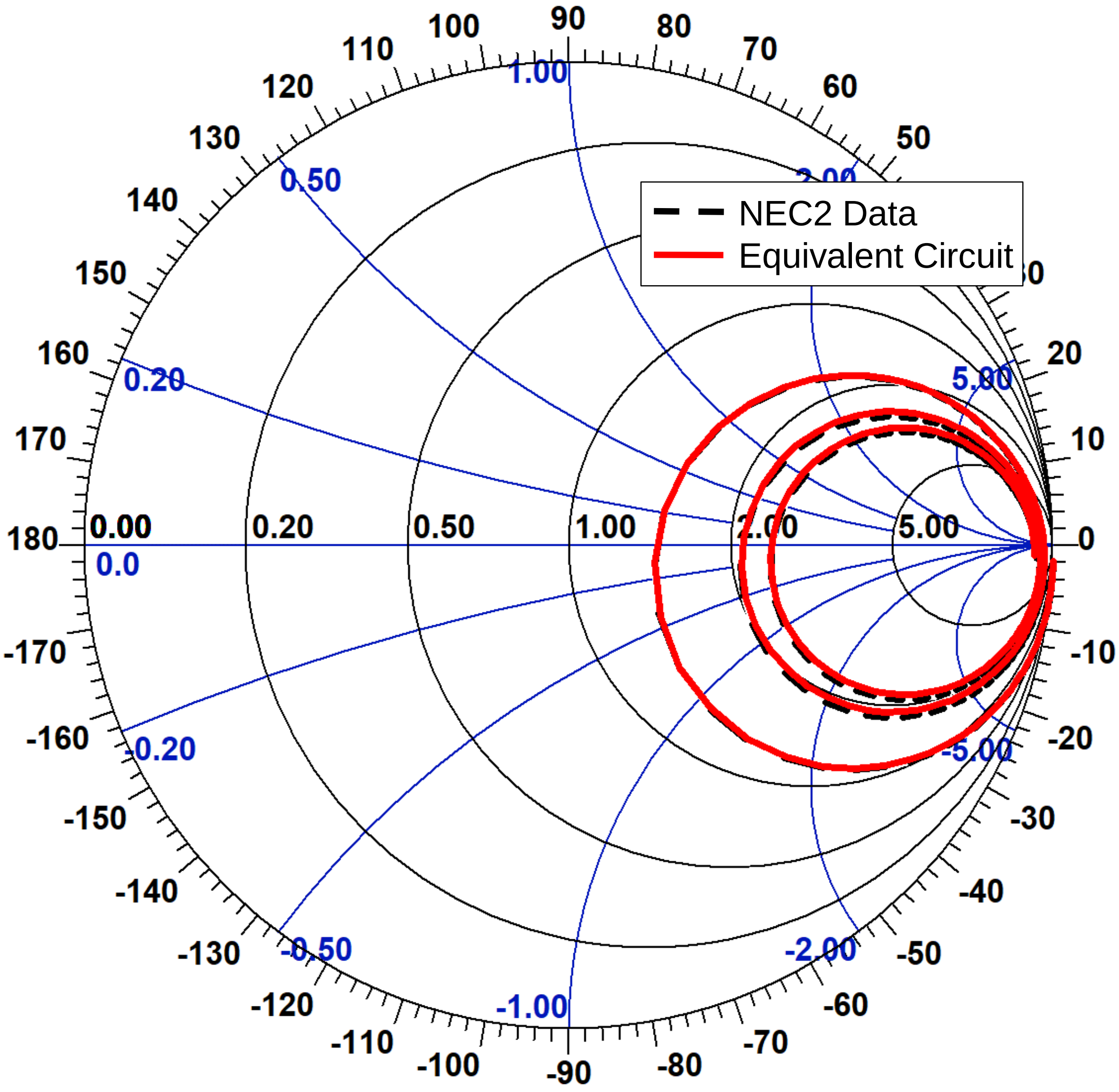
R11 = 5.06 Ω
C1 = 39.9 pF
L1 = 27.1 µH
R12 = 10.1 kΩ

R21 = 0 Ω
C2 = 4.64 pF
L2 = 24.9 µH
R22 = 50.1 kΩ

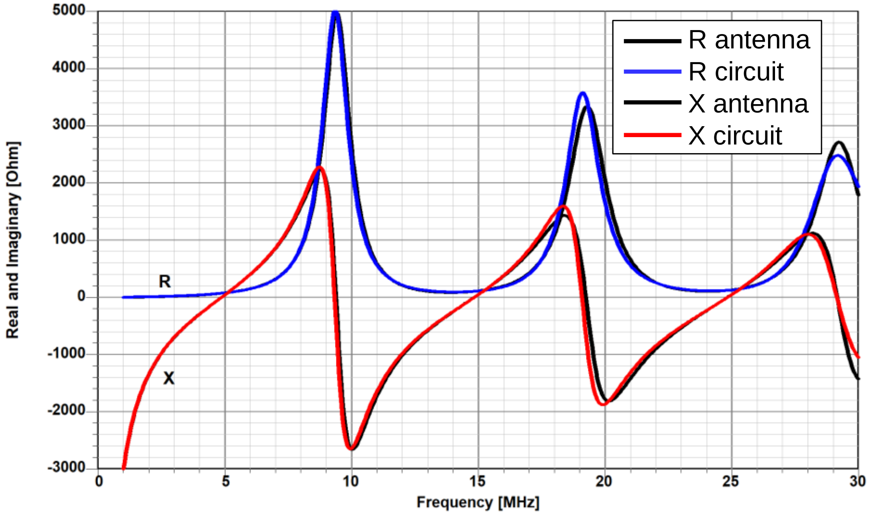
R31 = 25.5 Ω
C3 = 4.69 pF
L3 = 2.26 µH
R32 = 2.68 kΩ

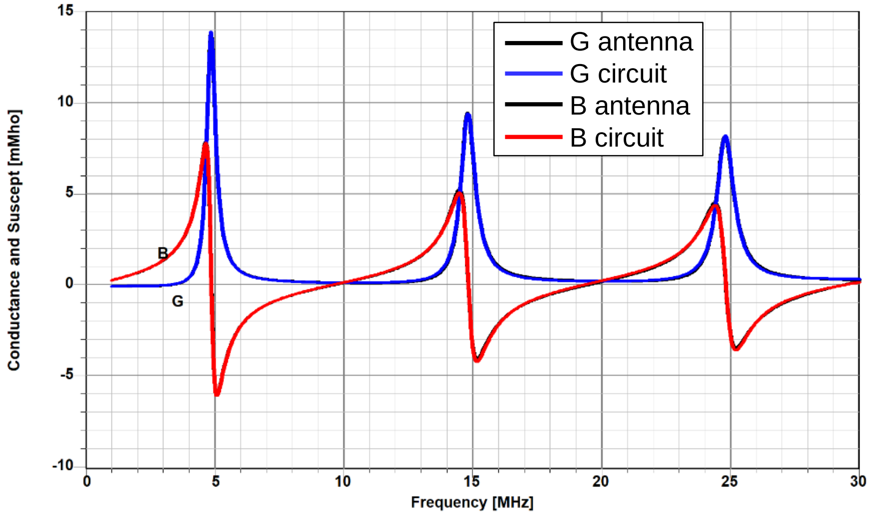
R41 = 0 Ω
C4 = 1.68 pF
L4 = 24.5 µH
R42 = 116 kΩ

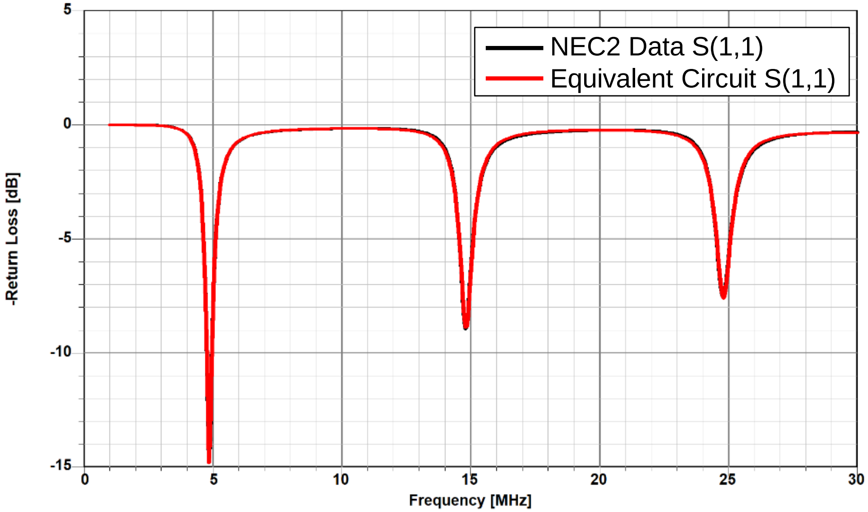
98.4-ft Dipole
Equivalent Circuit Performance from Zero to 30 MHz



98.4-ft Dipole
Equivalent Circuit Impedance



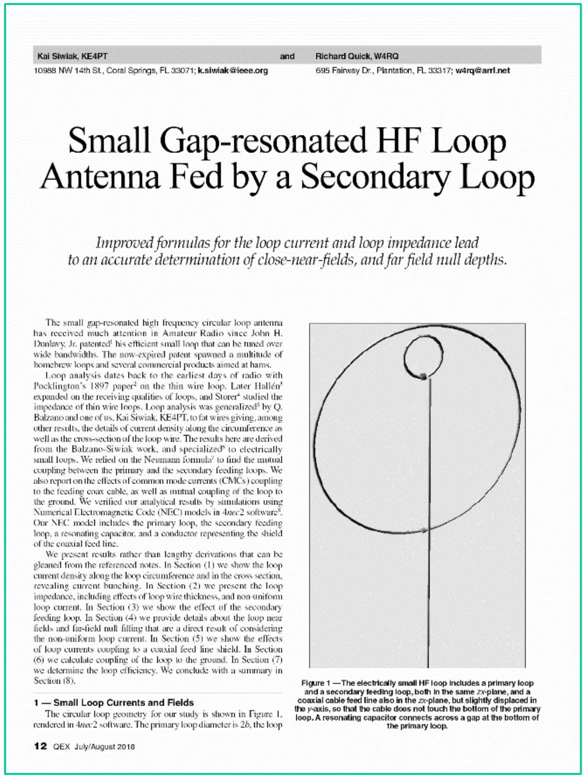




Example 4: Small Tuned Loop

Similar to AlexLoop

Small Gap-Resonated HF Loop Antenna



Kai Siwiak, KE4PT and **Richard Quick, W4RQ**

10988 NW 14th St., Coral Springs, FL 33071; **k.siwiak@ieee.org** 695 Fairway Dr., Plantation, FL 33317; **w4rq@arrl.net**

Small Gap-resonated HF Loop Antenna Fed by a Secondary Loop

Improved formulas for the loop current and loop impedance lead to an accurate determination of close-near-fields, and far field null depths.

The small gap-resonated high frequency circular loop antenna has received much attention in Amateur Radio since John H. Dunlavy, Jr. patented[1] his efficient small loop that can be tuned over wide bandwidths. The now-expired patent spawned a multitude of homebrew loops and several commercial products aimed at hams.

Loop analysis dates back to the earliest days of radio with Pocklington's 1897 paper[2] on the thin wire loop. Later Hallén[3] expanded on the receiving qualities of loops, and Storer[4] studied the impedance of thin wire loops. Loop analysis was generalized[5] by Q. Balzano and one of us, Kai Siwiak, KE4PT, to fat wires giving, among other results, the details of current density along the circumference as well as the cross-section of the loop wire. The results here are derived from the Balzano-Siwiak work, and specialized[6] to electrically small loops. We relied on the Neumann formula[7] to find the mutual coupling between the primary and the secondary feeding loops. We also report on the effects of common mode currents (CMCs) coupling to the feeding coax cable, as well as mutual coupling of the loop to the ground. We verified our analytical results by simulations using Numerical Electromagnetic Code (NEC) models in *4nec2* software[8]. Our NEC model includes the primary loop, the secondary feeding loop, a resonating capacitor, and a conductor representing the shield of the coaxial feed line.

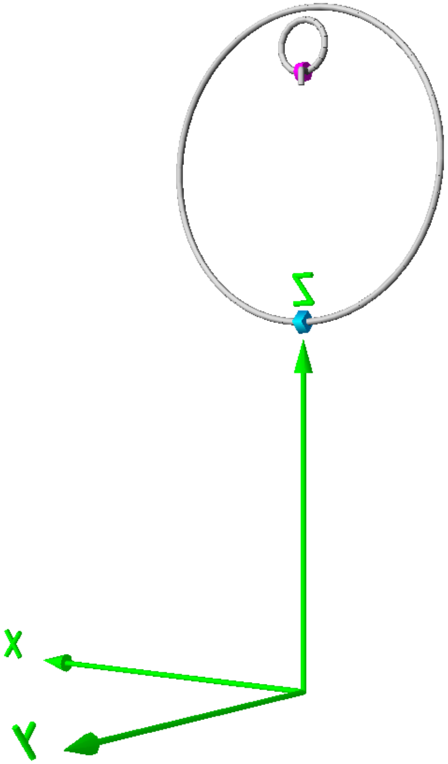
We present results rather than lengthy derivations that can be gleaned from the referenced notes. In Section (1) we show the loop current density along the loop circumference and in the cross section, revealing current bunching. In Section (2) we present the loop impedance, including effects of loop wire thickness, and non-uniform loop current. In Section (3) we show the effect of the secondary feeding loop. In Section (4) we provide details about the loop near fields and far-field null filling that are a direct result of considering the non-uniform loop current. In Section (5) we show the effects of loop currents coupling to a coaxial feed line shield. In Section (6) we calculate coupling of the loop to the ground. In Section (7) we determine the loop efficiency. We conclude with a summary in Section (8).

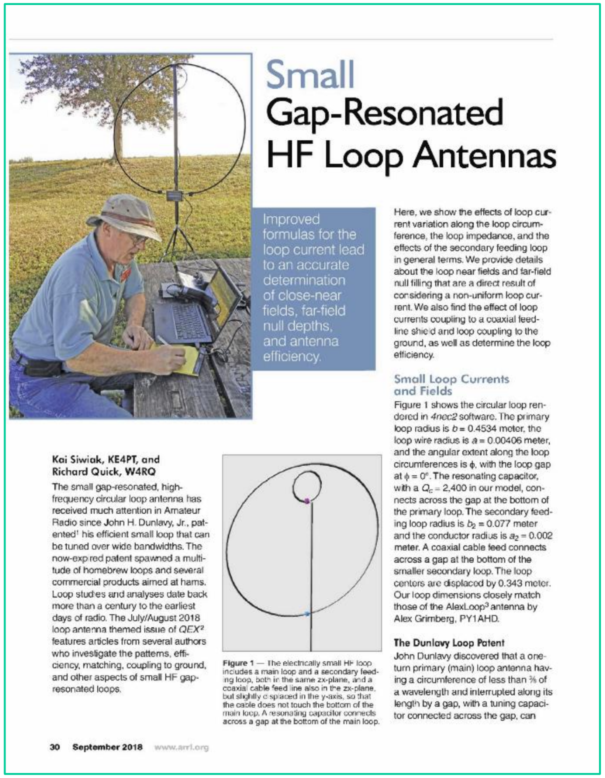
1 — Small Loop Currents and Fields

The circular loop geometry for our study is shown in Figure 1, rendered in *4nec2* software. The primary loop diameter is $2b$, the loop

Figure 1 — The electrically small HF loop includes a primary loop and a secondary feeding loop, both in the same *zx*-plane, and a coaxial cable feed line also in the *zx*-plane, but slightly displaced in the *y*-axis, so that the cable does not touch the bottom of the primary loop. A resonating capacitor connects across a gap at the bottom of the primary loop.

12 QEX July/August 2018





Small Gap-Resonated HF Loop Antennas

Improved formulas for the loop current lead to an accurate determination of close-near fields, far-field null depths, and antenna efficiency.

Kai Siwiak, KE4PT, and Richard Quick, W4RQ

The small gap-resonated, high-frequency circular loop antenna has received much attention in Amateur Radio since John H. Dunlavy, Jr., patented[1] his efficient small loop that can be tuned over wide bandwidths. The now-expired patent spawned a multitude of homebrew loops and several commercial products aimed at hams. Loop studies and analyses date back more than a century to the earliest days of radio. The July/August 2018 loop antenna themed issue of *QEX*[2] features articles from several authors who investigate the patterns, efficiency, matching, coupling to ground, and other aspects of small HF gap-resonated loops.

Figure 1 — The electrically small HF loop includes a main loop and a secondary feeding loop, both in the same zx-plane, and a coaxial cable feed line also in the zx-plane, but slightly displaced in the y-axis, so that the cable does not touch the bottom of the main loop. A resonating capacitor connects across a gap at the bottom of the main loop.

Here, we show the effects of loop current variation along the loop circumference, the loop impedance, and the effects of the secondary feeding loop in general terms. We provide details about the loop near fields and far-field null filling that are a direct result of considering a non-uniform loop current. We also find the effect of loop currents coupling to a coaxial feed-line shield and loop coupling to the ground, as well as determine the loop efficiency.

Small Loop Currents and Fields

Figure 1 shows the circular loop rendered in *4nec2* software. The primary loop radius is $b = 0.4534$ meter, the loop wire radius is $a = 0.00406$ meter, and the angular extent along the loop circumferences is ϕ, with the loop gap at $\phi = 0°$. The resonating capacitor, with a $Q_c = 2{,}400$ in our model, connects across the gap at the bottom of the primary loop. The secondary feeding loop radius is $b_2 = 0.077$ meter and the conductor radius is $a_2 = 0.002$ meter. A coaxial cable feed connects across a gap at the bottom of the smaller secondary loop. The loop centers are displaced by 0.343 meter. Our loop dimensions closely match those of the AlexLoop[3] antenna by Alex Grimberg, PY1AHD.

The Dunlavy Loop Patent

John Dunlavy discovered that a one-turn primary (main) loop antenna having a circumference of less than ⅔ of a wavelength and interrupted along its length by a gap, with a tuning capacitor connected across the gap, can

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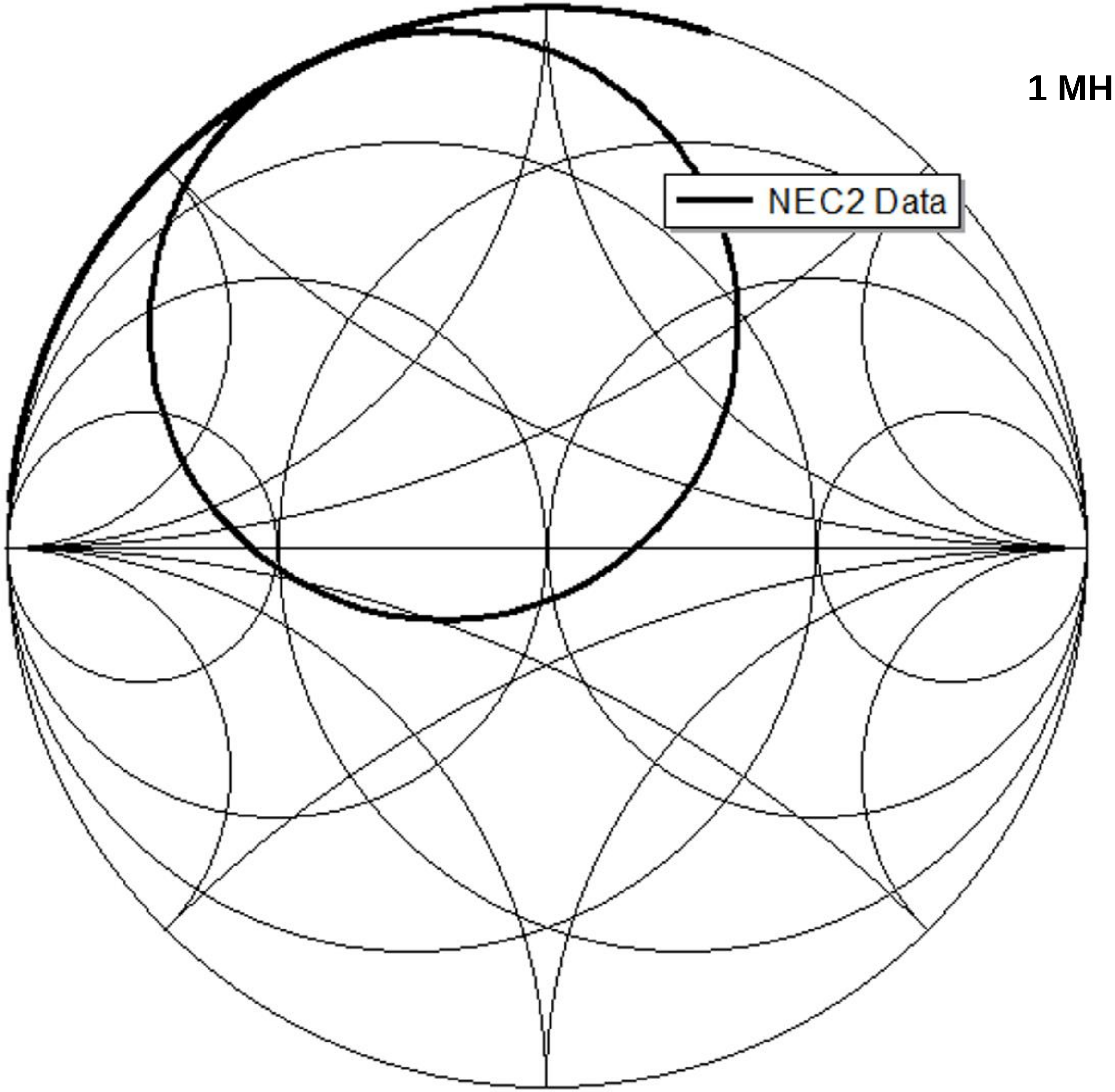
K. Siwiak, KE4PT, and R. Quick, W4RQ, *QEX*, July/August 2018

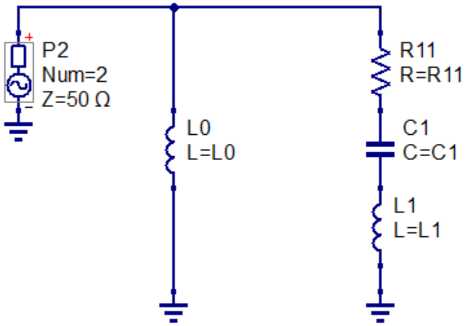
K. Siwiak, KE4PT, and R. Quick, W4RQ, *QST*, September 2018

Small Gap-Resonated Loop Tuned for 20-Meters



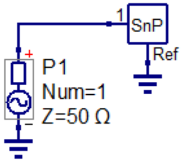
1 MHz to 30 MHz

Small Gap Resonated Loop Tuned for 20 Meters
Type 2b Equivalent Circuit – Optimization Setup



F1 MHz
14.12

antdata_S_dB
File=HF_Loop_Siwiak-Quick_QST_Sep_2018.s1p
Ports=1
domain=rectangular
interpol=cubic



equation
Outputs
Rant=real((1+S[1,1])/(1-S[1,1]))*50
Xant=imag((1+S[1,1])/(1-S[1,1]))*50
Gant=real((1-S[1,1])/(1+S[1,1]))/50
Bant=imag((1-S[1,1])/(1+S[1,1]))/50
Req=real((1+S[2,2])/(1-S[2,2]))*50
Xeq=imag((1+S[2,2])/(1-S[2,2]))*50
Geq=real((1-S[2,2])/(1+S[2,2]))/50
Beq=imag((1-S[2,2])/(1+S[2,2]))/50
F1=1e-06/(2*pi*sqrt(L1*C1))

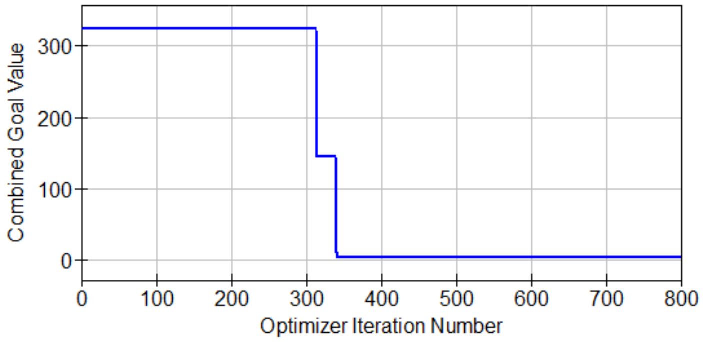
s-parameter simulation
SP1
Type=lin
Start=1 MHz
Stop=30 MHz
Points=29001

Optimization
Opt1
Sim=SP1
DE/bor/2/bin|40000|0.95|0.8|50
C1=0...1.17098e-12...1.2e-11 linear
L0=0...3.60877e-07...3.6e-06 linear
L1=0...0.000108522...1.1e-03 linear
R11=0...12.0474...120 linear
Mean_Square_Error_S_Mag=124000 MIN
Mean_Square_Error_S_Ang=46.5 MIN
Max_Square_Error_S_Mag=58500 MIN
Max_Square_Error_S_Ang=18.9 MIN

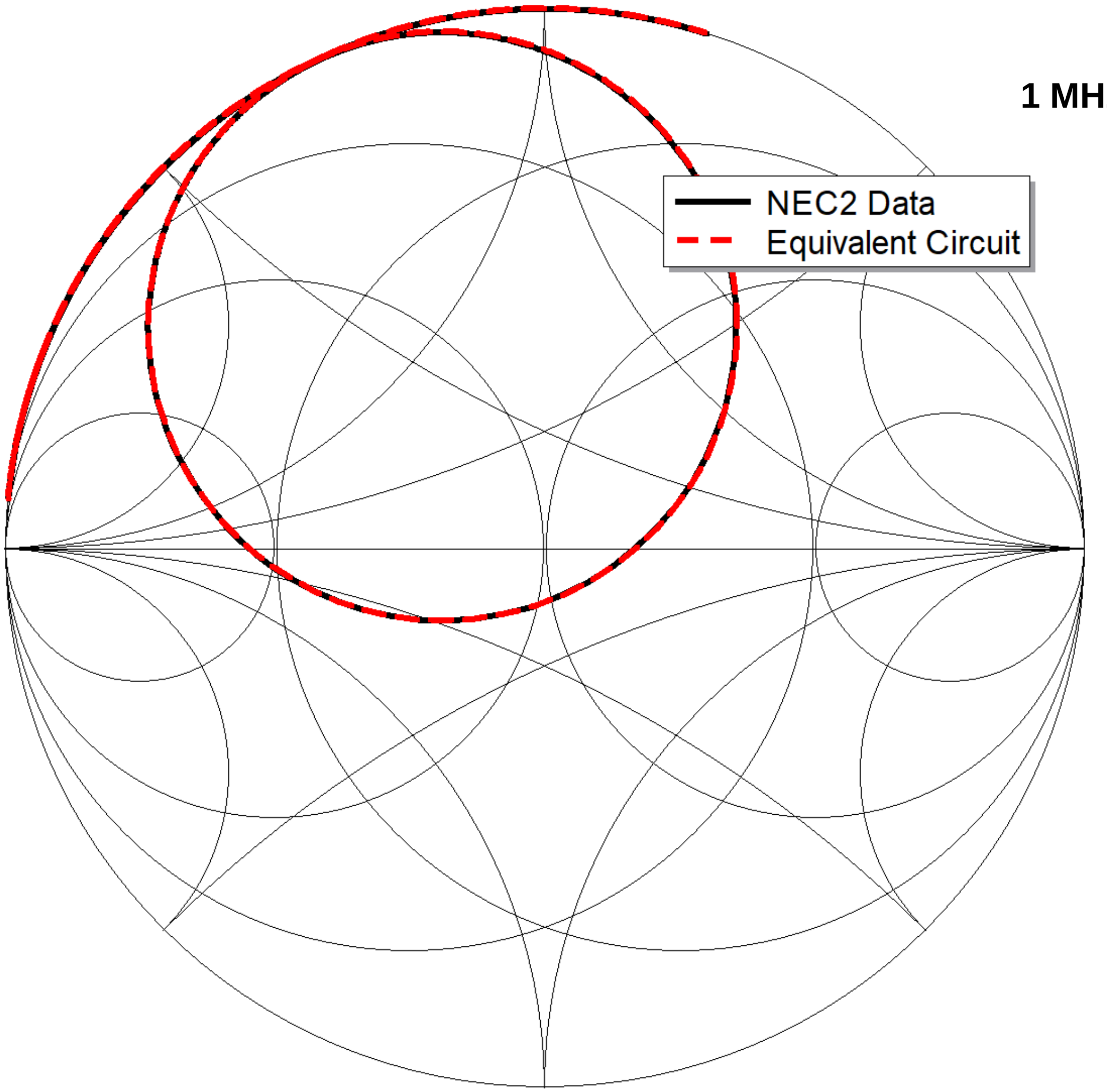
equation
Best_Values
C1=1.17098e-12
L0=360.877e-09
L1=108.522e-06
R11=12.0474

equation
Goals
Mean_Square_Error_S_Mag=average(range(mag(S[1,1]-S[2,2])^2, 1MHz, 30MHz))
Mean_Square_Error_S_Ang=average(range(mag(phase(S[1,1]/S[2,2]))^2, 1MHz, 30MHz))
Max_Square_Error_S_Mag=max(range(mag(S[1,1]-S[2,2])^2, 1MHz, 30MHz))
Max_Square_Error_S_Ang=max(range(mag(phase(S[1,1]/S[2,2]))^2, 1MHz, 30MHz))

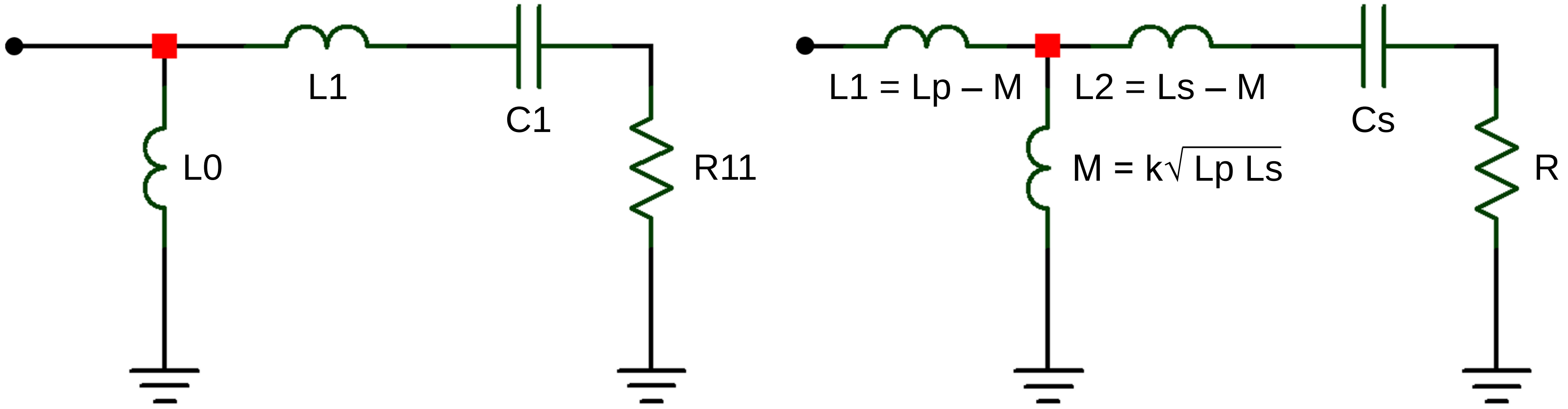
MeanSE |S|	MeanSE Ang(S)	MaxSE |S|	MaxSE Ang(S)
8.04e-06	0.0215	1.71e-05	0.0528



Small Gap-Resonated Loop 4-Element Equivalent Circuit Performance



Universal vs Transformer-Coupled Equivalent Circuits



L0 = 361 nH
L1 = 109 μH
C1 = 1.17 pF
R11 = 12.0 Ω

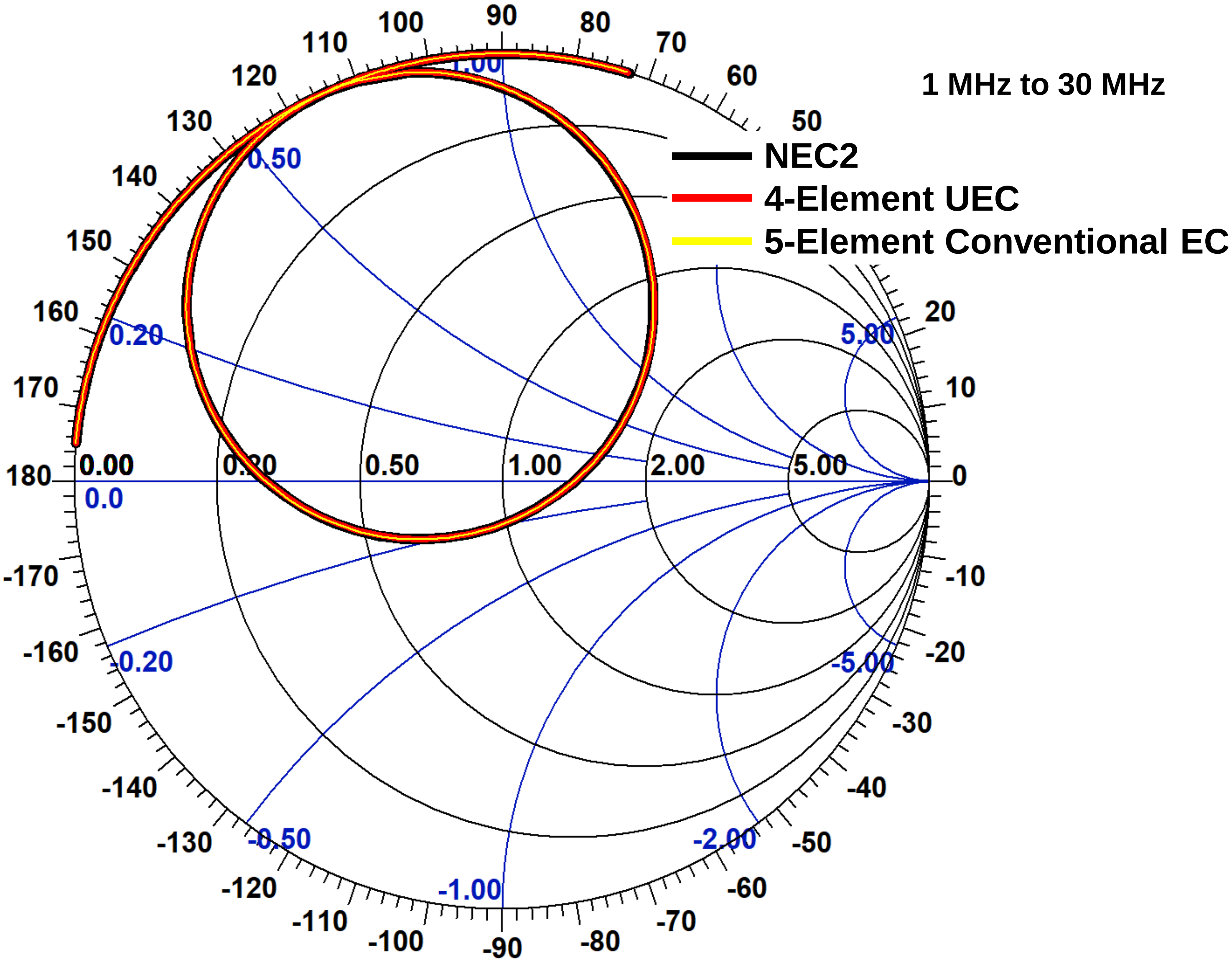
k = 0.0577
Lp = 359 nH
Ls = 2.23 μH
M = 51.6 nH

L1 = 308 nH
L2 = 2.17 μH
Cs = 57.3 pF
R = 0.247 Ω

- **Physical reasoning gives equivalent circuits that can be more complex than necessary. Minimum complexity is not the goal**
- **UEC theory gives equivalent circuits of minimal complexity**

M.E. Cram, W8NUE, “Small Transmitting Loops: a Different Perspective on Tuning and Determining Q and Efficiency,” *QEX*, July/August 2018.
A. Boswell; A.J. Tyler; A. White, “Performance of a Small Loop Antenna in the 3-10 MHz Band,” *IEEE Antennas and Propagation Magazine*, April 2005.

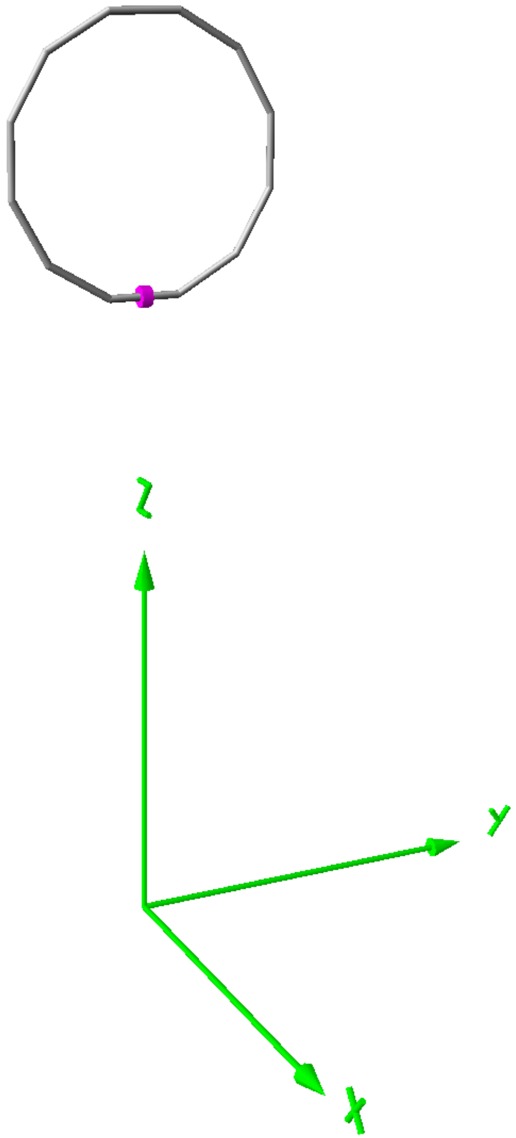
Comparison of Two Equivalent Circuits for Coupled Loop Antennas



Example 5: Large Untuned Loop

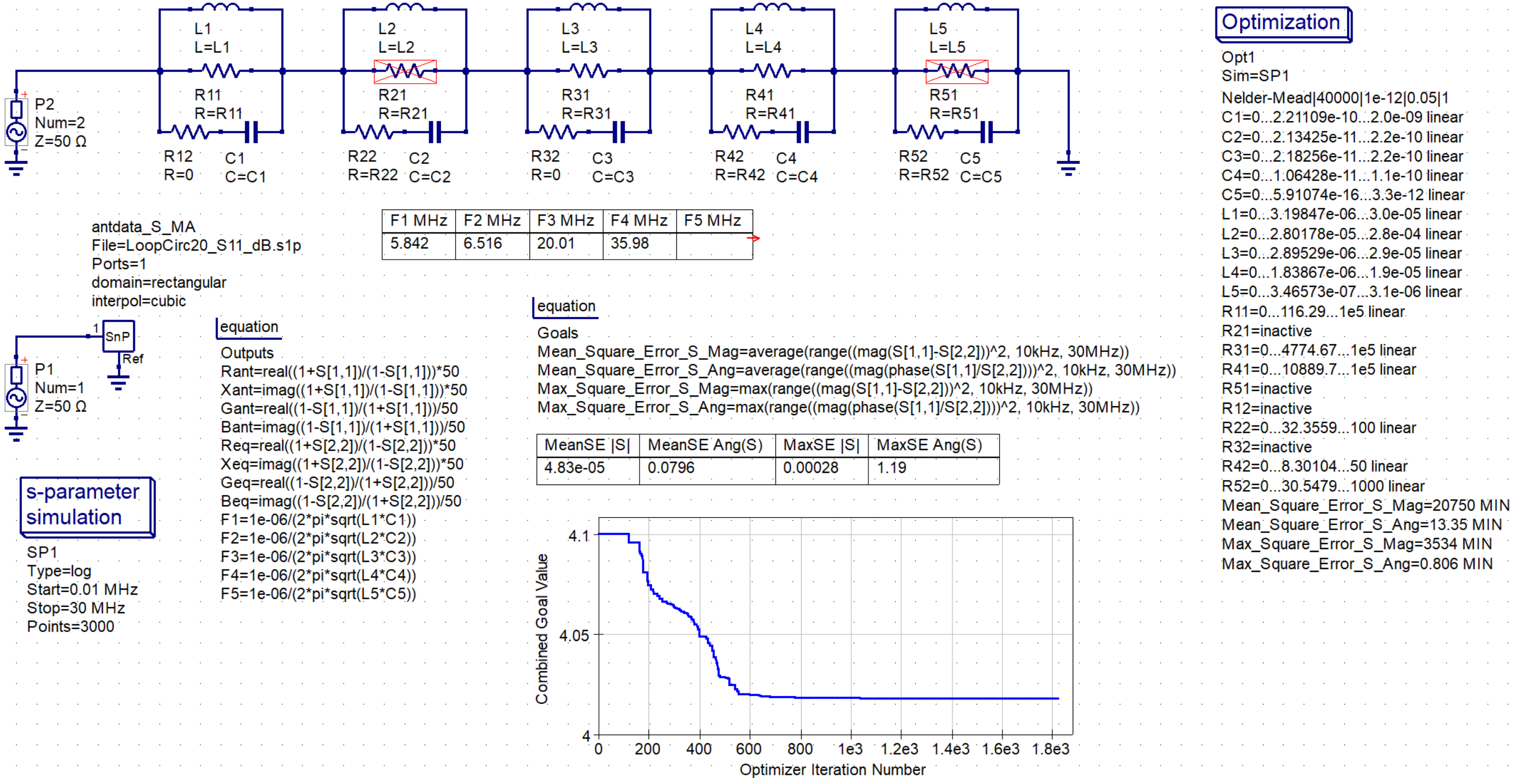
20-meter band single-turn circular loop

20-Meter Single-Turn Circular Loop



- 4nec2 model: LoopCirc20
- Loop diameter: 23.4 ft
- Wire diameter: AWG #12
- Free space, no ground

20-Meter Single-Turn Circular Loop
Type 1b Equivalent Circuit – Optimization Setup



antdata_S_MA
File=LoopCirc20_S11_dB.s1p
Ports=1
domain=rectangular
interpol=cubic

s-parameter simulation

SP1
Type=log
Start=0.01 MHz
Stop=30 MHz
Points=3000

equation

Outputs
Rant=real((1+S[1,1])/(1-S[1,1]))*50
Xant=imag((1+S[1,1])/(1-S[1,1]))*50
Gant=real((1-S[1,1])/(1+S[1,1]))/50
Bant=imag((1-S[1,1])/(1+S[1,1]))/50
Req=real((1+S[2,2])/(1-S[2,2]))*50
Xeq=imag((1+S[2,2])/(1-S[2,2]))*50
Geq=real((1-S[2,2])/(1+S[2,2]))/50
Beq=imag((1-S[2,2])/(1+S[2,2]))/50
F1=1e-06/(2*pi*sqrt(L1*C1))
F2=1e-06/(2*pi*sqrt(L2*C2))
F3=1e-06/(2*pi*sqrt(L3*C3))
F4=1e-06/(2*pi*sqrt(L4*C4))
F5=1e-06/(2*pi*sqrt(L5*C5))

F1 MHz	F2 MHz	F3 MHz	F4 MHz	F5 MHz
5.842	6.516	20.01	35.98	

equation

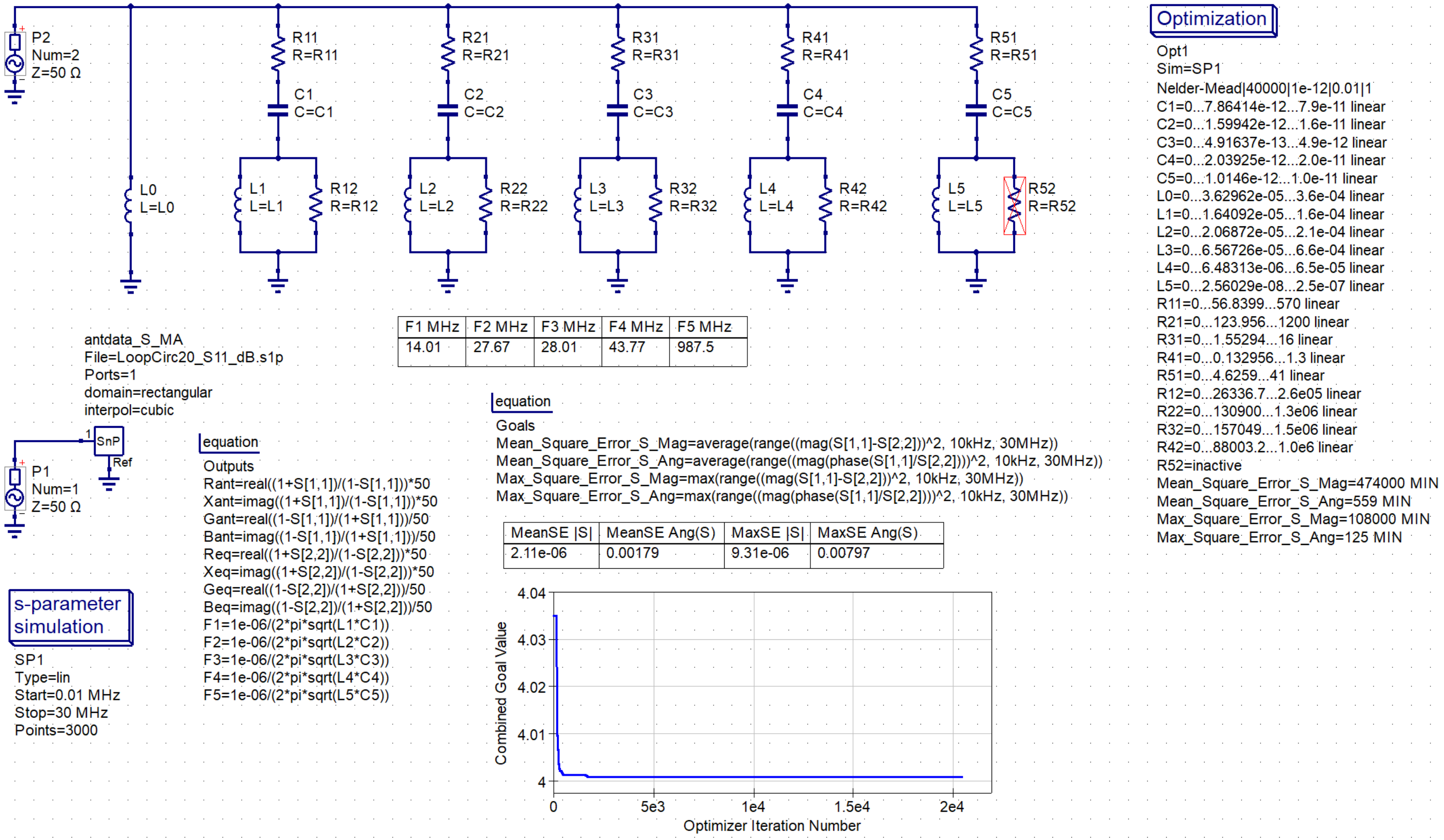
Goals
Mean_Square_Error_S_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))
Mean_Square_Error_S_Ang=average(range((mag(phase(S[1,1]/S[2,2])))^2, 10kHz, 30MHz))
Max_Square_Error_S_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))
Max_Square_Error_S_Ang=max(range((mag(phase(S[1,1]/S[2,2])))^2, 10kHz, 30MHz))

MeanSE |S|	MeanSE Ang(S)	MaxSE |S|	MaxSE Ang(S)
4.83e-05	0.0796	0.00028	1.19

Optimization

Opt1
Sim=SP1
Nelder-Mead|40000|1e-12|0.05|1
C1=0...2.21109e-10...2.0e-09 linear
C2=0...2.13425e-11...2.2e-10 linear
C3=0...2.18256e-11...2.2e-10 linear
C4=0...1.06428e-11...1.1e-10 linear
C5=0...5.91074e-16...3.3e-12 linear
L1=0...3.19847e-06...3.0e-05 linear
L2=0...2.80178e-05...2.8e-04 linear
L3=0...2.89529e-06...2.9e-05 linear
L4=0...1.83867e-06...1.9e-05 linear
L5=0...3.46573e-07...3.1e-06 linear
R11=0...116.29...1e5 linear
R21=inactive
R31=0...4774.67...1e5 linear
R41=0...10889.7...1e5 linear
R51=inactive
R12=inactive
R22=0...32.3559...100 linear
R32=inactive
R42=0...8.30104...50 linear
R52=0...30.5479...1000 linear
Mean_Square_Error_S_Mag=20750 MIN
Mean_Square_Error_S_Ang=13.35 MIN
Max_Square_Error_S_Mag=3534 MIN
Max_Square_Error_S_Ang=0.806 MIN

20-Meter Single-Turn Circular Loop
Type 2b Equivalent Circuit – Optimization Setup



Optimization

Opt1
Sim=SP1
Nelder-Mead|40000|1e-12|0.01|1
C1=0...7.86414e-12...7.9e-11 linear
C2=0...1.59942e-12...1.6e-11 linear
C3=0...4.91637e-13...4.9e-12 linear
C4=0...2.03925e-12...2.0e-11 linear
C5=0...1.0146e-12...1.0e-11 linear
L0=0...3.62962e-05...3.6e-04 linear
L1=0...1.64092e-05...1.6e-04 linear
L2=0...2.06872e-05...2.1e-04 linear
L3=0...6.56726e-05...6.6e-04 linear
L4=0...6.48313e-06...6.5e-05 linear
L5=0...2.56029e-08...2.5e-07 linear
R11=0...56.8399...570 linear
R21=0...123.956...1200 linear
R31=0...1.55294...16 linear
R41=0...0.132956...1.3 linear
R51=0...4.6259...41 linear
R12=0...26336.7...2.6e05 linear
R22=0...130900...1.3e06 linear
R32=0...157049...1.5e06 linear
R42=0...88003.2...1.0e6 linear
R52=inactive
Mean_Square_Error_S_Mag=474000 MIN
Mean_Square_Error_S_Ang=559 MIN
Max_Square_Error_S_Mag=108000 MIN
Max_Square_Error_S_Ang=125 MIN

antdata_S_MA
File=LoopCirc20_S11_dB.s1p
Ports=1
domain=rectangular
interpol=cubic

F1 MHz	F2 MHz	F3 MHz	F4 MHz	F5 MHz
14.01	27.67	28.01	43.77	987.5

equation

Goals
Mean_Square_Error_S_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))
Mean_Square_Error_S_Ang=average(range((mag(phase(S[1,1]/S[2,2])))^2, 10kHz, 30MHz))
Max_Square_Error_S_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 10kHz, 30MHz))
Max_Square_Error_S_Ang=max(range((mag(phase(S[1,1]/S[2,2])))^2, 10kHz, 30MHz))

equation

Outputs
Rant=real((1+S[1,1])/(1-S[1,1]))*50
Xant=imag((1+S[1,1])/(1-S[1,1]))*50
Gant=real((1-S[1,1])/(1+S[1,1]))/50
Bant=imag((1-S[1,1])/(1+S[1,1]))/50
Req=real((1+S[2,2])/(1-S[2,2]))*50
Xeq=imag((1+S[2,2])/(1-S[2,2]))*50
Geq=real((1-S[2,2])/(1+S[2,2]))/50
Beq=imag((1-S[2,2])/(1+S[2,2]))/50
F1=1e-06/(2*pi*sqrt(L1*C1))
F2=1e-06/(2*pi*sqrt(L2*C2))
F3=1e-06/(2*pi*sqrt(L3*C3))
F4=1e-06/(2*pi*sqrt(L4*C4))
F5=1e-06/(2*pi*sqrt(L5*C5))

MeanSE |S|	MeanSE Ang(S)	MaxSE |S|	MaxSE Ang(S)
2.11e-06	0.00179	9.31e-06	0.00797

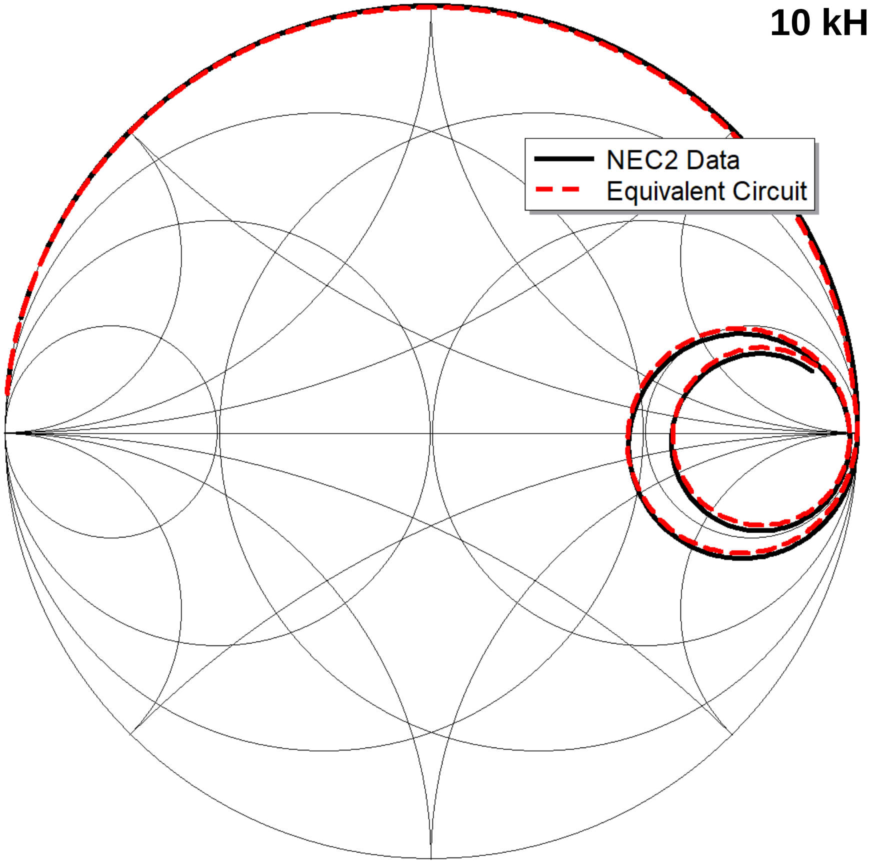
s-parameter simulation

SP1
Type=lin
Start=0.01 MHz
Stop=30 MHz
Points=3000

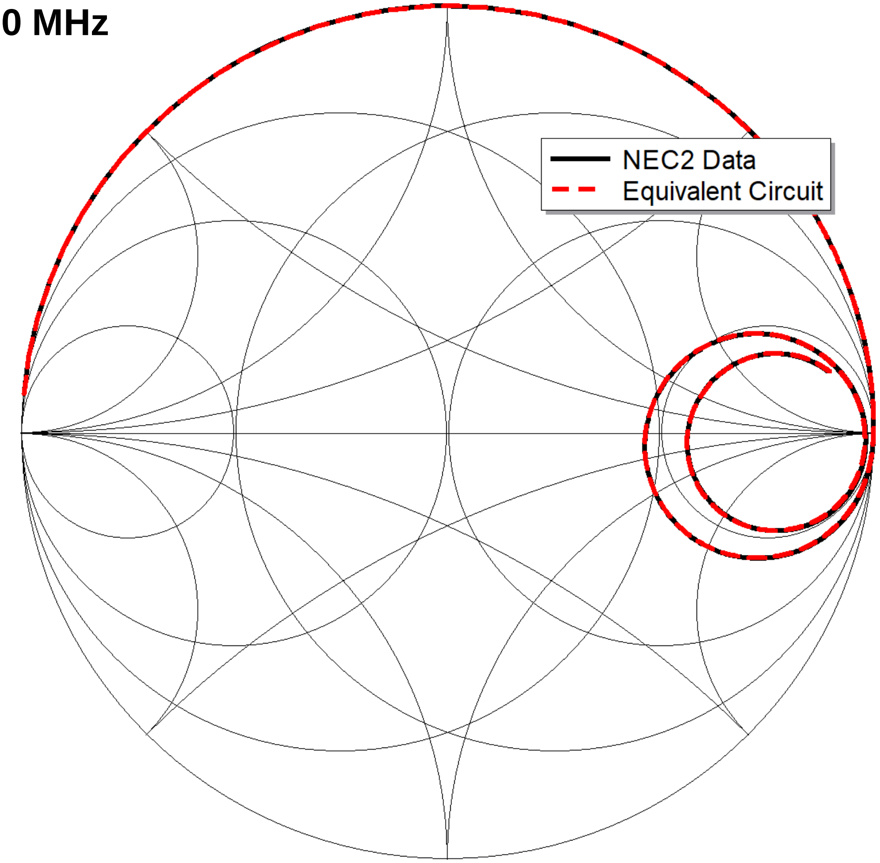
20-Meter Circular Loop Equivalent Circuit Performance

10 kHz to 30 MHz

Type 1b Series Ladder

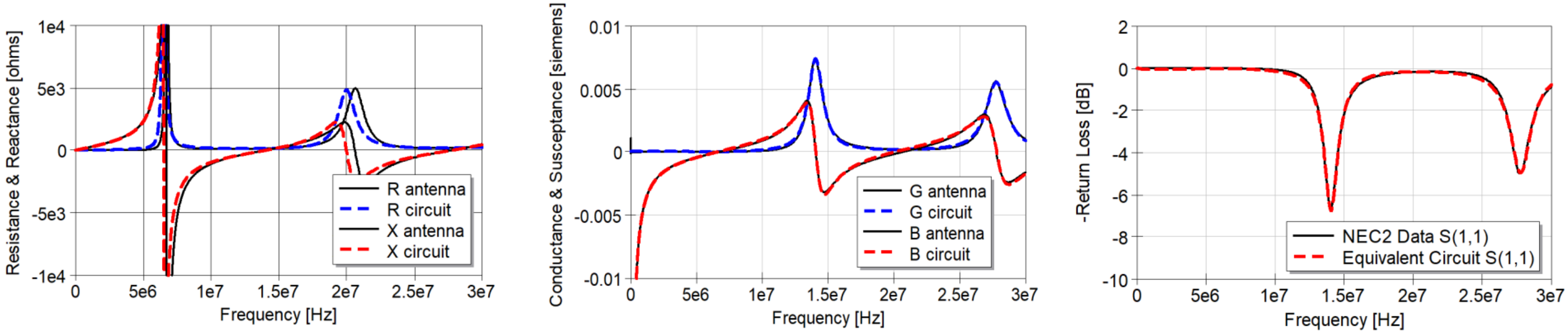


Type 2b Parallel Ladder

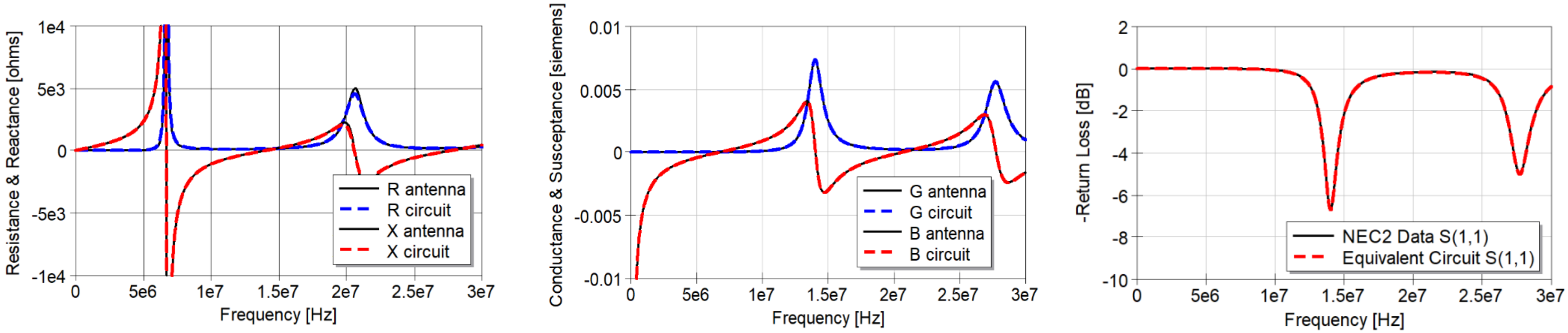


20-Meter Circular Loop Equivalent Circuit Performance

- ## Type 1b Equivalent Circuit

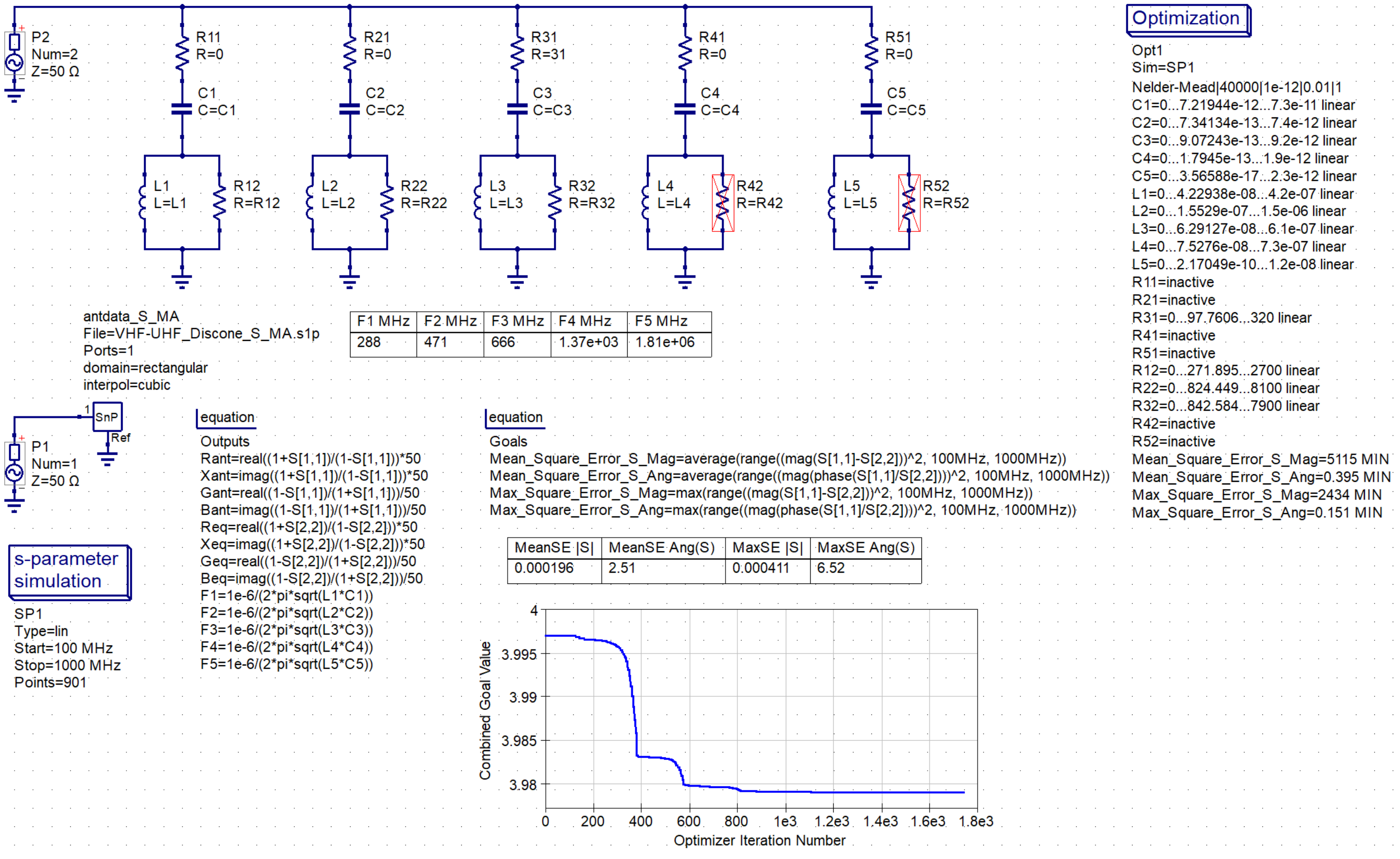


- ## Type 2b Equivalent Circuit



Example 6: VHF-UHF Discone

VHF-UHF Discone
Broadband Equivalent Circuit – Optimization Setup



antdata_S_MA
File=VHF-UHF_Discone_S_MA.s1p
Ports=1
domain=rectangular
interpol=cubic

F1 MHz	F2 MHz	F3 MHz	F4 MHz	F5 MHz
288	471	666	1.37e+03	1.81e+06

equation
Outputs
Rant=real((1+S[1,1])/(1-S[1,1]))*50
Xant=imag((1+S[1,1])/(1-S[1,1]))*50
Gant=real((1-S[1,1])/(1+S[1,1]))/50
Bant=imag((1-S[1,1])/(1+S[1,1]))/50
Req=real((1+S[2,2])/(1-S[2,2]))*50
Xeq=imag((1+S[2,2])/(1-S[2,2]))*50
Geq=real((1-S[2,2])/(1+S[2,2]))/50
Beq=imag((1-S[2,2])/(1+S[2,2]))/50
F1=1e-6/(2*pi*sqrt(L1*C1))
F2=1e-6/(2*pi*sqrt(L2*C2))
F3=1e-6/(2*pi*sqrt(L3*C3))
F4=1e-6/(2*pi*sqrt(L4*C4))
F5=1e-6/(2*pi*sqrt(L5*C5))

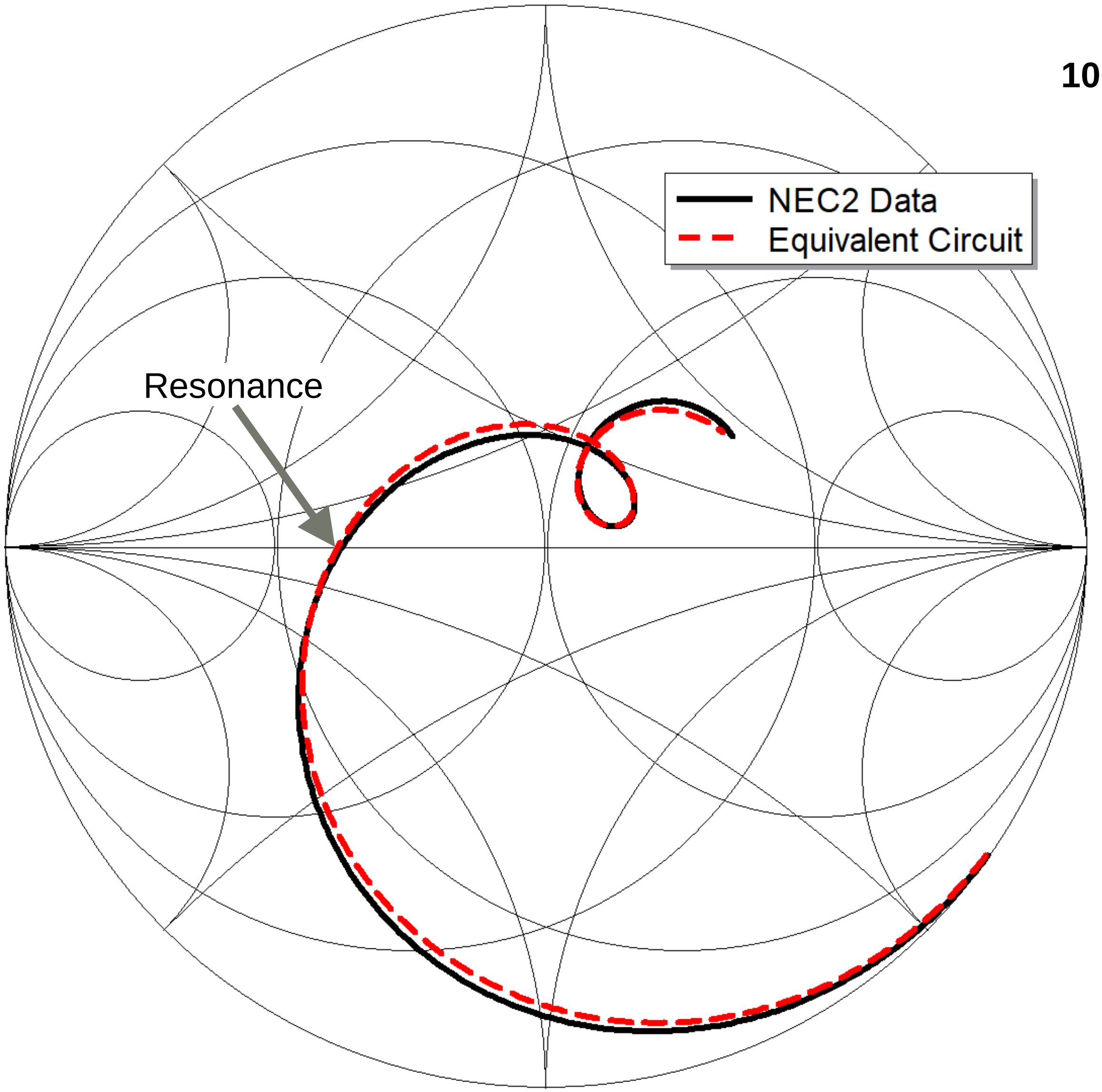
equation
Goals
Mean_Square_Error_S_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 100MHz, 1000MHz))
Mean_Square_Error_S_Ang=average(range((mag(phase(S[1,1]/S[2,2])))^2, 100MHz, 1000MHz))
Max_Square_Error_S_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 100MHz, 1000MHz))
Max_Square_Error_S_Ang=max(range((mag(phase(S[1,1]/S[2,2])))^2, 100MHz, 1000MHz))

MeanSE |S|	MeanSE Ang(S)	MaxSE |S|	MaxSE Ang(S)
0.000196	2.51	0.000411	6.52

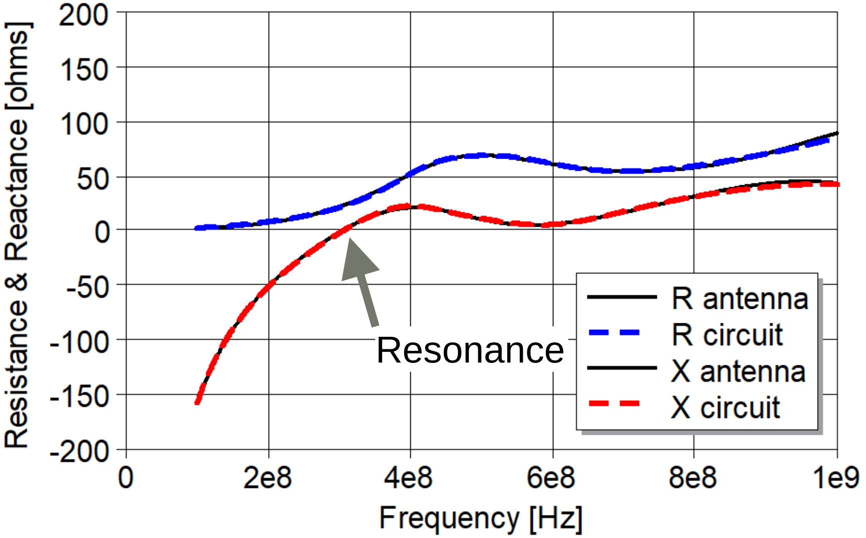
s-parameter simulation
SP1
Type=lin
Start=100 MHz
Stop=1000 MHz
Points=901

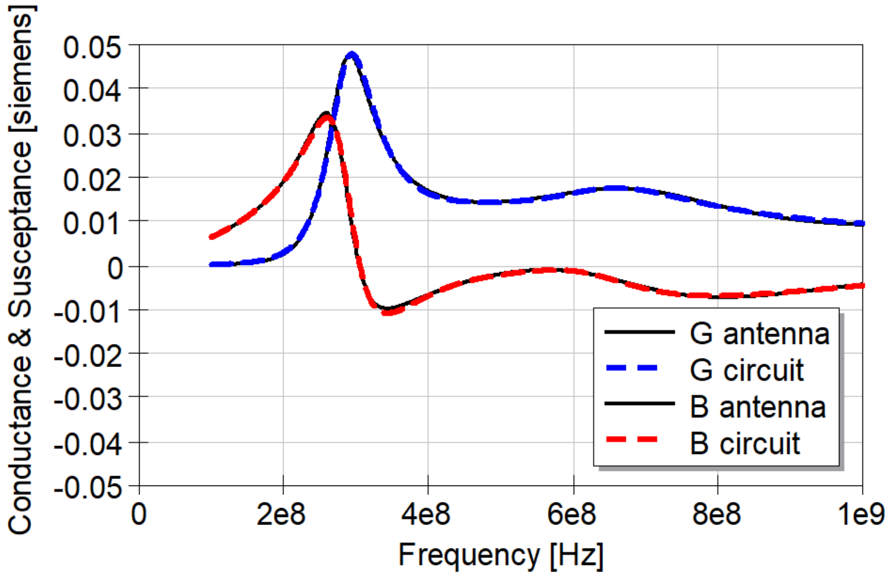
Optimization
Opt1
Sim=SP1
Nelder-Mead|40000|1e-12|0.01|1
C1=0...7.21944e-12...7.3e-11 linear
C2=0...7.34134e-13...7.4e-12 linear
C3=0...9.07243e-13...9.2e-12 linear
C4=0...1.7945e-13...1.9e-12 linear
C5=0...3.56588e-17...2.3e-12 linear
L1=0...4.22938e-08...4.2e-07 linear
L2=0...1.5529e-07...1.5e-06 linear
L3=0...6.29127e-08...6.1e-07 linear
L4=0...7.5276e-08...7.3e-07 linear
L5=0...2.17049e-10...1.2e-08 linear
R11=inactive
R21=inactive
R31=0...97.7606...320 linear
R41=inactive
R51=inactive
R12=0...271.895...2700 linear
R22=0...824.449...8100 linear
R32=0...842.584...7900 linear
R42=inactive
R52=inactive
Mean_Square_Error_S_Mag=5115 MIN
Mean_Square_Error_S_Ang=0.395 MIN
Max_Square_Error_S_Mag=2434 MIN
Max_Square_Error_S_Ang=0.151 MIN

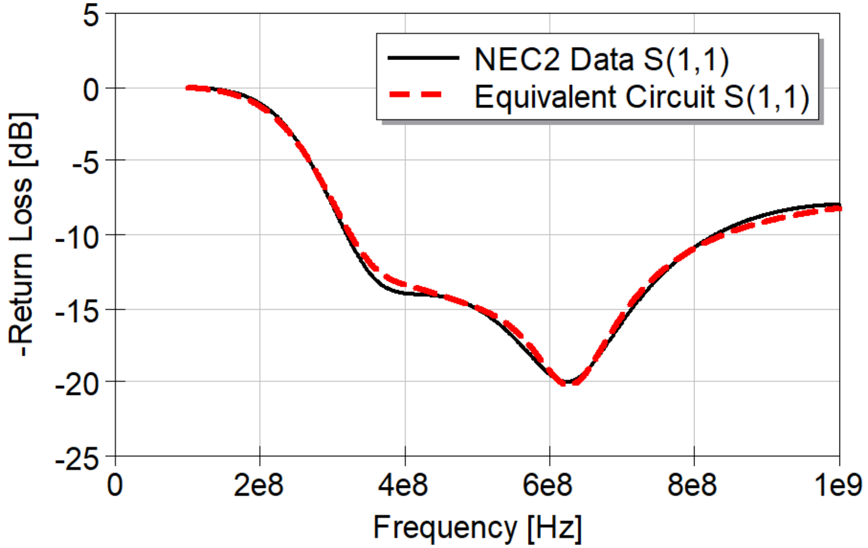
Discone Equivalent Circuit Performance



Discone Equivalent Circuit Performance



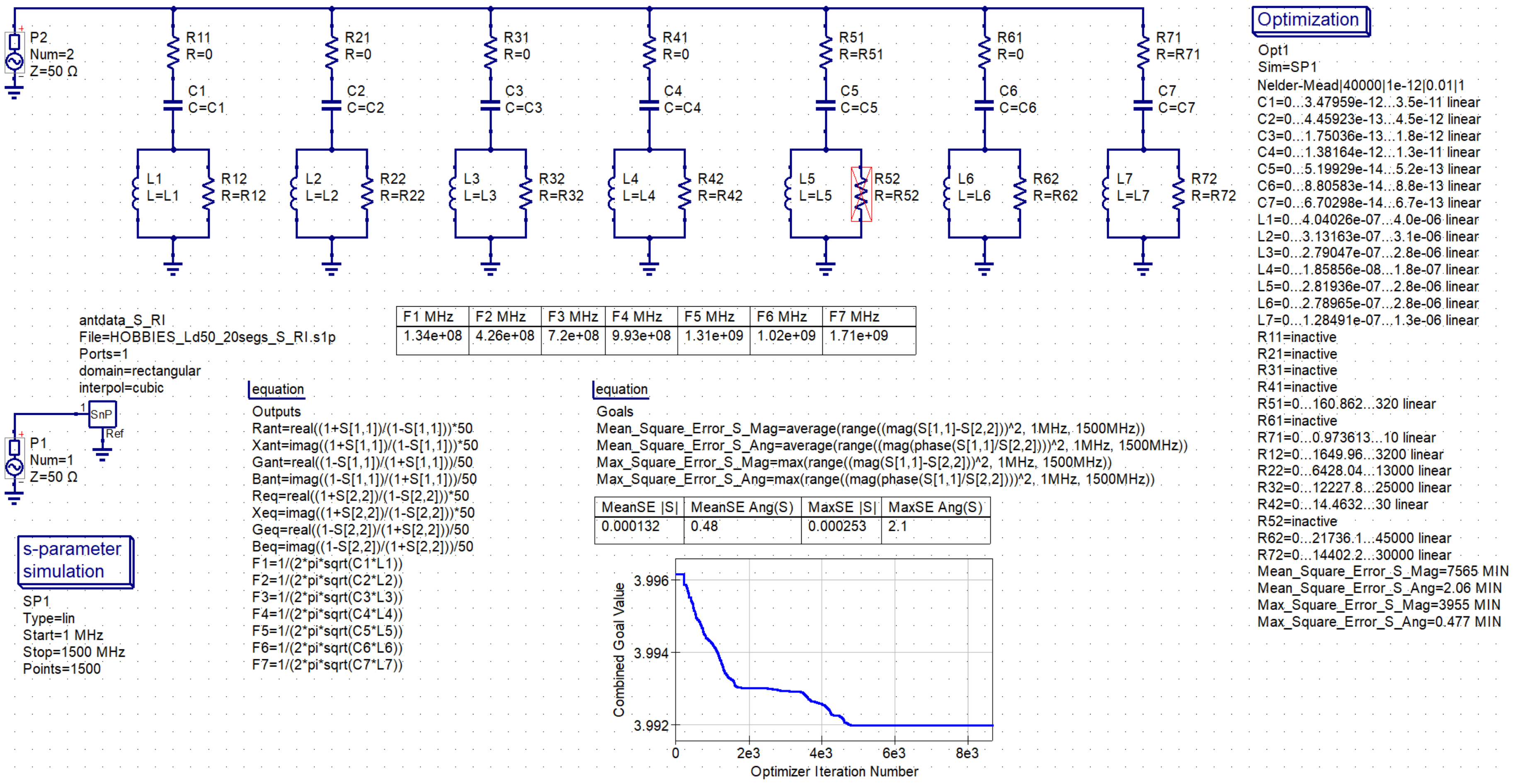




Example 7: Fat VHF Dipole

L/d = 50

1-Meter Long Dipole
Broadband Equivalent Circuit – Optimization Setup



antdata_S_RI
File=HOBBIES_Ld50_20segs_S_RI.s1p
Ports=1
domain=rectangular
interpol=cubic

s-parameter simulation

SP1
Type=lin
Start=1 MHz
Stop=1500 MHz
Points=1500

F1 MHz	F2 MHz	F3 MHz	F4 MHz	F5 MHz	F6 MHz	F7 MHz
1.34e+08	4.26e+08	7.2e+08	9.93e+08	1.31e+09	1.02e+09	1.71e+09

equation

Outputs
Rant=real((1+S[1,1])/(1-S[1,1]))*50
Xant=imag((1+S[1,1])/(1-S[1,1]))*50
Gant=real((1-S[1,1])/(1+S[1,1]))/50
Bant=imag((1-S[1,1])/(1+S[1,1]))/50
Req=real((1+S[2,2])/(1-S[2,2]))*50
Xeq=imag((1+S[2,2])/(1-S[2,2]))*50
Geq=real((1-S[2,2])/(1+S[2,2]))/50
Beq=imag((1-S[2,2])/(1+S[2,2]))/50
F1=1/(2*pi*sqrt(C1*L1))
F2=1/(2*pi*sqrt(C2*L2))
F3=1/(2*pi*sqrt(C3*L3))
F4=1/(2*pi*sqrt(C4*L4))
F5=1/(2*pi*sqrt(C5*L5))
F6=1/(2*pi*sqrt(C6*L6))
F7=1/(2*pi*sqrt(C7*L7))

equation

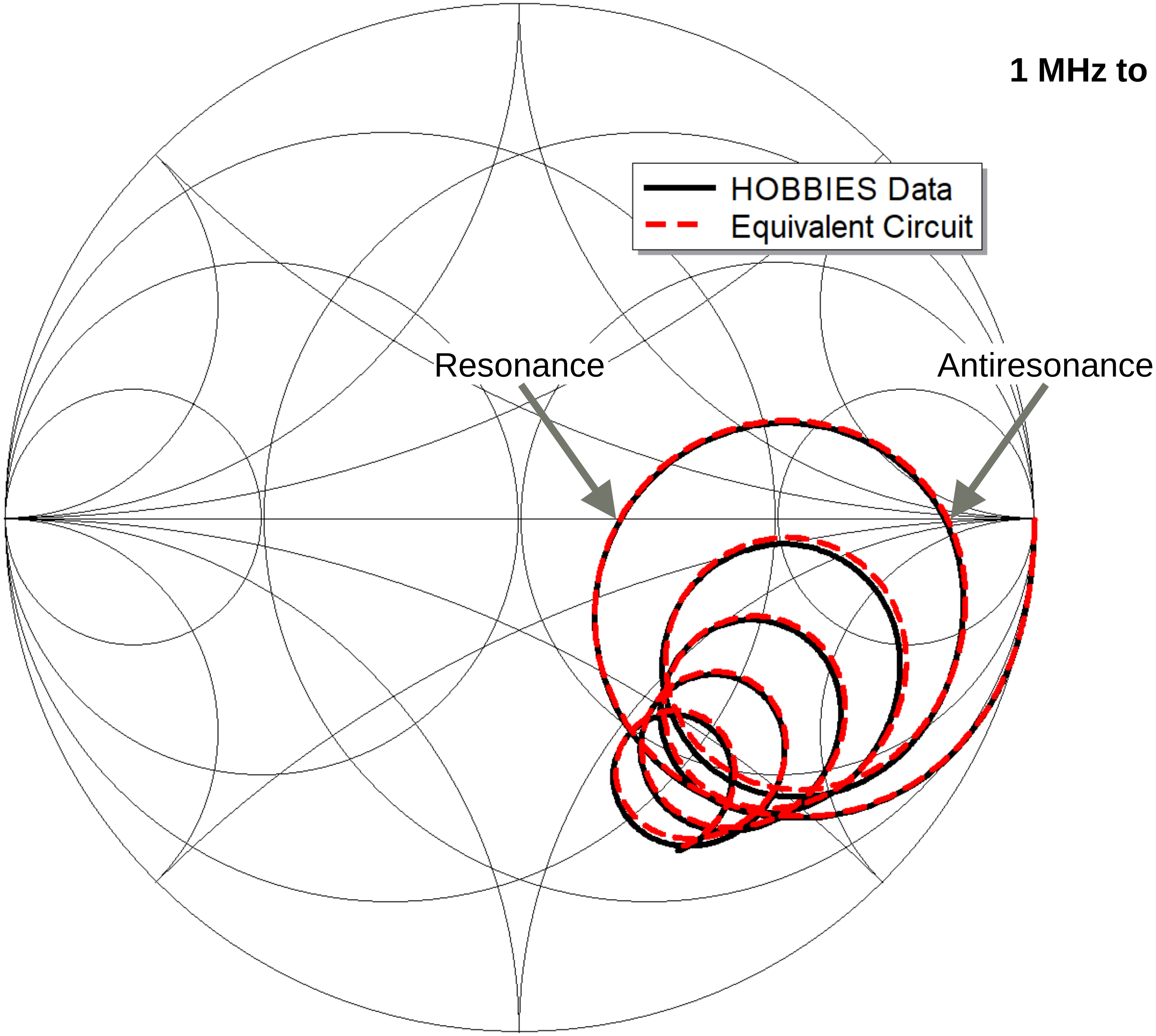
Goals
Mean_Square_Error_S_Mag=average(range((mag(S[1,1]-S[2,2]))^2, 1MHz, 1500MHz))
Mean_Square_Error_S_Ang=average(range((mag(phase(S[1,1]/S[2,2])))^2, 1MHz, 1500MHz))
Max_Square_Error_S_Mag=max(range((mag(S[1,1]-S[2,2]))^2, 1MHz, 1500MHz))
Max_Square_Error_S_Ang=max(range((mag(phase(S[1,1]/S[2,2])))^2, 1MHz, 1500MHz))

MeanSE |S|	MeanSE Ang(S)	MaxSE |S|	MaxSE Ang(S)
0.000132	0.48	0.000253	2.1

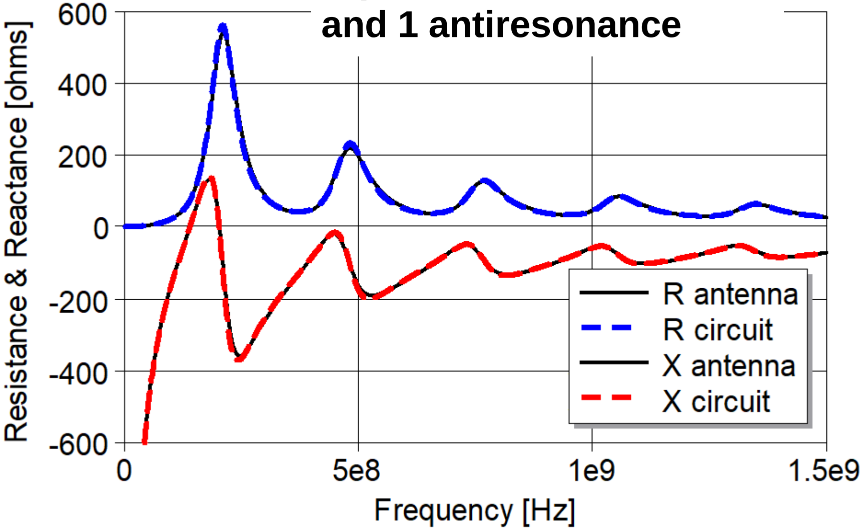
Optimization

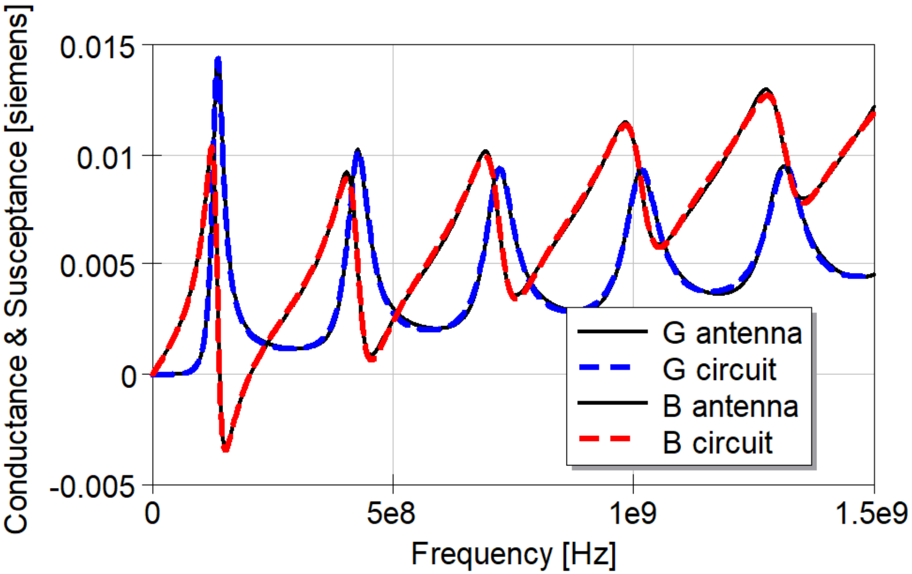
Opt1
Sim=SP1
Nelder-Mead|40000|1e-12|0.01|1
C1=0...3.47959e-12...3.5e-11 linear
C2=0...4.45923e-13...4.5e-12 linear
C3=0...1.75036e-13...1.8e-12 linear
C4=0...1.38164e-12...1.3e-11 linear
C5=0...5.19929e-14...5.2e-13 linear
C6=0...8.80583e-14...8.8e-13 linear
C7=0...6.70298e-14...6.7e-13 linear
L1=0...4.04026e-07...4.0e-06 linear
L2=0...3.13163e-07...3.1e-06 linear
L3=0...2.79047e-07...2.8e-06 linear
L4=0...1.85856e-08...1.8e-07 linear
L5=0...2.81936e-07...2.8e-06 linear
L6=0...2.78965e-07...2.8e-06 linear
L7=0...1.28491e-07...1.3e-06 linear
R11=inactive
R21=inactive
R31=inactive
R41=inactive
R51=0...160.862...320 linear
R61=inactive
R71=0...0.973613...10 linear
R12=0...1649.96...3200 linear
R22=0...6428.04...13000 linear
R32=0...12227.8...25000 linear
R42=0...14.4632...30 linear
R52=inactive
R62=0...21736.1...45000 linear
R72=0...14402.2...30000 linear
Mean_Square_Error_S_Mag=7565 MIN
Mean_Square_Error_S_Ang=2.06 MIN
Max_Square_Error_S_Mag=3955 MIN
Max_Square_Error_S_Ang=0.477 MIN

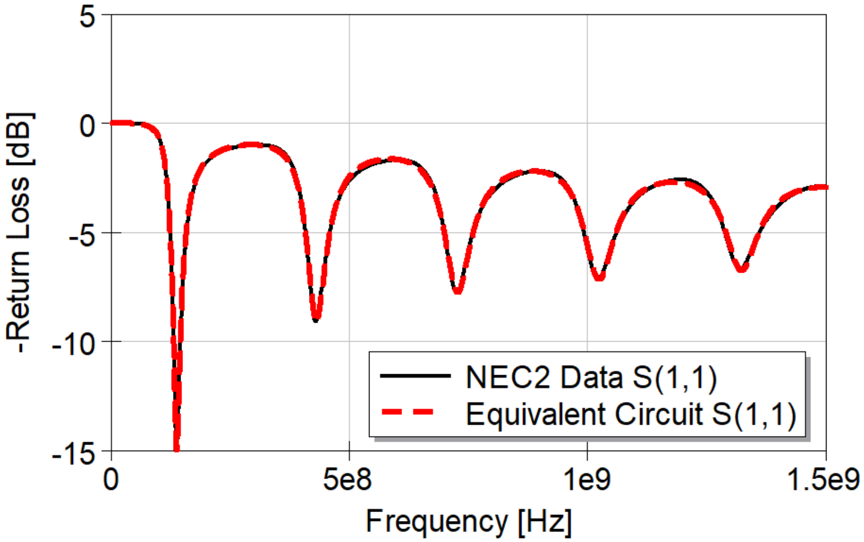
1-Meter Dipole Equivalent Circuit Performance



1-Meter Dipole Equivalent Circuit Performance


Dipole has 1 resonance and 1 antiresonance





Comments on How to Export Impedance Data from CEM Programs to EDA Programs

- **CEM programs compute immittance Z or Y data in real and imaginary form as R, X or G, B**
- **Data should be converted to S data to make a Touchstone file, .s1p or .s2p**
- **Touchstone allows data in three forms**
  - MA (magnitude-angle)
  - dB (magnitude in dB-angle)
  - RI (real-imaginary)
- **For S data near the boundary of the Smith chart, RI and dB are more accurate than MA**
- **Import data files into an EDA project one of two ways**
  - Use the “Project → Add File to Project” command, or
  - Copy/paste the file to a project folder before opening the project

Best results are achieved by importing S data in dB or RI forms and interpolating with a cubic spline interpolator.

Summary and Conclusions

- **Immittance (admittance or impedance) of 1-port distributed electromagnetic structures like antennas have broadband lumped-element equivalent circuits**
- **Antenna impedance can always be represented by a generalized ladder network of just four possible topologies**
    - Number of stages depends on desired accuracy of approximation
- **Element values may be set by either of two methods**
    - Semi-analytical: Compute element values from the poles and residues found by SEM analysis
    - Numerical: Use a circuit optimizer to fit one of four ladder circuits to measured or computed impedance data. Fit a small bandwidth first. Add stages to increase bandwidth
- **If making a 2-port antenna emulator for non-radiating transmission testing**
    - Use Darlington synthesis to convert the network to a reactance 2-port with terminating resistor
    - Load resistor at Port 2 represents radiation plus loss

Further Reading

- A.D. Yaghjian, “A Simplified Derivation of Causality From Passivity for the Impedance Representation of Transmitting Antennas,” *IEEE Trans. Antennas and Propagation*, Jan 2022.
- A.D. Yaghjian, “Physical Unrealizability of a Series Reactance and Resistance of a Passive Causal Input Impedance,” *Int. Conf. Electromagnetics in Advanced Applications*, Verona, Italy, Sep 11-15, 2017.
- S. Licul and W.A. Davis, “Unified Frequency and Time-Domain Antenna Modeling and Characterization,” *IEEE Trans. Antennas and Propagation*, Sep 2005.
- T.K. Sarkar and O. Pereira, “Using the Matrix Pencil Method to Estimate the Parameters of a Sum of Complex Exponentials,” *IEEE Antennas and Propagation Magazine*, Feb 1995.
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- C. E. Baum, “The Singularity Expansion Method - Background and Developments,” *IEEE Antennas and Propagation Society Newsletter*, Aug 1986
- A.G. Ramm, “Theoretical and Practical Aspects of Singularity and Eigenmode Expansion Methods,” *IEEE Trans. Antennas and Propagation*, Nov 1980.
- C.E. Baum, “Emerging Technology for Transient and Broad-Band Analysis and Synthesis of Antennas and Scatterers,” *Proc. IEEE*, Nov 1976.
- M.L. Van Blaricum and R. Mittra, “A Technique for Extracting the Poles and Residues of a System Directly from Its Transient Response,” *IEEE Trans. Antennas and Propagation*, Nov 1975.
- C.E. Baum, *On the Singularity Expansion Method for the Solution of Electromagnetic Interaction Problems*, Interaction Note 88, AFWL, Dec 1971.
- D.C. Youla, “Physical Realizability Criteria,” *IRE Trans. Circuit Theory*, Aug 1960.
- M.K. Zinn, “Network Representation of Transcendental Impedance Functions,” *Bell System Tech J.*, Mar 1952.
- S.A. Schelkunoff, “Representation of Impedance Functions in Terms of Resonant Frequencies,” *Proc. IRE*, Feb 1944.

The End

This presentation incorporating revisions and errata will be archived at https://www.fars.k6ya.org/docs/k6oik