Weird Waves
Exotic Electromagnetic Phenomena

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ARRL Pacificon Presentations by K6OIK

Year	Title	Archived at http://www.fars.k6ya.org
1999	Mysteries of the Smith Chart	
2000	Jam-Resistant Repeater Technology	
2001	Mysteries of the Smith Chart	✓
2002	How-to-Make Better RFI Filters Using Stubs	
2003	Twin-Lead J-Pole Design	
2004	Antenna Impedance Models – Old and New	✓
2005	Novel and Strange Ideas in Antennas and Impedance Matching	
2006	Novel and Strange Ideas in Antennas and Impedance Matching II	✓
2007	New Results on Antenna Impedance Models and Matching	✓
2008	Antenna Modeling for Radio Amateurs	✓
2009	(convention held in Reno)	
2010	Facts About SWR, Reflected Power, and Power Transfer on Real Transmission Lines with Loss	✓
2011	Conjugate Match Myths	✓
2012	Transmission Line Filters Beyond Stubs and Traps	✓
2013	Bode, Chu, Fano, Wheeler – Antenna Q and Match Bandwidth	✓
2014	A Transmission Line Power Paradox and Its Resolution	✓
2015	Weird Waves: Exotic Electromagnetic Phenomena	✓
2015	The Joy of Matching: How to Design Multi-Frequency and Multi-Band Match Networks	✓

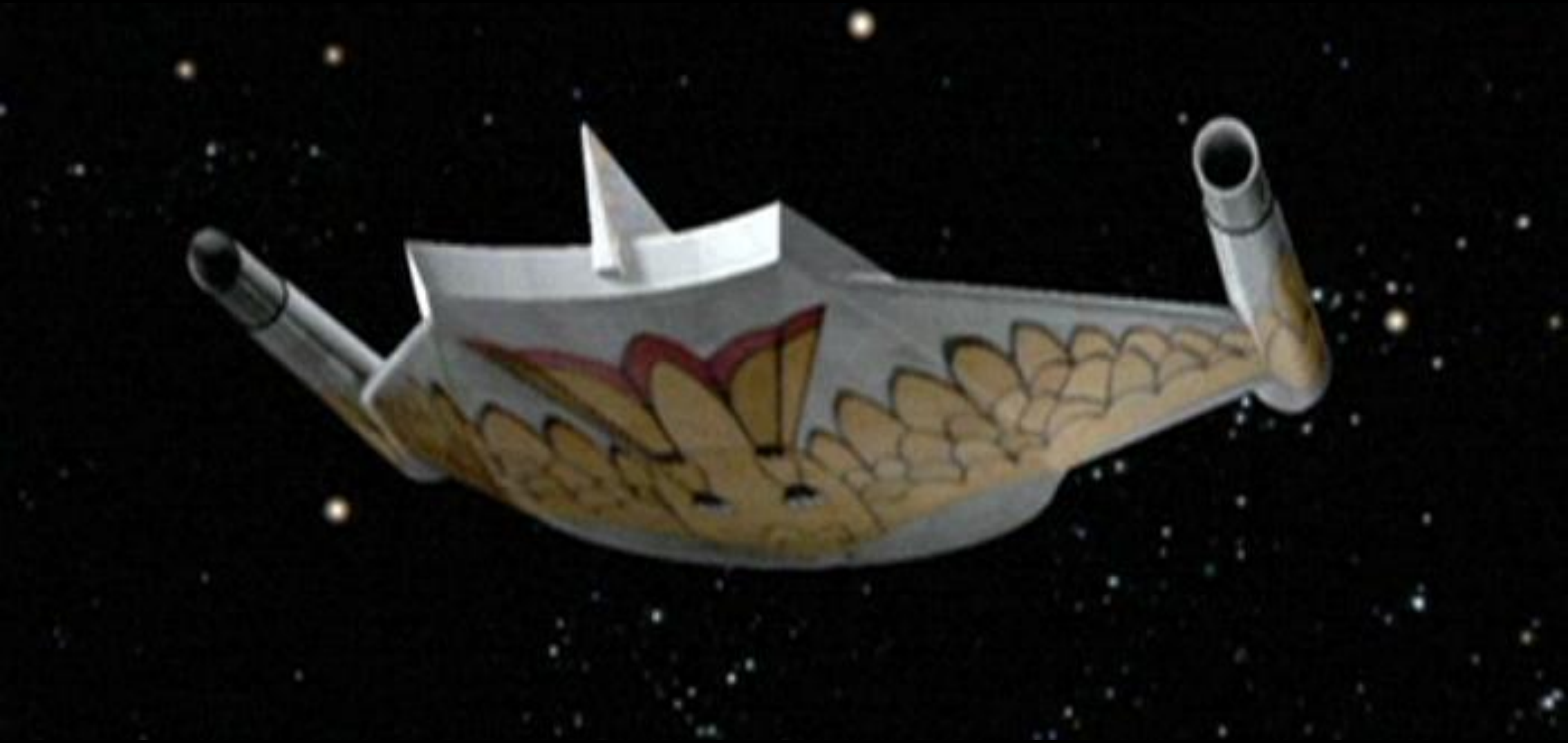
Outline

- **Electromagnetic Cloaking**
- **Free-space “localized” waves**
  - Knotted waves, linked waves, and vortex waves
- **Vortex waves as Bessel modes**
  - Constant phase surface
  - Wavelength
  - Phase velocity
  - Polarization
  - Poynting vector, power flow, momentum
  - Velocities: phase, energy, signaling, information
- **Comments, interpretation and speculation**
  - Reconciliation with Einstein

Electromagnetic Cloaking

Cloaking versus Invisibility

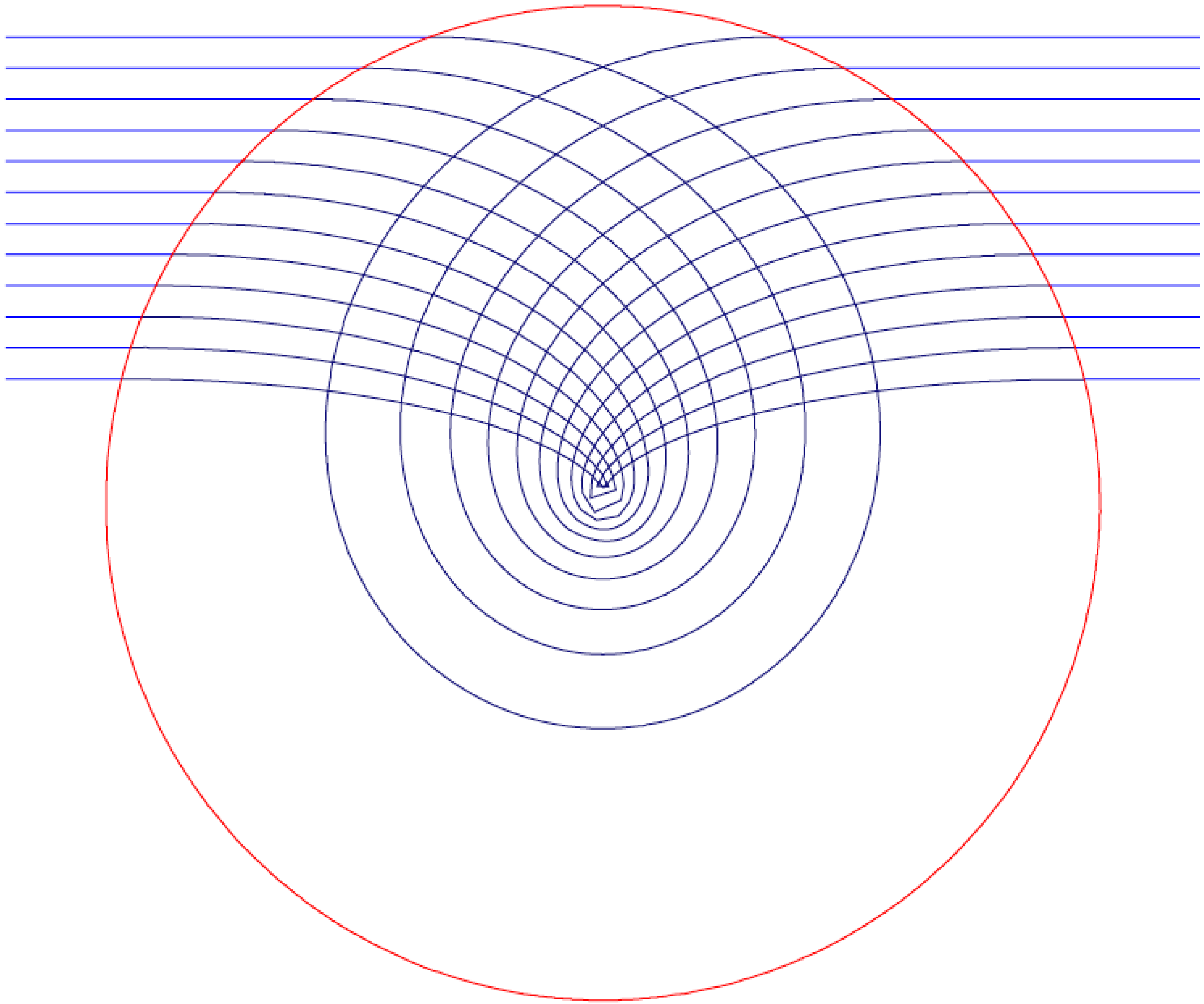
- **Invisibility requires more than transparency and anti-reflective surfaces**
  - Optical lenses are transparent and can have anti-reflective surfaces; yet you see them
- **Cloaking requires more than invisiblity**
  - An Eaton lens can be invisible, but its volume is filled
  - No room for a "payload"
- **Cloaking hides a region of space which can contain a hidden payload**
- **Idea was introduced in the *Star Trek* television series, season 1, episode 9, on December 15, 1966, which featured a Romulan Bird Of Prey**



An Invisible Sphere

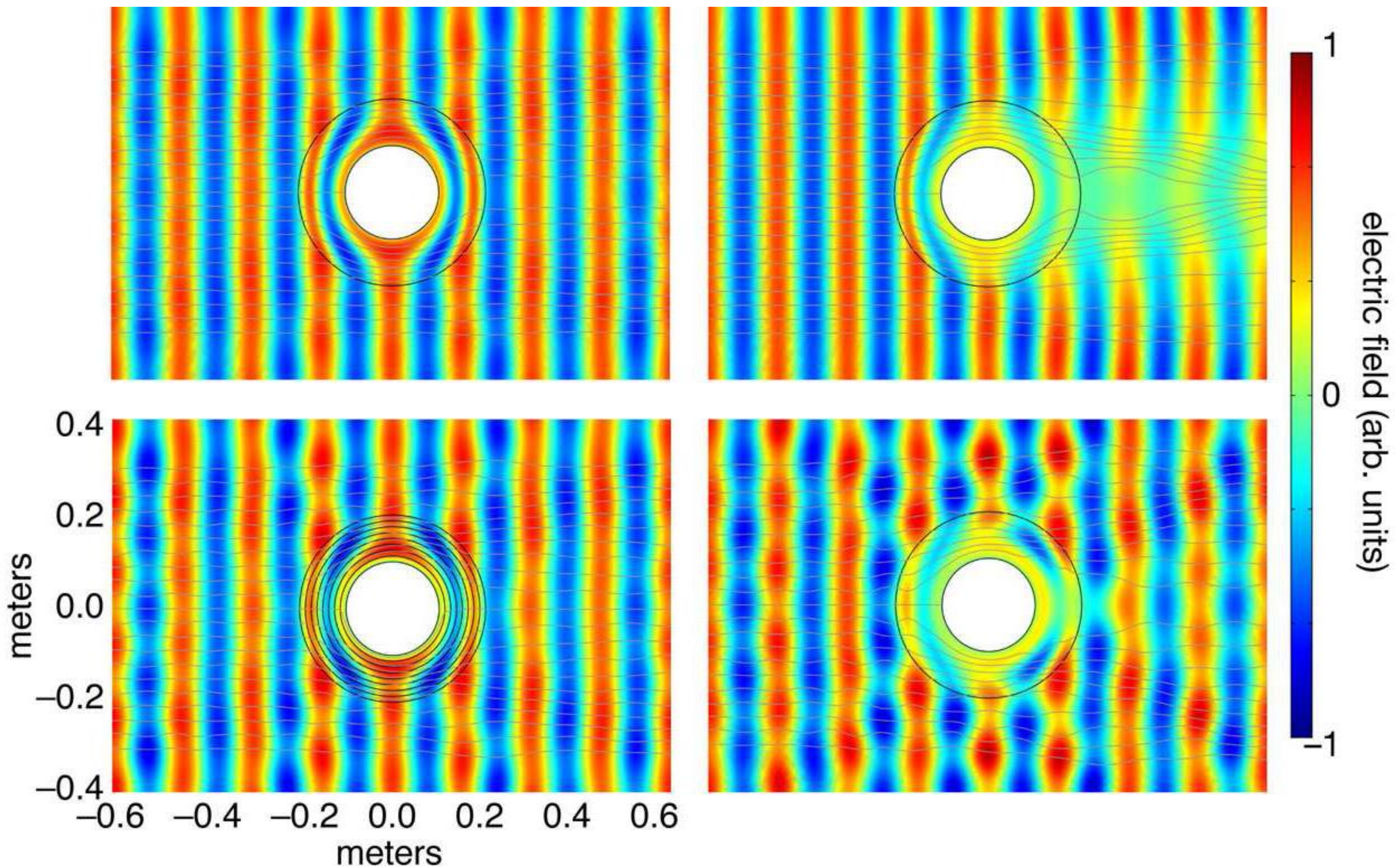
A.J. Danner, "Visualizing Invisibility: Photorealistic Depictions of Optical Cloaking," *IEEE CLEO* and *OSA QELS*, May 16-21, 2010

How It Works
Ray Paths Through an Invisible Eaton Lens



U. Leonhard and T. Philbin, *Geometry and Light: The Science of Invisibility*, Dover, 2010

Computer Simulations of Cloaking at 3 GHz



S.A. Cummer, et al., "Full-Wave Simulations of Electromagnetic Cloaking Structures," *arXiv* 0607242, July 2006

Summary

- **Cloaking requires controlling of scattered and reaction fields**
- **Methods**
  - Axial cloaking using ordinary materials (mirrors and lenses)
  - Transformation optics using ordinary or meta materials
  - Reflection (carpet) cloaking using a metasurface or Pendry lens
  - Surround cloaking using metamaterial shell(s): Pendry 2006
- **Cloak bandwidth set by metamaterial properties**
- **Objects inside the cloak cannot see or communicate out at the cloaked wavelengths but can communicate out at other wavelengths**
- **Applications:**
  - Radar invisibility (avoid traffic tickets)
  - Stealth antennas
  - Ground independent antennas (cloak the earth under an antenna)
  - Sports – invisible balls (invisible baseballs, basketballs, golf balls, ping-pong balls, tennis balls,…)

Weird Waves

Heaviside's Vector "Duplex" Equations for Maxwell's Theory

$$\nabla \times \mathbf{E} = -\mathbf{M} - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

$$\nabla \cdot \mathbf{B} = \rho_m$$

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{J} = \sigma_e \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{M} = \sigma_m \mathbf{H}$$

"And God said, Let there be light; and there was light." *Genesis* 1:3

D'Alembert's Wave Equations in Free Space

- **Time domain**

$$\nabla^2 \mathbf{E} = \mu \varepsilon \frac{\partial^2}{\partial t^2} \mathbf{E}$$

$$\nabla^2 \mathbf{H} = \varepsilon \mu \frac{\partial^2}{\partial t^2} \mathbf{H}$$

- **Frequency domain, complex phasor form**

$$\nabla^2 \mathbf{E} + \beta^2 \mathbf{E} = 0$$

$$\nabla^2 \mathbf{H} + \beta^2 \mathbf{H} = 0$$

- **Solved analytically by a technique called "separation of variables"**
- **Solved numerically by computational electromagnetics software (CEM) – aka "antenna modeling"**

Misconceptions About Waves

Journal of Scientific Exploration, Vol. 16, No. 3, pp. 359–362, 2002 0892-3310/02

Can Longitudinal Electromagnetic Waves Exist?

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Abstract—In discussions on electro smog K. Meyl has proposed to consider the "dangerous" scalar waves (1) in addition to Hertzian waves. But we have already shown in a previous paper (2) that, indeed, Meyl's scalar waves cannot cause any harm, to anybody—since *they do not exist*. Some readers have interpreted Meyl's scalar waves to be identical with longitudinal electromagnetic waves, but this is not clear due to Meyl's inconsistencies; e.g., his splitting the wave equation is erroneous. Therefore, to calm down our worried readers, below we shall prove that longitudinal electromagnetic waves are harmless as well by recalling a well-known classical result: Plane longitudinal electromagnetic waves *do not exist*. We supplement this by showing that longitudinal spherical electromagnetic waves have the same pleasant property: *They don't exist.*

Keywords: electromagnetism

"In case the medium is nondispersive, *u* (group velocity) coincides with the phase velocity *v*, but otherwise is a function of the wave number k_0. ...

The group velocity *u* differs from the phase velocity only in dispersive media."

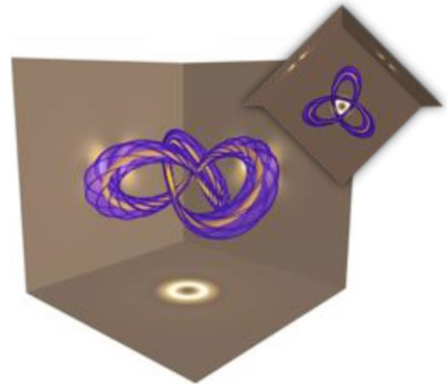
[J.A. Stratton, pp. 332, 339]

"In a nondispersive medium, the phase and group velocities are equal."

[J.G. Van Bladel, p. 830]

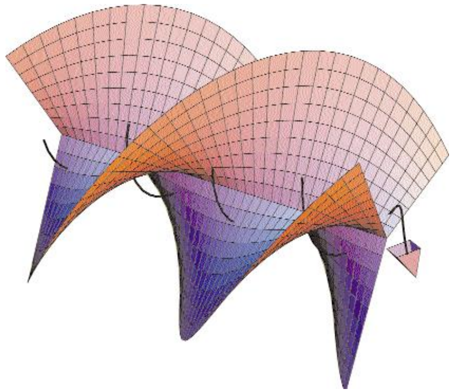
Three Kinds of Localized Waves

- **Knotted waves**



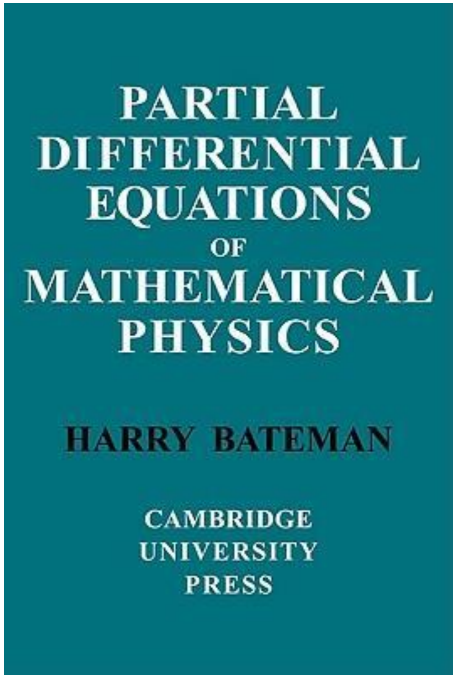
- **Linked waves**

- **Vortex waves**



Weird Waves from Maxwell's Equations

Harry Bateman, 1882 - 1946

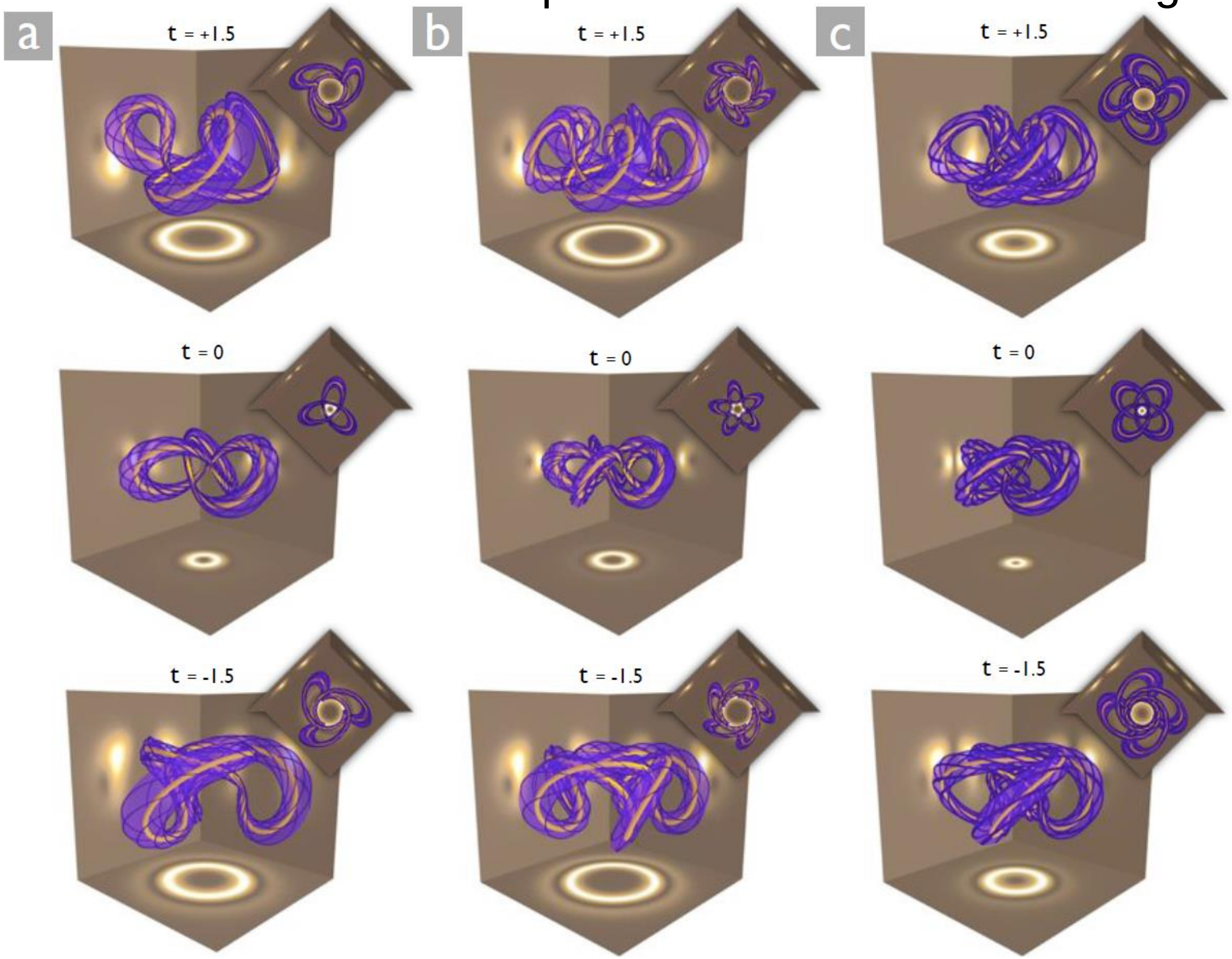


1915

- **Knotted waves and linked waves are obtained by using Bateman's (forgotten) method to solve Maxwell's equations**
- **Bateman's solutions were rediscovered in fluid dynamics**

Knotted Waves

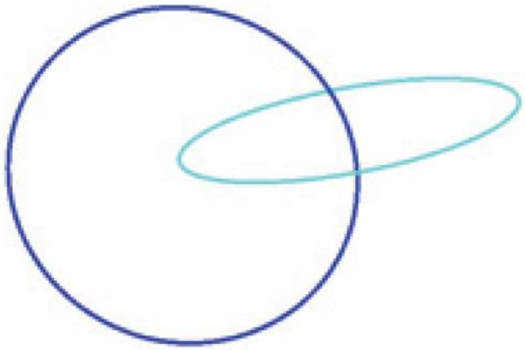
Trefoil Knot | Cinquefoil Knot | 4 Linked Rings

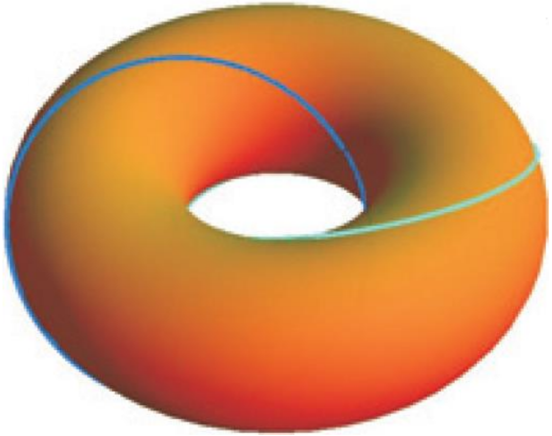


H. Kedia, I. Bialynicki-Birula, D. Peralta-Salas, and W.T.M. Irvine, "Tying Knots in Light Fields," *arXiv* 1302.0342v1, Feb. 2013, and *Physical Review Letters*, vol. 111, no. 15, Oct. 10, 2013

Linked Waves – Hopf Fibrations

- **Hopf fibrations are made of linked "unknots" or circles**
- **Two circles nested on a torus as shown here are linked**
- **Mutually linked circles fill the torus**





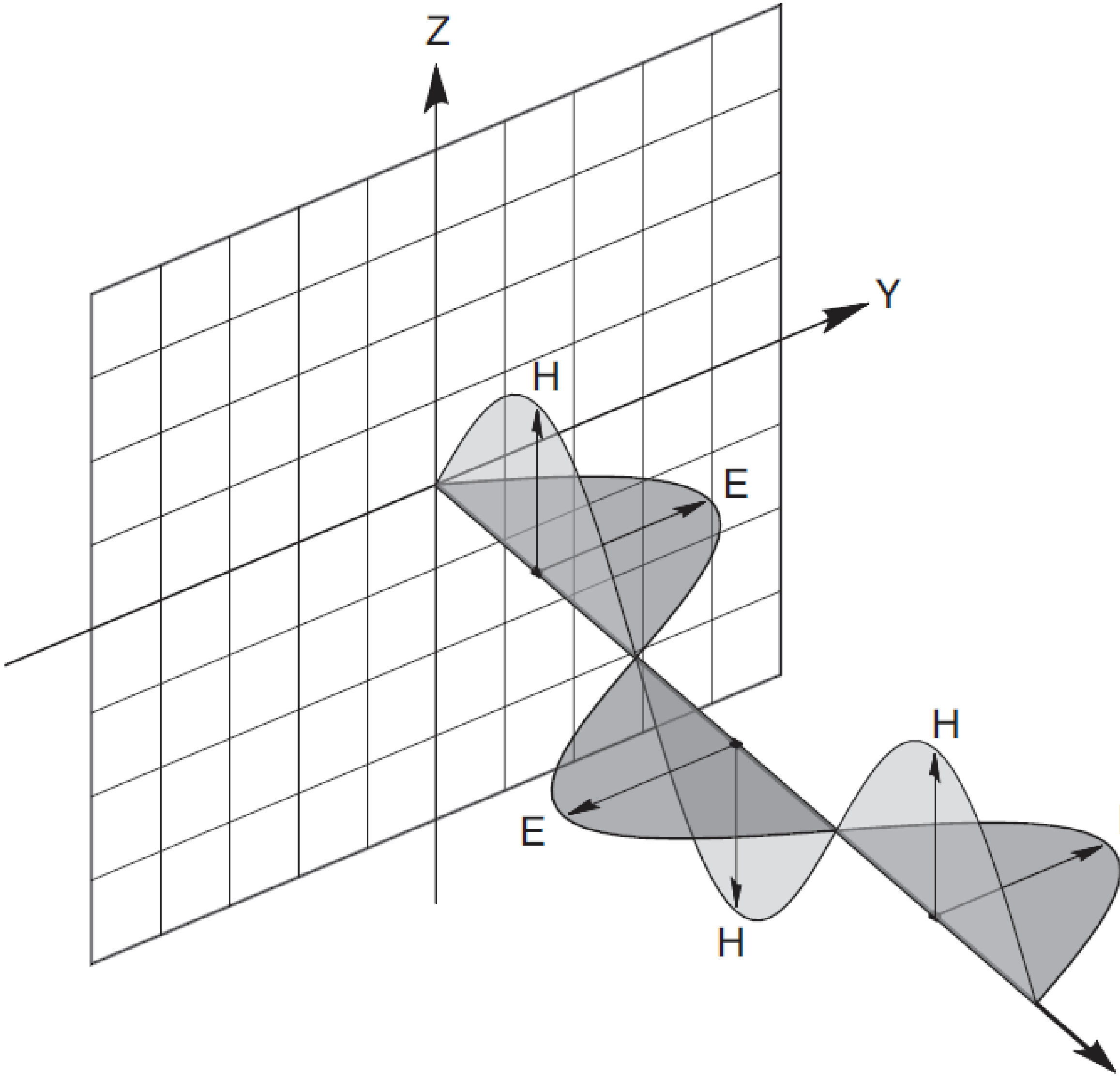
W.T.M. Irvine and D. Bouwmeester, "Linked and Knotted Beams of Light," *Nature Physics*, Sept. 2008

Electromagnetic Behavior

- Bateman's solutions provide mutually perpendicular "dual" fields having fixed amplitude ratio (wave impedance)
- The Poynting vector forms a closed loop, i.e. traces the knot ad infinitum; energy flow follows the knot
- If the knot drift velocity is zero, energy remains confined and spatially localized; the wave propagates locally only
- Some knotted wave solutions are perturbationally stable through time and in space
- Ordinary concepts of "near field" and "far field" do not apply

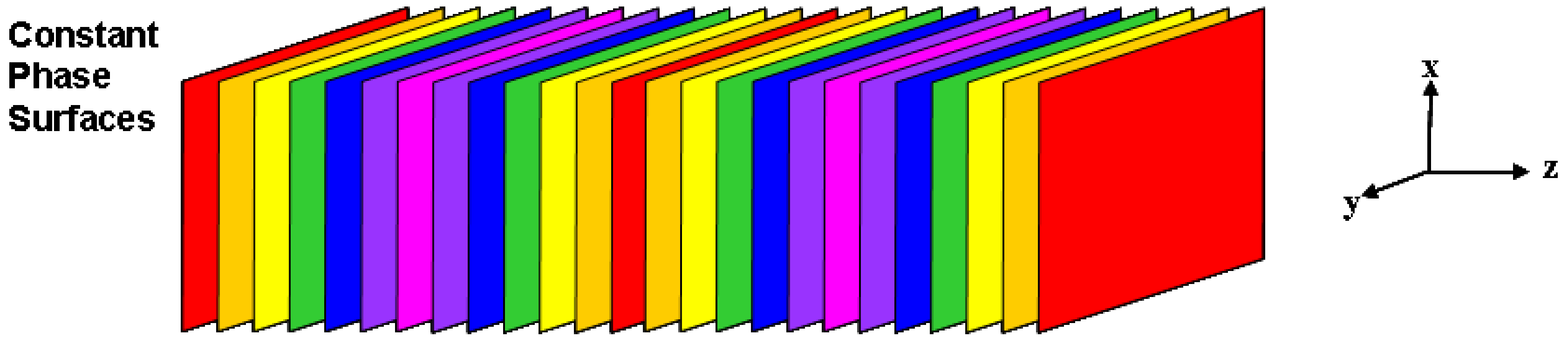
Electromagnetic Vortex Waves

Simple Wave Propagation



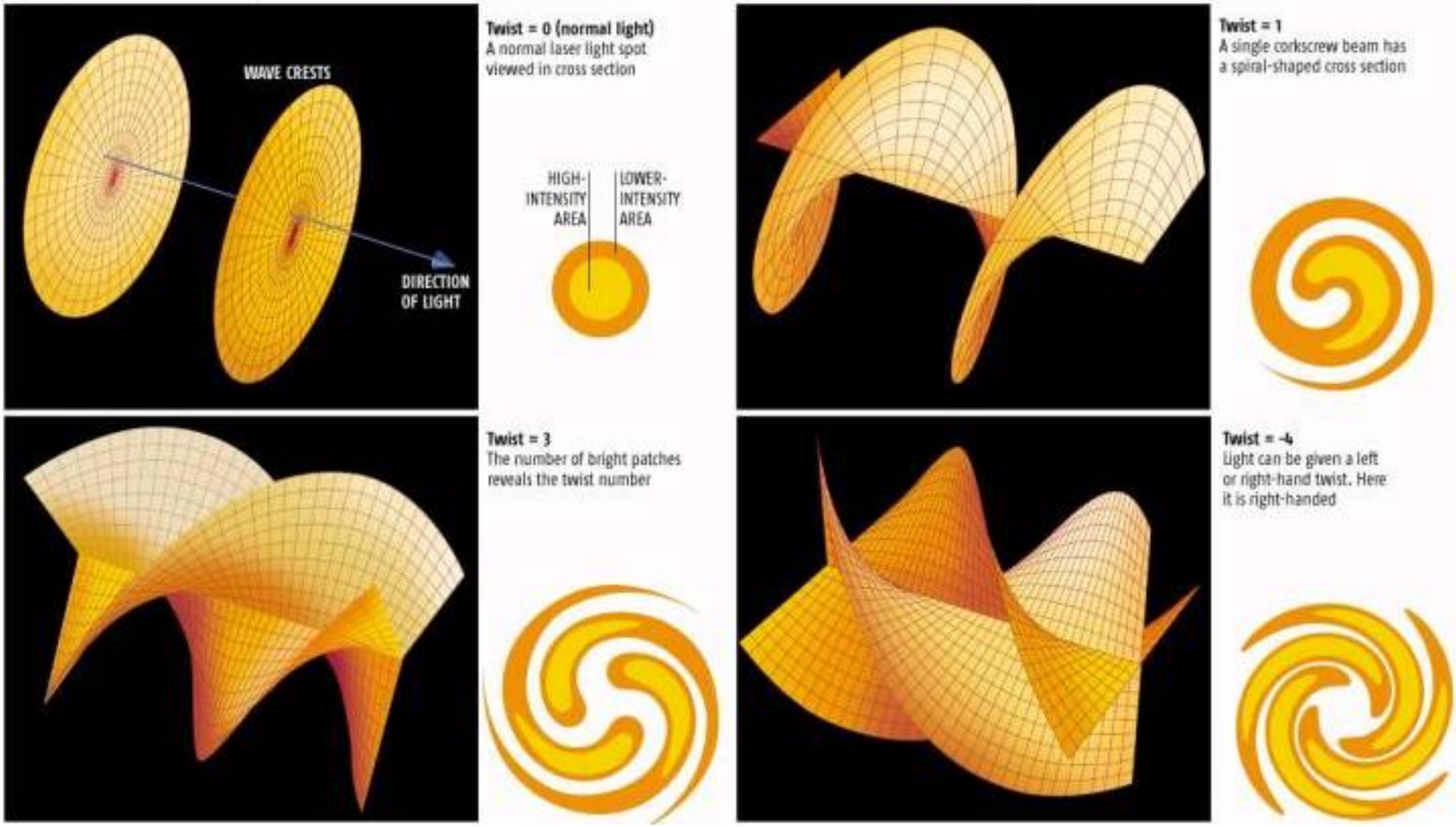
Uniform Transverse Electromagnetic (TEM) Plane Waves

- "A uniform plane wave is a wave, which does not depend on two of the three spatial coordinates in a Cartesian system"



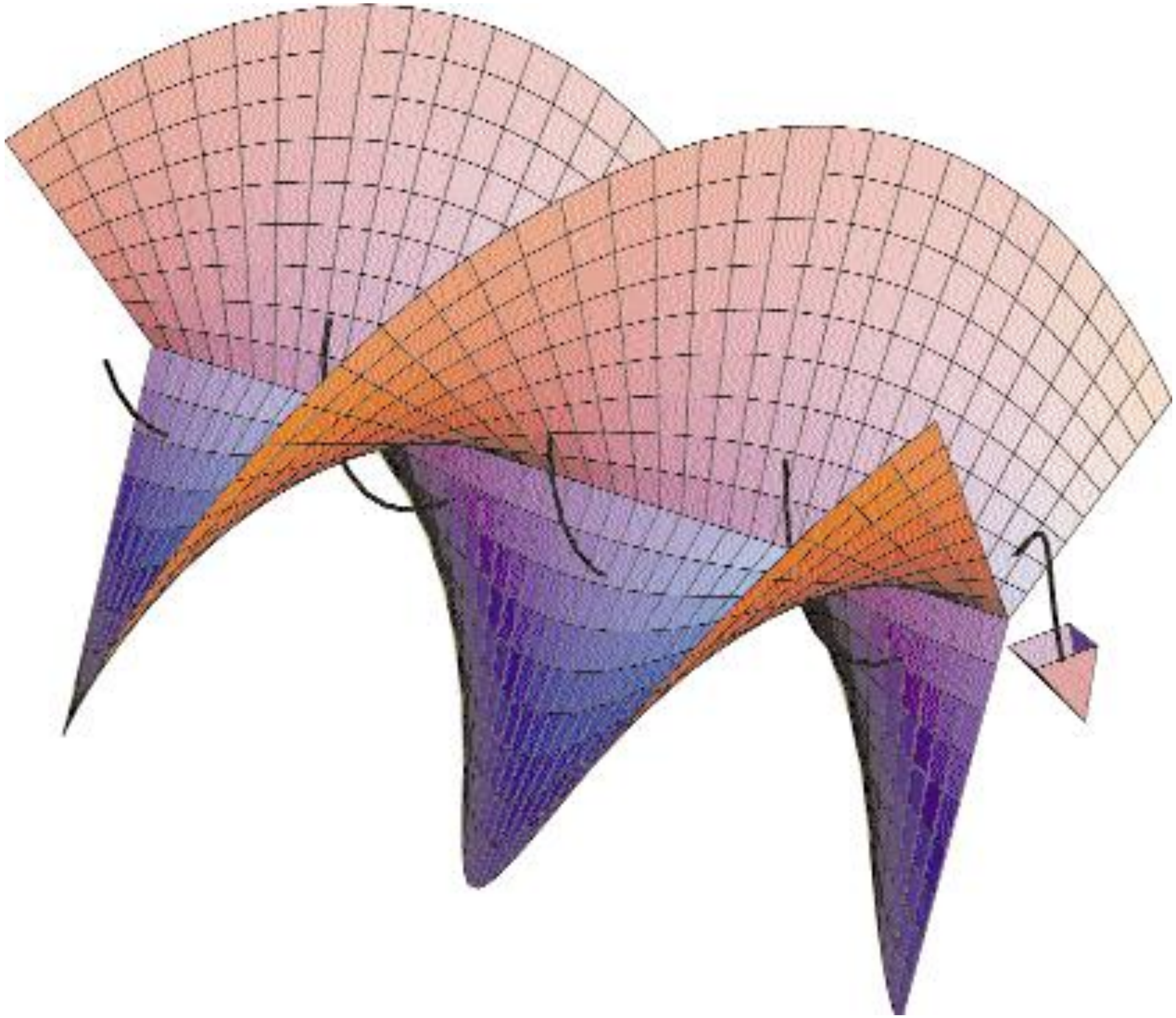
- We frequently assume far field radiation is uniform TEM plane waves
- But other kinds of waves can satisfy the wave equation and propagate in free space

Vortex Beam Phase Surfaces

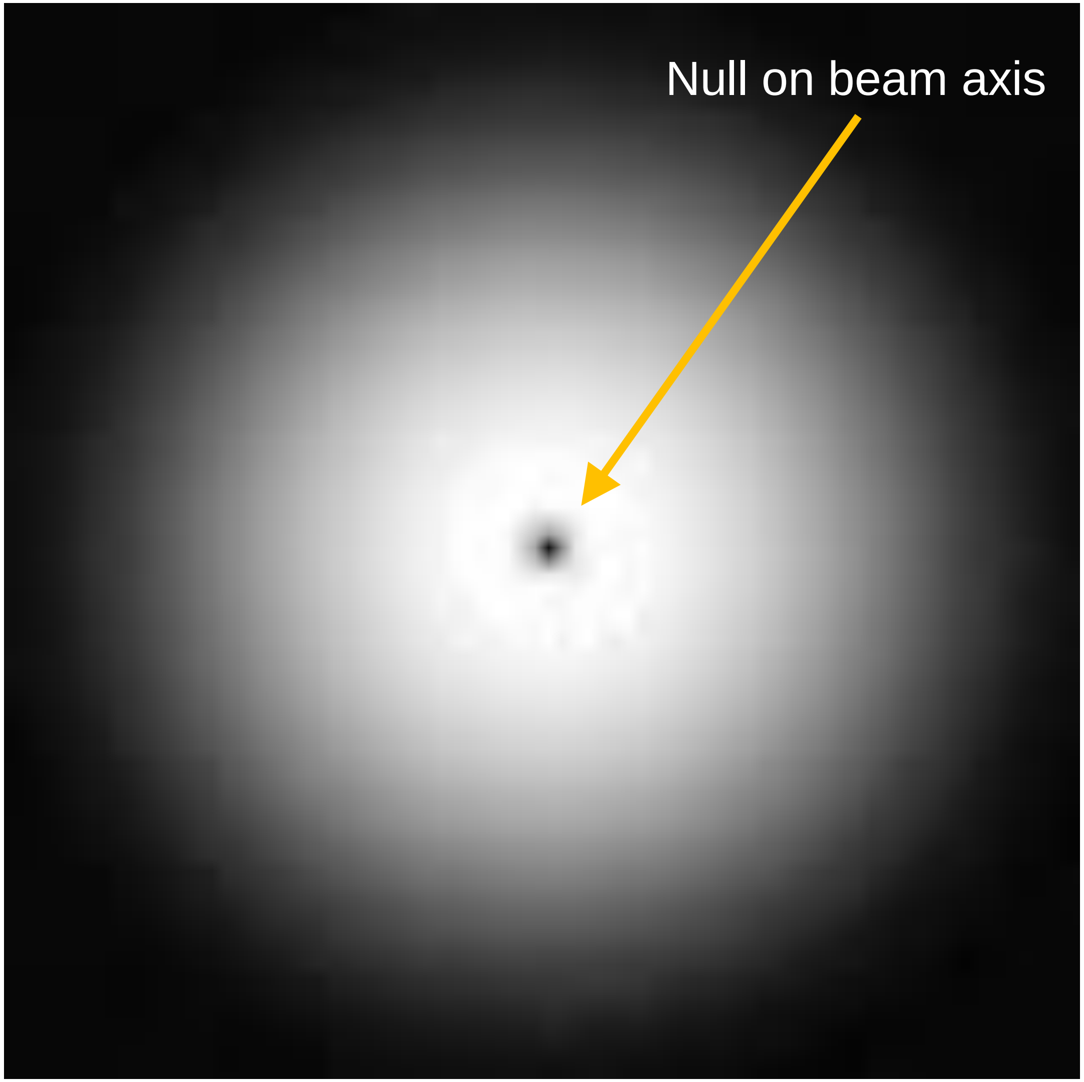


S. Battersby, "Twisting the Light Away," *New Scientist*, June 12, 2004

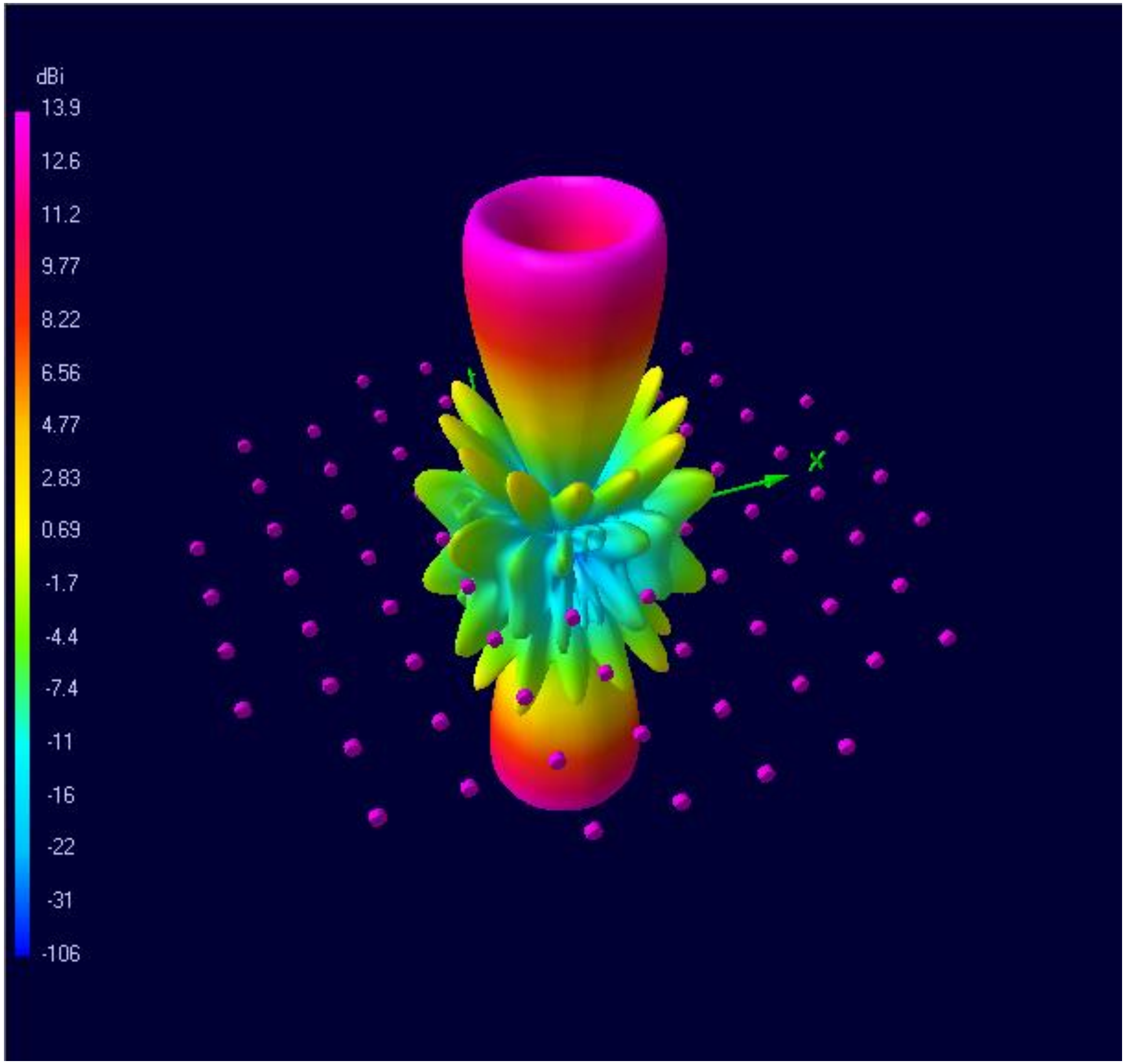
m = 3 Helicoidal Phase Surface



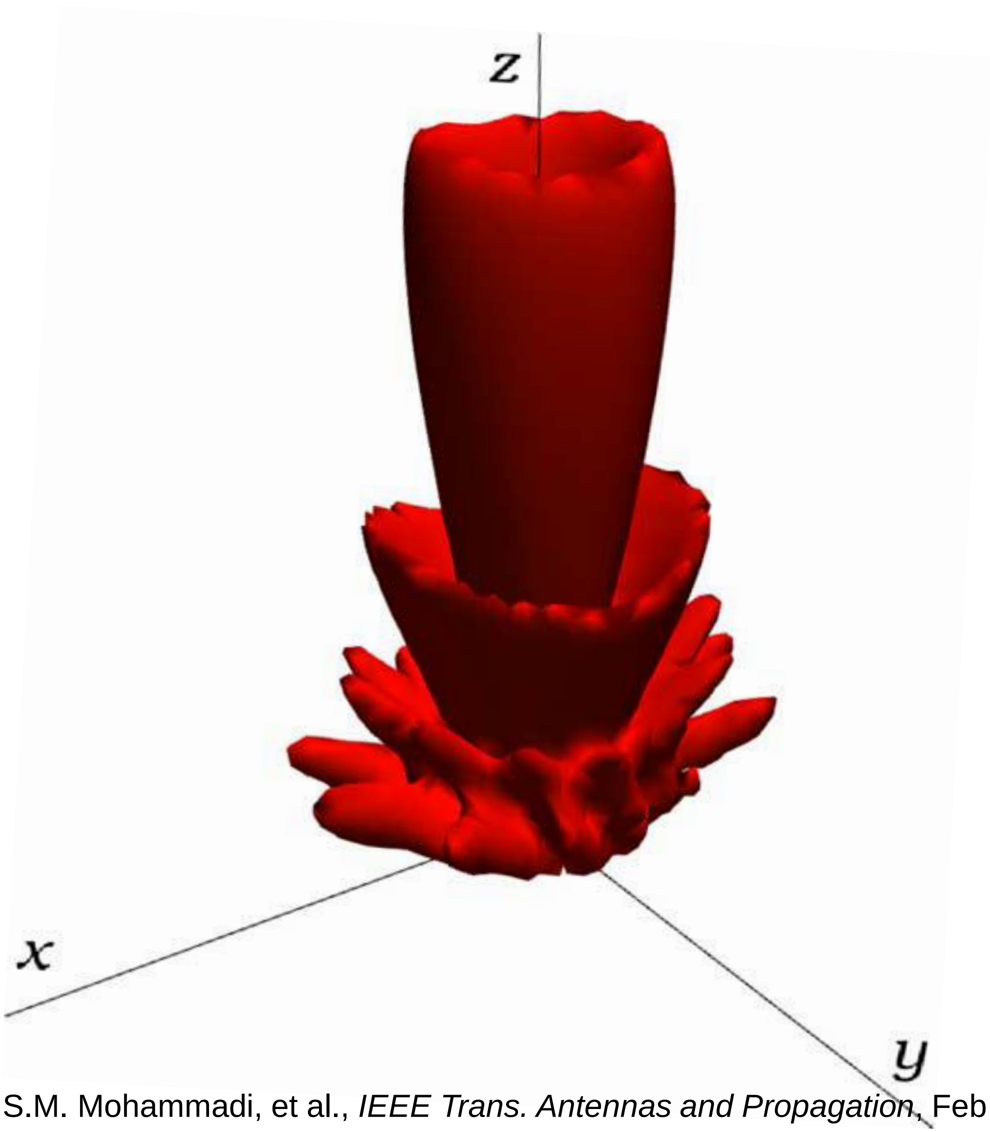
Intensity of a Vortex Wave – Viewed on Axis



m = 2 Vortex, Pattern of 80-Element 3λ Array



m = 1 Vortex, Pattern of 12-Element 4λ CDAA



S.M. Mohammadi, et al., *IEEE Trans. Antennas and Propagation*, Feb. 2010

Fields in Terms of Vector Potentials

- **In charge-free space, the electric and magnetic fields are a superposition of terms due to electric and magnetic potentials**

$$\boldsymbol{E} = \boldsymbol{E}_A + \boldsymbol{E}_F = -j\omega\boldsymbol{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \bullet \boldsymbol{A}) - \frac{1}{\varepsilon}\nabla\times\boldsymbol{F}$$

$$\boldsymbol{H} = \boldsymbol{H}_A + \boldsymbol{H}_F = \frac{1}{\mu}\nabla\times\boldsymbol{A} - j\omega\boldsymbol{F} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla \bullet \boldsymbol{F})$$

- **Our interest is in collimated, non-diffracting TE^z and TM^z vortex waves traveling in the *z*-axis (axial) direction**
- **Collimated, non-diffracting beams are derived as field solutions to the wave equation in cylindrical coordinates**

C.A.Balanis, *Advanced Engineering Electromagnetics*, Wiley

Fields are Derivatives of Potentials (Calculus ugh!)

$$E_\rho = -j\omega A_\rho - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\left(\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)$$

$$E_\phi = -j\omega A_\phi - j\frac{1}{\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right)$$

Zero for TE^z

$$E_z = -j\omega A_z - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}\right] - \frac{1}{\varepsilon}\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho F_\phi) - \frac{\partial F_\rho}{\partial\phi}\right)$$

$$H_\rho = -j\omega F_\rho - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial\rho}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right)$$

$$H_\phi = -j\omega F_\phi - j\frac{1}{\omega\mu\varepsilon}\frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)$$

Zero for TM^z

$$H_z = -j\omega F_z - j\frac{1}{\omega\mu\varepsilon}\frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}\right] + \frac{1}{\mu}\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right)$$

TE^z Wave Solution

- **Transverse electric (TE^z) waves traveling in the *z* direction are described by**

$$\mathbf{A} = 0 \quad \text{and} \quad \mathbf{F} = \mathbf{a}_z F_z(\rho, \phi, z)$$

- **Wave equation for electric vector potential**

$$\nabla^2 F_z(\rho, \phi, z) + \beta^2 F_z(\rho, \phi, z) = 0$$

$$\frac{\partial^2 F_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 F_z}{\partial \phi^2} + \frac{\partial^2 F_z}{\partial z^2} + \beta^2 F_z = 0$$

- **Solution by separation of variables**

$$F_z(\rho, \phi, z) = \left[C_1 J_m(\beta_\rho \rho) + D_1 Y_m(\beta_\rho \rho)\right] \times \left[C_2 e^{-jm\phi} + D_2 e^{jm\phi}\right] \times \left[C_3 e^{-j\beta_z z} + D_3 e^{j\beta_z z}\right]$$

- **Specific solution for vortex wave traveling in *z* direction with phase advancing in ϕ direction**

$$F_z(\rho, \phi, z) = C\, J_m(\beta_\rho \rho)\, e^{-j(m\phi + \beta_z z)}$$

TE^z Vortex Fields

$$E_\rho = -\frac{1}{\varepsilon\rho}\frac{\partial F_z}{\partial\phi} = j\frac{C\eta\omega m}{\beta\rho} J_m(\beta_\rho\rho)\, e^{-j(m\phi+\beta_z z)}$$

$$E_\phi = \frac{1}{\varepsilon}\frac{\partial F_z}{\partial\rho} = \frac{C\eta\omega\beta_\rho}{\beta} J'_m(\beta_\rho\rho)\, e^{-j(m\phi+\beta_z z)}$$

$$E_z = 0 = 0$$

$$H_\rho = -j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2}{\partial\rho\,\partial z}F_z = -\frac{C\omega\beta_\rho\beta_z}{\beta^2} J'_m(\beta_\rho\rho)\, e^{-j(m\phi+\beta_z z)}$$

$$H_\phi = -j\frac{1}{\omega\mu\varepsilon\rho}\frac{\partial^2}{\partial\phi\,\partial z}F_z = j\frac{C\omega m\beta_z}{\beta^2\rho} J_m(\beta_\rho\rho)\, e^{-j(m\phi+\beta_z z)}$$

$$H_z = -j\frac{1}{\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z^2}+\beta^2\right)F_z = -j\frac{C\beta_\rho^2}{\beta^2} J_m(\beta_\rho\rho)\, e^{-j(m\phi+\beta_z z)}$$

TM^z Vortex Fields

$$E_\rho = -j\frac{1}{\omega\mu\varepsilon}\frac{\partial^2}{\partial\rho\,\partial z}A_z = -\frac{A\omega\beta_\rho\beta_z}{\beta^2}J'_m(\beta_\rho\rho)\,e^{-j(m\phi+\beta_z z)}$$

$$E_\phi = -j\frac{1}{\omega\mu\varepsilon\rho}\frac{\partial^2}{\partial\phi\,\partial z}A_z = j\frac{A\omega m\beta_z}{\beta^2\rho}J_m(\beta_\rho\rho)\,e^{-j(m\phi+\beta_z z)}$$

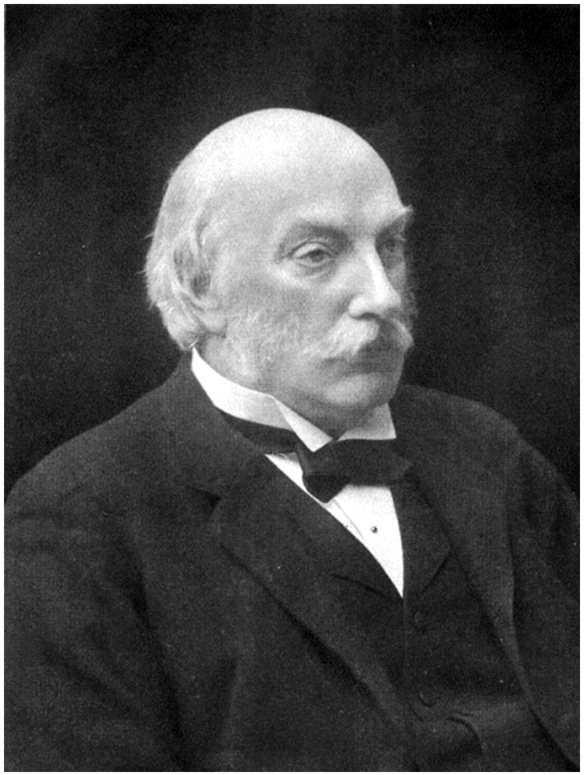
$$E_z = -j\frac{1}{\omega\mu\varepsilon}\left(\frac{\partial^2}{\partial z^2}+\beta^2\right)A_z = -j\frac{A\beta_\rho^2}{\beta^2}J_m(\beta_\rho\rho)\,e^{-j(m\phi+\beta_z z)}$$

$$H_\rho = \frac{1}{\mu\rho}\frac{\partial A_z}{\partial\phi} = -j\frac{A\omega m}{\eta\beta\rho}J_m(\beta_\rho\rho)\,e^{-j(m\phi+\beta_z z)}$$

$$H_\phi = -\frac{1}{\mu}\frac{\partial A_z}{\partial\rho} = -\frac{A\omega\beta_\rho}{\eta\beta}J'_m(\beta_\rho\rho)\,e^{-j(m\phi+\beta_z z)}$$

$$H_z = 0 = 0$$

Lord Rayleigh Circular Waveguide Solution 1897



Lord Rayleigh
John William Strutt
1842 – 1919

XVIII. *On the Passage of Electric Waves through Tubes, or the Vibrations of Dielectric Cylinders.* By LORD RAYLEIGH, *F.R.S.**

General Analytical Investigation.

THE problem here proposed bears affinity to that of the vibrations of a cylindrical solid treated by Pochhammer† and others, but when the bounding conductor is

* Communicated by the Author.
† Crelle, vol. xxxi. 1876.

Phil. Mag. S. 5. Vol. 43. No. 261. *Feb.* 1897. L

Wave Impedances

- **TE^z**

$$\frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \eta \frac{\beta}{\beta_z} = \frac{\eta}{\cos\delta}$$

- **TM^z**

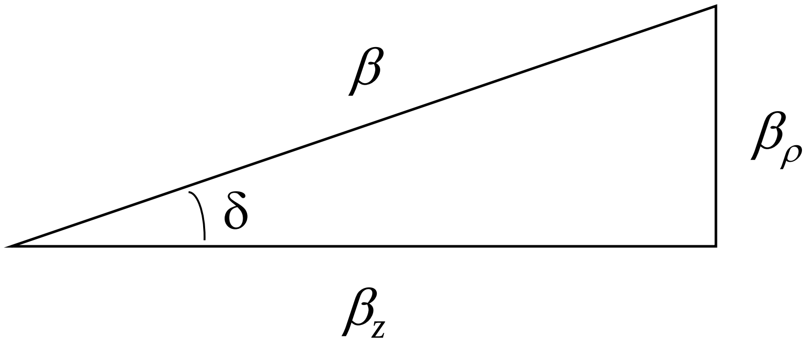
$$\frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \eta \frac{\beta_z}{\beta} = \eta\cos\delta$$

Relationships Among Phase Constants

- The radial and axial phase constants satisfy

$$\beta_\rho^2 + \beta_z^2 = \beta^2 = \frac{\omega^2}{c^2}$$

- Right triangle relation



- Hence

$$\beta_\rho = \beta \sin\delta \quad \text{and} \quad \beta_z = \beta \cos\delta$$

$$\tan\delta = \frac{\beta_\rho}{\beta_z}$$

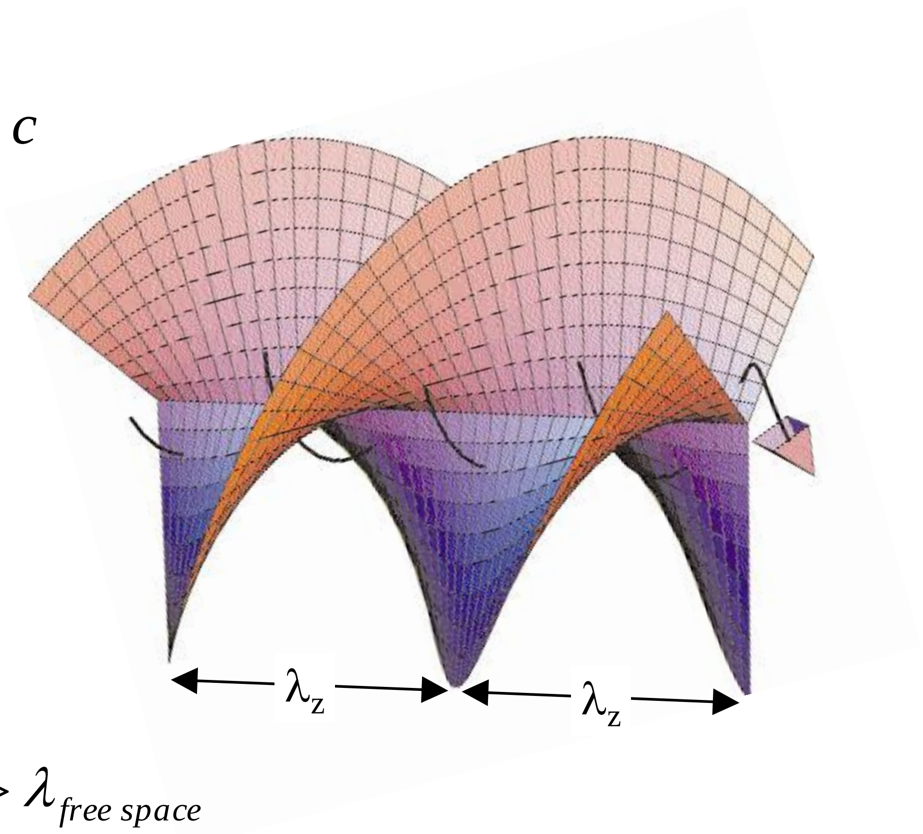
Guided versus Free-Space Wave Solutions

- **Circular waveguide modes**
  - TEM^z mode does not exist
  - Vortex modes do not exist
  - TE^z and TM^z non-vortex modes exist that satisfy Dirichlet or Neumann boundary conditions
  - Mode parameter δ assumes discrete values
  - Modes form a countable set
- **Free space vortex modes**
  - TEM^z mode exists $\Rightarrow m = 0$
  - TE^z and TM^z vortex modes exist $\Rightarrow m \geq 1$
  - Mode cutoff frequency phenomenon is absent
  - Modes parameterized by m and δ are nondenumerable

Axial Phase Velocity and Wavelength

- **Axial phase velocity**

$$v_{phase} = \frac{\omega}{\beta_z} = \frac{\omega}{\beta\cos\delta} = \frac{c}{\cos\delta} > c$$



- **Axial wavelength**

$$\lambda_z = \frac{2\pi}{\beta_z} = \frac{2\pi}{\beta\cos\delta} = \frac{\lambda_{free\ space}}{\cos\delta} > \lambda_{free\ space}$$

S.D. Stearns, "More Unusual Features of the Microwave Vortex," *IEEE APS-URSI*, July 2012

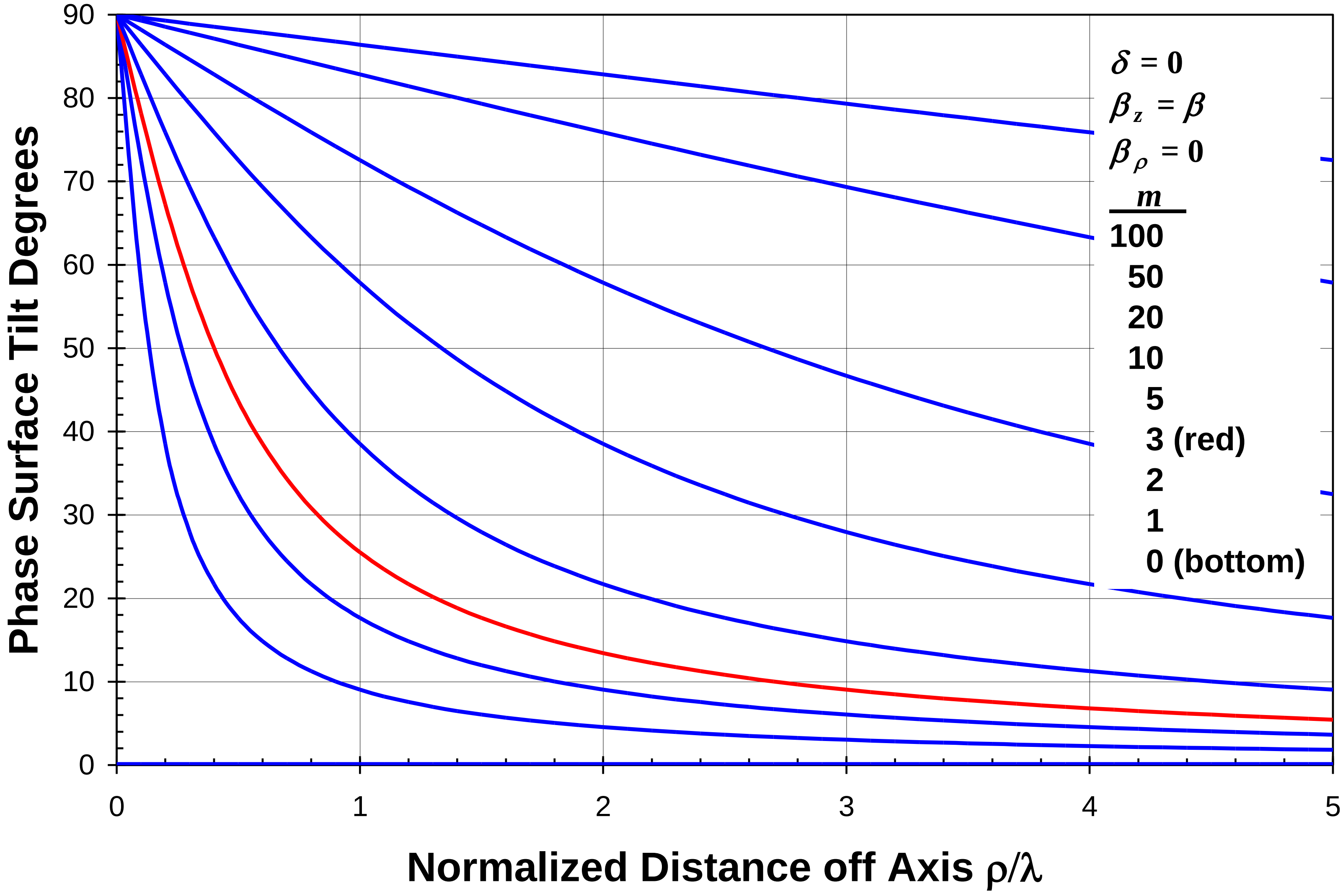
Phase Surface Tilt

- **Phase surface tilt varies as a function of position according to**

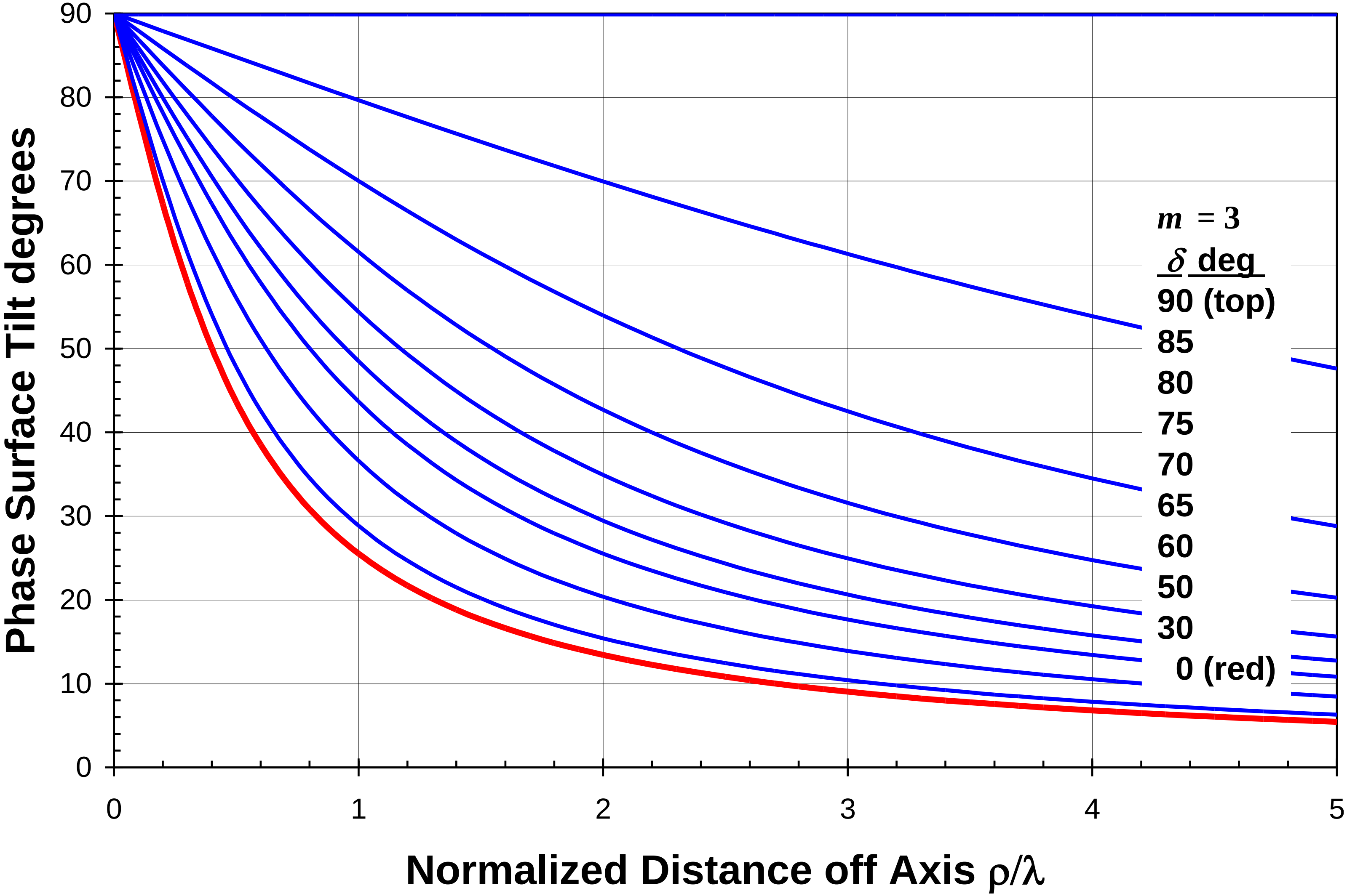
$$\text{Tilt Angle} = \cos^{-1}\gamma_z = \cos^{-1}\frac{\frac{\rho}{\lambda}}{\sqrt{\left(\frac{\rho}{\lambda}\right)^2 + \left(\frac{m}{2\pi\cos\delta}\right)^2}}$$

- **Phase tilt depends on three variables**
  - Distance off axis ρ
  - Mode number (topological charge) m
  - Phase constant ratio β_ρ/β_z or angle δ

Phase Surface Tilt Angle from z Axis



Phase Surface Tilt Angle, *m* = 3



Polarization Varies Across the Wavefront

- **The polarization of a Bessel mode varies from point to point in the transverse plane**
- **Polarization depends on distance off axis**
- **TE^z modes**
  - Linearly polarized in the ϕ direction at radial distances corresponding to the zeros of $J_m(\beta_\rho \rho)$
  - Linearly polarized in the ρ direction at radial distances corresponding to the zeros of $J'_m(\beta_\rho \rho)$
- **TM^z modes**
  - Linearly polarized in the ρ direction at radial distances corresponding to the zeros of $J_m(\beta_\rho \rho)$
  - Linearly polarized in the ϕ direction at radial distances corresponding to the zeros of $J'_m(\beta_\rho \rho)$
- **Between rings, the polarization transitions through elliptical polarizations with varying axial ratio**

Poynting Vector, Power Flow, and Momentum

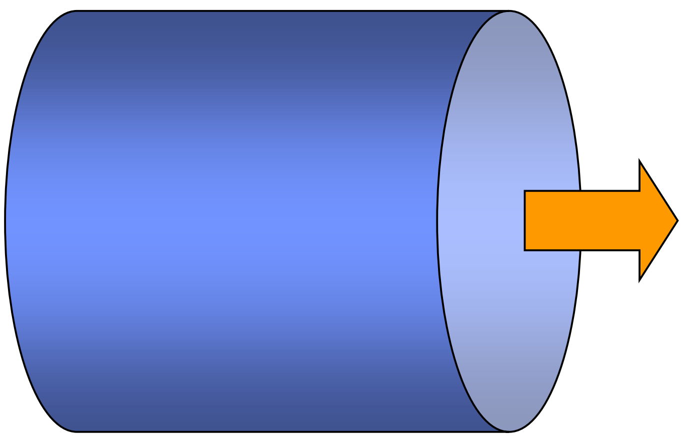
$$\mathbf{S}_{TE^z} = \mathbf{a}_\rho \, j\left(\frac{C^2\eta\omega\beta_\rho^3}{2\beta^3}\right) J_m'(\beta_\rho\rho)\, J_m(\beta_\rho\rho) \; + \; \mathbf{a}_\phi\left(\frac{C^2\eta\omega\beta_\rho^2\, m}{2\beta^3\rho}\right) J_m^2(\beta_\rho\rho) \; + \; \mathbf{a}_z \frac{C^2\eta\omega^2\beta_z}{2\beta}\left\{\left(\frac{\beta_\rho}{\beta}\right)^2 J_m'^2(\beta_\rho\rho) + \left(\frac{m}{\beta\rho}\right)^2 J_m^2(\beta_\rho\rho)\right\}$$

$$\mathbf{S}_{TM^z} = \mathbf{a}_\rho \, j\left(\frac{-A^2\omega\beta_\rho^3}{2\eta\beta^3}\right) J_m'(\beta_\rho\rho)\, J_m(\beta_\rho\rho) \; + \; \mathbf{a}_\phi\left(\frac{A^2\omega\beta_\rho^2 m}{2\eta\beta^3\rho}\right) J_m^2(\beta_\rho\rho) \; + \; \mathbf{a}_z \frac{A^2\omega^2\beta_z}{2\eta\beta}\left\{\left(\frac{\beta_\rho}{\beta}\right)^2 J_m'^2(\beta_\rho\rho) + \left(\frac{m}{\beta\rho}\right)^2 J_m^2(\beta_\rho\rho)\right\}$$

- **Real power and linear momentum have axial and azimuthal components**
- **Radial power is reactive and represents stored energy with direction along any radial, alternately in and out as the sign of $J'_m\, J_m$**
- **For given m, a combination of TE^z plus TM^z exists, viz. $A/C = \eta$, that has no radial power nor stored field energy**
- **Real power spirals through space around the vortex axis, CW or CCW according to the sign of m**

Wavefront Energy Velocity

- **Determined by the rate of total energy crossing a transverse plane or "port"**
- **The energy or momentum velocity is given by the ratio of real Poynting flux to real energy density per unit length, both defined over a transverse plane**



$$v_{energy} = \frac{P_z}{U_E + U_H}$$

Energy Velocity is Subluminal

$$v_{energy}^{TE^z} = \frac{\int_0^\infty \int_0^{2\pi} S_z(\rho,\phi,z_0)\,\rho\,d\phi\,d\rho}{\int_0^\infty \int_0^{2\pi} \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\phi\,d\rho}$$

$$= \frac{\frac{1}{2}\int_0^\infty \left(E_\rho H_\phi^* - E_\phi H_\rho^*\right)\rho\,d\rho}{\int_0^\infty \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\rho}$$

$$= \frac{\frac{\cos\delta}{\varepsilon_0\eta}\int_0^\infty \frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)\rho\,d\rho}{\int_0^\infty \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\rho}$$

$$= c\cos\delta\left(\frac{U_E}{U_E+U_H}\right)$$

$$< c$$

$$v_{energy}^{TM^z} = \frac{\int_0^\infty \int_0^{2\pi} S_z(\rho,\phi,z_0)\,\rho\,d\phi\,d\rho}{\int_0^\infty \int_0^{2\pi} \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\phi\,d\rho}$$

$$= \frac{\frac{1}{2}\int_0^\infty \left(E_\rho H_\phi^* - E_\phi H_\rho^*\right)\rho\,d\rho}{\int_0^\infty \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\rho}$$

$$= \frac{\frac{\eta\cos\delta}{\mu_0}\int_0^\infty \frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\rho\,d\rho}{\int_0^\infty \left[\frac{\varepsilon_0}{2}\left(\mathbf{E}\bullet\mathbf{E}^*\right)+\frac{\mu_0}{2}\left(\mathbf{H}\bullet\mathbf{H}^*\right)\right]\rho\,d\rho}$$

$$= c\cos\delta\left(\frac{U_H}{U_E+U_H}\right)$$

$$< c$$

OAM-Multiplexed Communication Demo

- Public demonstration in Venice, Italy, July 24, 2011
- Organized by Prof. F. Tamburini, University of Bologna, and Prof. Bo Thidé, Uppsala University
- Attended by Princess Elettra Marconi, relative of Gugielmo Marconi, and more than 2,000 spectators
- Simultaneous transmission of two signals on 2.414 GHz
- Colocated transmitters and receivers
- Signal 1 used Yagi antennas for Tx and Rx
- Signal 2 used spiral-ramp reflector antennas for OAM Tx and Rx
- Both signals were received without interference
- Debate between communication theorists and physicists as to the proper explanation of the result; both sides missed important points

OAM reflector antenna

Ionospheric Heater Experiments

- **HAARP, Gakona, Alaska**
  - U.S. Navy
  - HF: 2.8 to 10 MHz, often 3.39 and 6.99 MHz
- **HIPAS, Fairbanks, Alaska**
  - UCLA
  - HF: 2.85 and 4.53 MHz
- **EISCAT, Ramfjordmoen near Tromsø, Norway**
  - Norway, Sweden, Finland, Japan, China, UK, and Germany
  - VHF/UHF: 224, 500 and 931 MHz
- **Sura, Vasilsursk, Russia**
  - HF: 4.5 to 9.3 MHz
- **Arecibo, Puerto Rico**
  - NSF and Cornell Univ.
  - HF: 5.1 and 8.175 MHz

The Norway spiral, Dec. 9, 2009

Summary of Vortex Bessel Modes

- **Vortex Bessel modes satisfy Maxwell's equations and the wave equation exactly – no paraxial approximation**
- **Differences from circular waveguide modes**
  - No metal wall boundary condition $\Rightarrow$ no cutoff phenomenon
  - Modes are indexed by two parameters, one integer and one real number
- **Constant-phase surfaces are multi-sheet helicoids**
- **Phase surface tilt depends on mode parameters and distance off axis**
- **Polarization varies across the wavefront**
- **Wavelength is dilated**
- **Phase velocity of the wave is superluminal (greater than *c*)**
- **Energy velocity varies across the wavefront but is everywhere subluminal (less than *c*)**
- **All energy velocities between 0 and *c* are achievable by choosing mode parameter δ**
- ***c* becomes an *upper bound* on the speed of electromagnetic waves in free space**

How can light travel slower than the speed of light?

Final Comments

- Electromagnetic radiation can have either spin or orbital angular momentum (SAM or OAM)
- Photons have SAM or OAM
- Wave fields have SAM or OAM density

Wave behavior ≠ Photon behavior

- According to Einstein, photons travel at speed *c*, period!
- Waves can travel at speeds less than *c* if photons travel on curved paths
- Photons having OAM apparently travel on curved paths
- Localized waves, such as knotted, linked, and vortex waves, depend on OAM for their weird properties

Understanding photon entanglement may explain the mystery.

So What?

- **What are localized waves good for, and how do you make them?**
- **Research world wide is focused on how to make localized waves**
  - Spiral ramp reflector antenna (Italian communications demo)
  - Phased arrays (HAARP)
  - Metasurface reflector antennas
  - Optical gratings and lenses
- **Applications**
  - Manufacturing – Optical tweezers for moving, twisting atoms, molecules and nano objects
  - Communications – Increased communication capacity of free space for point-to-point communications (×*m* instead of ×2 for polarization multiplexing)
  - Military – Communication and radar ECM countermeasures
  - Energy storage – Electromagnetic flywheel using knotted waves
  - Astrophysics – If knotted or linked localized waves exist in space as local circulating energy, they would be invisible (dark) and have mass – dark matter?
  - Amateur Radio – Curved propagation path communication for DX and NVIS without ionosphere or tropo ducts
- **Caveat**
  - OAM is a nuisance to laser designers, who design filters to remove it
  - Otherwise, a laser ruler might measure distance wrong!

Further Reading

- **S.D. Stearns, "Transverse and Longitudinal Structure of Bessel Vortex Beam Solutions to Maxwell's Equations,"** ***IEEE International Symposium on Antennas and Propagation*****, July 2014**
- **S.D. Stearns, "More Unusual Features of the Microwave Vortex,"** ***IEEE International Symposium on Antennas and Propagation*****, July 2012**
- **H. Kedia, et al, "Tying Knots in Light Fields,"** ***arXiv*** **1302.0342v1, Feb. 2013**
- **W.T.M. Irvine, "Linked and Knotted Beams of Light, Conservation of Helicity and the Flow of Null Electromagnetic Fields,"** ***arXiv*** **1110.5408v1, Oct. 2011**
- **W.T.M. Irvine and D. Bouwmeester, "Linked and Knotted Beams of Light,"** ***Nature Physics*****, Aug. 2008**

The End

This presentation and revisions will be archived at

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